

Stringy Instantons and Fermion Masses
in Intersecting D-brane models

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On The Fermion Mass Problem

★The vast mass hierarchy among the three fermion generations:

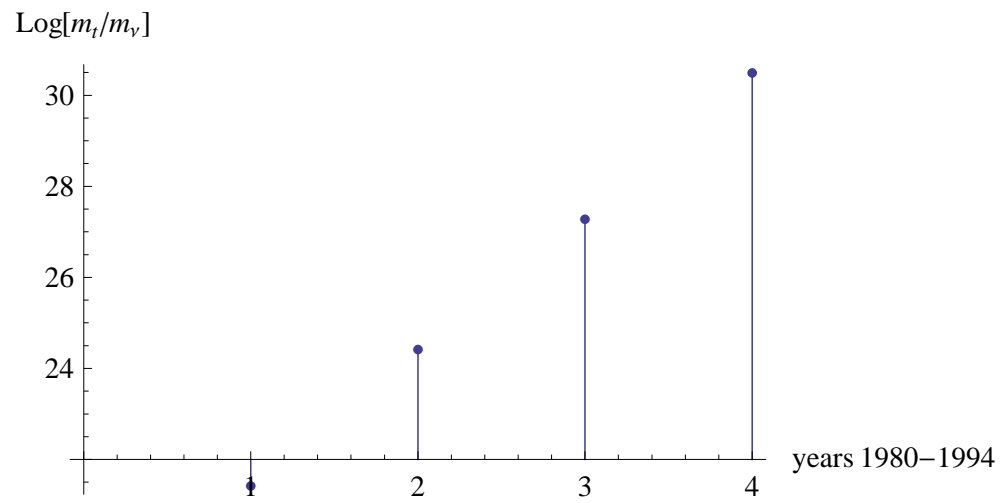
$$\begin{aligned}\frac{m_t}{m_u} &\sim 10^5 \\ \frac{m_t}{m_{\nu_e}} &\sim \frac{M_{GUT}}{M_W}\end{aligned}\tag{1}$$

has no satisfactory explanation in SM and SUSY-GUTs

*The second ratio in (1) is the well known **hierarchy problem***

Early attempts: Additional ‘family’ symmetry. (Froggatt & Nielsen, NPB147(1979)277 etc...)

Strings: Predict additional $U(1)$ factors “incarnating” naturally the family symmetries.



★ String Approach: The Main Features

▲ Restricted Superpotential couplings, due to selection rules and $U(1)$'s (*Normally only 3^d generation present at tree-level*)

$$\mathcal{W}_{tree} \supset \lambda_t Q_t t^c h_u + \lambda_b Q_t b^c h_d + \lambda_l L_\tau \tau^c h_d$$

▲ Gauge-Yukawa coupling relations, ($\lambda_t \sim g_{unif}$) fix $m_t \sim 180$ GeV.

▲ Anomalous $U(1)_A$ & D/F - flatness fix various singlet vevs $\langle \Phi_i \rangle$

$$\frac{\partial \mathcal{W}}{\partial \Phi_j} = 0; \quad \sum_j Q_j^k |\langle \Phi_j \rangle|^2 - \xi_{FI}^2 \delta_{A k} = 0, \quad (A = \text{Anomalous } U(1))$$

▲ Calculable NR-terms to all orders!

▲ Fermion mass entries expressed in terms of

$$\epsilon_i = \frac{\langle \Phi_i \rangle}{\mathcal{M}} < 1$$

Stringy Inspired Classification: **Texture zeros:** (see e.g. *Ramond-Roberts-Ross, Nucl.Phys.B406:19-42,1993 ... etc*)

General Idea: Minimal structure, maximum information...

$$m_U = \begin{pmatrix} 0 & \epsilon^6 & 0 \\ \epsilon^6 & 0 & \epsilon^2 \\ 0 & \epsilon^2 & 1 \end{pmatrix} \quad m_D = \begin{pmatrix} 0 & 2\epsilon^4 & 0 \\ 2\epsilon^4 & 2\epsilon^3 & 2\epsilon^3 \\ 0 & 2\epsilon^3 & 1 \end{pmatrix}$$

$$m_U = \begin{pmatrix} 0 & 0 & \sqrt{2}\epsilon^4 \\ 0 & \epsilon^4 & 0 \\ \sqrt{2}\epsilon^4 & 0 & 1 \end{pmatrix} \quad m_D = \begin{pmatrix} 0 & 2\epsilon^4 & 0 \\ 2\epsilon^4 & 2\epsilon^3 & 4\epsilon^3 \\ 0 & 4\epsilon^3 & 1 \end{pmatrix}$$

$$m_U = \begin{pmatrix} 0 & \sqrt{2}\epsilon^6 & 0 \\ \sqrt{2}\epsilon^6 & \sqrt{3}\epsilon^4 & \epsilon^2 \\ 0 & \epsilon^2 & 1 \end{pmatrix} \quad m_D = \begin{pmatrix} 0 & 2\epsilon^4 & 0 \\ 2\epsilon^4 & 2\epsilon^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$m_U = \begin{pmatrix} 0 & \sqrt{2}\epsilon^6 & 0 \\ \sqrt{2}\epsilon^6 & \epsilon^4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad m_D = \begin{pmatrix} 0 & 2\epsilon^4 & 0 \\ 2\epsilon^4 & 2\epsilon^3 & 4\epsilon^3 \\ 0 & 4\epsilon^3 & 1 \end{pmatrix}$$

$$m_U = \begin{pmatrix} 0 & 0 & \epsilon^4 \\ 0 & \sqrt{2}\epsilon^4 & \frac{\epsilon^2}{\sqrt{2}} \\ \epsilon^4 & \frac{\epsilon^2}{\sqrt{2}} & 1 \end{pmatrix} \quad m_D = \begin{pmatrix} 0 & 2\epsilon^4 & 0 \\ 2\epsilon^4 & 2\epsilon^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

However....

Subtle is the real World!

▼ No zeros in realistic string constructions...

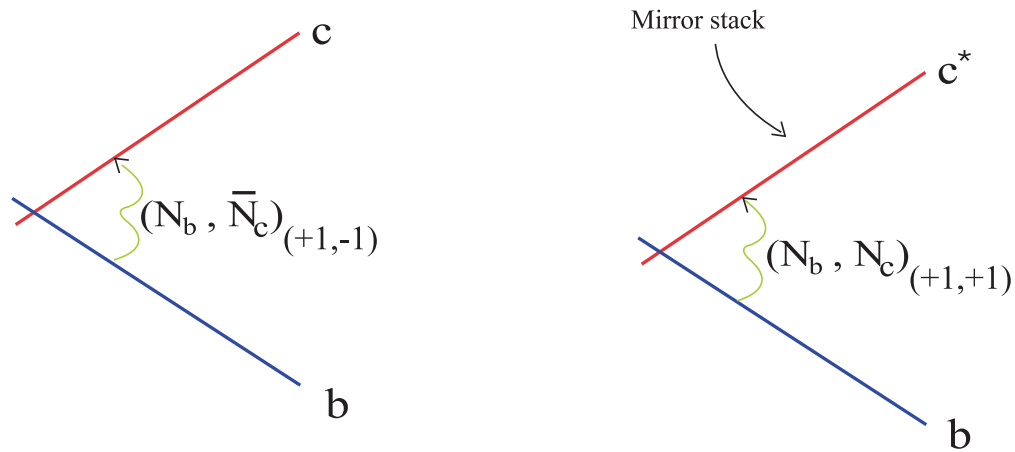
▼ Non-symmetric textures is usually the case!!!

♠ Need of unified treatment of symmetric & non-symmetric mass matrices

Mass textures in a class of D-brane Standard-like Models

Preliminaries

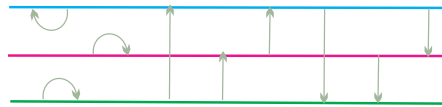
Matter multiplets



A string stretched between:

- i)* D_b, D_c intersecting brane stacks giving rise to bifundamental $(N_b, \bar{N}_c)_{(+1,-1)}$, and
- ii)* D_b, D_{c^*} (mirror), incarnating the $(N_b, N_c)_{(+1,+1)}$

U(3) gauge bosons



Quark doublets

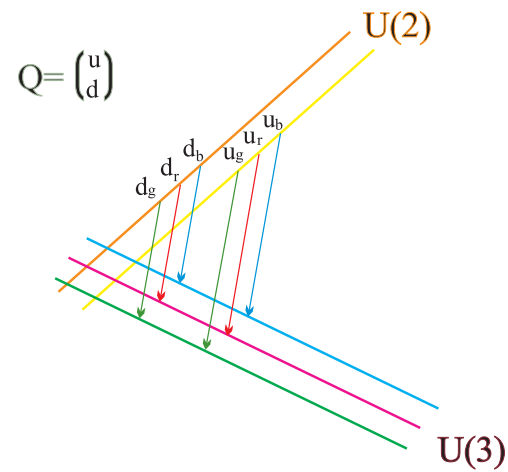


Figure 1: The 9 gauge bosons of $U(3)$ and the quark-doublets.

D-brane **SM** analogue with P extra $U(1)$ branes

$$G = U(3)_C \times U(2)_L \times U(1)^P \quad (2)$$

Antoniadis et al, Nucl. Phys. B 660 (2003) 81; Ibanez et al, JHEP 0111 (2001) 002; R. Blumenhagen et al, Nucl. Phys. B 616 (2001) 3...

For $P = 0$, notice that

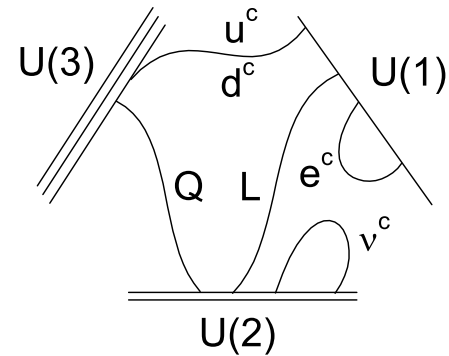
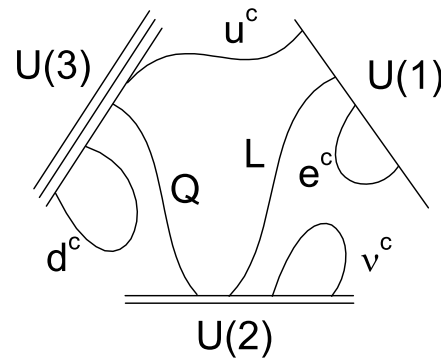
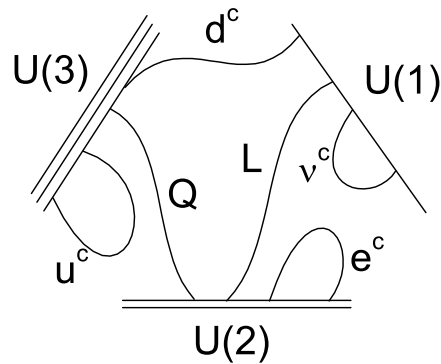
$$U(3)_C \times U(2)_L \rightarrow SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L$$

Hypercharge $U(1)_Y \not\subseteq U(1)_C \times U(1)_L$,

$$\Rightarrow P \geq 1$$

Three **SM** variants for the simplest configuration $P = 1$:

(*Antoniadis and Dimopoulos, Nucl. Phys. B* **715** (2005) 120)



\implies **SM** embedding is not unique!

Most general hypercharge

$$Y = k_3 Q_3 + k_2 Q_2 + \sum_{i=1}^P k'_i Q'_i \quad (3)$$

and gauge coupling condition

$$\frac{1}{g_Y^2} = \frac{6k_3^2}{g_3^2} + \frac{4k_2^2}{g_2^2} + 2 \sum_{i=1}^P \frac{k'_i{}^2}{g_i^2} \quad (4)$$

The requirement to obtain the correct $U(1)_Y$ charges for the SM spectrum determines the coefficients $k_{2,3}, k'_i$.

- $k_{2,3}$ are due to the contributions of the abelian factors of $U(3)_C \rightarrow SU(3)_C \times U(1)_C$ and $U(2)_L \rightarrow SU(2)_L \times U(1)_L$
- k'_i are due to the abelian factors (the $U(1)$ branes).
- For $P = 1, 2, 3$ the results for $k_{2,3}, k'_i$ shown in table below:
(D.V. Gioutsos, **GKL** and A. Psallidas, **Phys Rev D74:075007,2006**)

GKL, N. Vlachos and N. Tracas [Phys Rev D76:115009,2007](#))

P		$ k_3 $	$ k_2 $	$ k'_1 $	$ k'_2 $	$ k'_3 $
1	a_1	$\frac{1}{3}$	$\frac{1}{2}$	0	—	—
	b_1	$\frac{1}{6}$	0	$\frac{1}{2}$	—	—
2	a_2	$\frac{1}{6}$	0	$\frac{1}{2}$	$\frac{1}{2}$	—
	b_2	$\frac{2}{3}$	$\frac{1}{2}$	1	0	—
	c_2	$\frac{1}{3}$	$\frac{1}{2}$	0	1	—
3	a_3	$\frac{1}{6}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	b_3	$\frac{1}{3}$	$\frac{1}{2}$	0	1	1
	c_3	$\frac{2}{3}$	$\frac{1}{2}$	1	0	0

Table 1: Simplest $U(3) \times U(2) \times U(1)^P$ hypercharge embeddings for $P = 1, 2, 3$

Analysis of various phenomenological issues of the above models shows that for models where the $U(1)$ branes are aligned to $SU(3)$ or $SU(2)$ branes:

(*Gioutsos, GKL, Rizos, EPJC 45, 241 (2006), hep-ph/0508120*)

• With respect to the predictions of the string scale, there are three classes of viable models: (P is the number of abelian branes)

1: There are $P = 1, 3$ models with string scale $M_S \sim 10^{16}$ GeV.

(see also *GKL arXiv:0903.3691* for intersecting branes)

2: There are $P = 2$ models with $M_S \sim 10^{7-8}$ GeV;

In addition, the condition $m_b^0 = m_\tau^0$ at M_S is fulfilled.

3: There are $P = 3$ models with M_S as low as a few TeV.

Uniqueness could arise from the embedding in a higher gauge group

A closer look to a viable $P = 1$ case

GKL, arXiv:0903.3691

Conditions on wrappings:

$$(N_a, \bar{N}_b) \rightarrow I_{ab} = \prod_{i=1}^3 (m_{ai}n_{bi} - m_{bi}n_{ai})$$

$$(N_a, N_b) \rightarrow I_{ab^*} = - \prod_{i=1}^3 (m_{ai}n_{bi} + m_{bi}n_{ai})$$

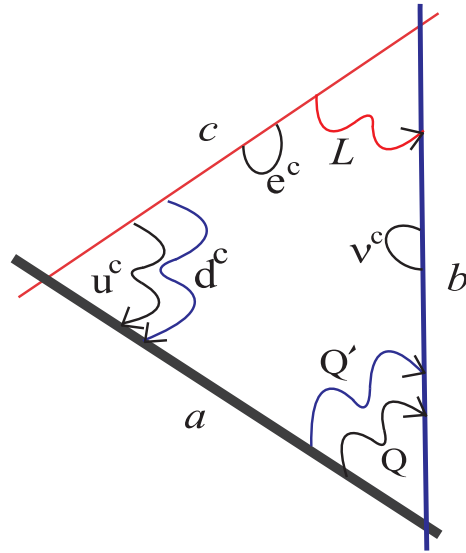
$$A \rightarrow I_{aa^*}^A = 8m_{a1}m_{a2}m_{a3}$$

$$A\&S \rightarrow I_{aa^*}' = 4m_{a1}m_{a2}m_{a3} (n_{a1}n_{a2}n_{a3} - 1)$$

Anomaly cancelation conditions imply $\#N_a = \#\bar{N}_a, \forall U(N_a)$

SPECTRUM

Inters.	$SU(3) \times SU(2)$	Q_a	Q_b	Q_c	Y
ab	$1 \times Q(3, \bar{2})$	1	-1	0	$\frac{1}{6}$
ab^*	$2 \times Q'(3, 2)$	1	1	0	$\frac{1}{6}$
ac	$3 \times u^c(\bar{3}, 1)$	-1	0	1	$-\frac{2}{3}$
ac^*	$3 \times d^c(\bar{3}, 1)$	-1	0	-1	$\frac{1}{3}$
bc	$3 \times L(1, \bar{2})$	0	-1	1	$-\frac{1}{2}$
cc^*	$3 \times e^c(1, 1)$	0	0	-2	1
bb^*	$3 \times \nu^c(1, 1)$	0	-2	0	0
bc^*	$1 \times H_d(1, 2)$	0	1	1	$-\frac{1}{2}$
	$1 \times H_u(1, \bar{2})$	0	-1	-1	$\frac{1}{2}$



Depiction of the $U(3) \times U(2) \times U(1)$ intersecting D-brane configuration. ($a \equiv U(3)$, $b \equiv U(2)$, $c \equiv U(1)$)

Blue string representing the quark doublet Q' is stretched between the $D6_a$ and $D6_{b^*}$.

One endpoint of the “ d^c -string” is attached on the mirror $D6_{c^*}$.

Effective field theory model

String scale:

$$M_S = e^{\frac{\pi}{2} \left(\frac{\sin^2 \theta_W}{\alpha} - \frac{1}{\alpha_3} \right)} \left(\frac{m_Z}{m_s} \right)^{\frac{13}{12}} m_Z \sim \mathcal{O}(10^{16}) \text{ GeV} \quad (5)$$

Yukawa couplings:

$$\begin{aligned} \mathcal{W} \supset & \lambda_{1j}^u Q'_1 u_j^c H_u + \lambda_{2j}^u Q'_2 u_j^c H_u \rightarrow \text{up - quarks} \\ & + \lambda_j^d Q_3 d_j^c \rightarrow \text{down - quarks} \\ & + \lambda_{ij}^l L_i e_j^c H_d \rightarrow \text{charged leptons} \end{aligned}$$

Resulting up and down quark Yukawa textures.

We have the following distinct possibilities

$$m_Q = \begin{pmatrix} 0 & 0 & 0 \\ \lambda_{21}^u & \lambda_{22}^u & \lambda_{23}^u \\ \lambda_{31}^u & \lambda_{32}^u & \lambda_{33}^u \end{pmatrix} \langle H_u \rangle, \quad m_d = \begin{pmatrix} \lambda_{11}^d & \lambda_{12}^d & \lambda_{13}^d \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \langle H_d \rangle$$

or

$$m_Q = \begin{pmatrix} \lambda_{11}^u & \lambda_{12}^u & \lambda_{13}^u \\ 0 & 0 & 0 \\ \lambda_{31}^u & \lambda_{32}^u & \lambda_{33}^u \end{pmatrix} \langle H_u \rangle, \quad m_d = \begin{pmatrix} 0 & 0 & 0 \\ \lambda_{21}^d & \lambda_{22}^d & \lambda_{23}^d \\ 0 & 0 & 0 \end{pmatrix} \langle H_d \rangle$$

or

$$m_Q = \begin{pmatrix} \lambda_{11}^u & \lambda_{12}^u & \lambda_{13}^u \\ \lambda_{21}^u & \lambda_{22}^u & \lambda_{23}^u \\ 0 & 0 & 0 \end{pmatrix} \langle H_u \rangle \quad (6)$$

$$m_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \lambda_{31}^d & \lambda_{32}^d & \lambda_{33}^d \end{pmatrix} \langle H_d \rangle \quad (7)$$

⇒ at tree-level ... mass textures...not viable.

Possible solutions:

- * NR-terms (extending the spectrum)
- * Additional Higgs pair
- * Stringy Instanton Effects

* NR-terms and/or Additional Higgs pair

Inters.	$SU(3) \times SU(2)$	\mathcal{Q}_a	\mathcal{Q}_b	\mathcal{Q}_c	Y
bb^*	$\nu'^c (1, 1)$	0	+2	0	0
bc	$H'_d(1, 2)$	0	-1	1	$-\frac{1}{2}$
	$H'_u(1, \bar{2})$	0	1	-1	$\frac{1}{2}$

ν^c and H'_u, H'_d superpotential contributions

$$\mathcal{W}' = \lambda'^u_j Q u_j^c H'_u + \lambda'^d_j Q'_p d_j^c H'_d + \lambda'^\nu_{ij} \mathcal{L}_i \nu'^c_j H_u \quad (8)$$

Instanton Effects

▷ **YM Instantons:** Classical Euclidean Solutions of F.T. Equations
($F_{\mu\nu} = \tilde{F}_{\mu\nu}$)

Physical Meaning: Tunneling event between degenerate classical vacua in the presence of fermion zero-modes.

▷ **N=1 SUSY:** Non-perturbative Superpotential couplings.
(a surrogate for the non-renormalizable terms.)

★ **Intersecting D-brane models:**

Non-perturbative string effects generate missing **Yukawa Couplings**

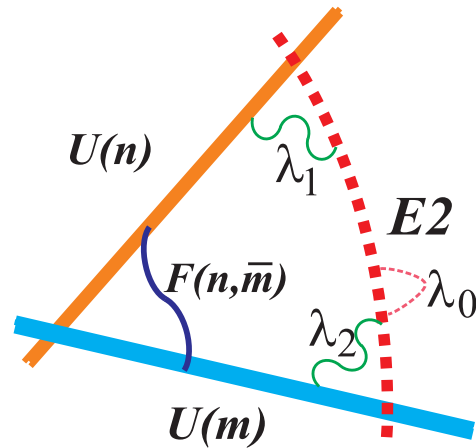
Blumenhagen et al, Nucl. Phys. B 771 (2007) 113,

Ibanez et al, JHEP 0703 (2007) 052,

M. Bianchi et al, JHEP 0707 (2007) 038

Depiction

of a Euclidean Brane intersecting $U(m), U(n)$ D-branes



- a) $E2 - E2$ (uncharged) instantons (broken SUSY).
- b) $D6 - E2$ (charged) instantons in the intersections.

The Stringy Mechanism:

Certain desired Yukawa couplings $\prod_j \Phi_j$ violate some $U(1)_a$
 ($U(n)_a \rightarrow SU(n)_a \times U(1)_a$)

Under $U(1)_a$, the transformation property of the exponential
 instanton action is ($U(1)_a$ is broken by the instanton!)

$$e^{-S_{\mathcal{E}}} \rightarrow e^{-S_{\mathcal{E}}} e^{i Q_a(\mathcal{E}2) \Lambda_a} \quad (9)$$

$Q_a(\mathcal{E}2)$ depends on the $\mathcal{E}2$ intersections

$$Q_a(\mathcal{E}2) = -\mathcal{N}_a \pi_{\mathcal{E}} \circ \pi_a \equiv -\mathcal{N}_a I_{\mathcal{E}a} \quad (10)$$

$U(1)_a$ charge violation of $\prod_j \Phi_j$ compensated by a S_{inst} -shift:

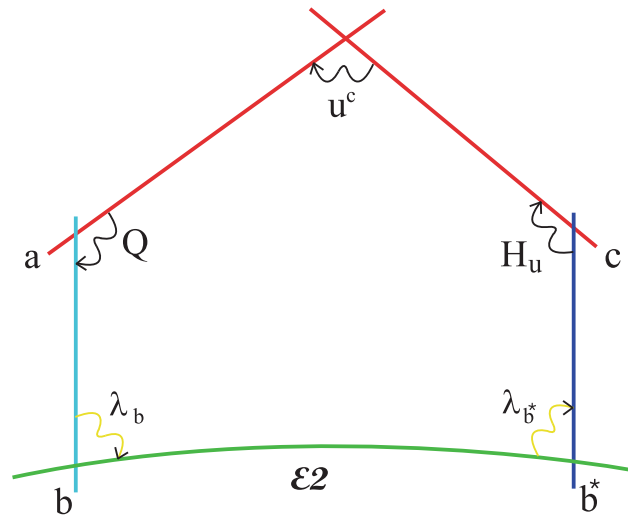
$$\mathcal{W}_{n.p.} \sim \prod_j \Phi_j e^{-S_{inst}}$$

Up Quark coupling $Qu_j^c H_u$ violates the $U(1)_b$ charge by two units

$$Q_{b_Q} + Q_{b_{u^c}} + Q_{b_{H_u}} = -2$$

Choose # of $\bar{\lambda}_b^i, \lambda_b^i, i = 1, 2, 3$ so that $I_{\mathcal{E}b}$ satisfies

$$Q_b(\mathcal{E}2) = -\mathcal{N}_b I_{\mathcal{E}b} \quad (11)$$



Integration over zero modes

$$\int \{d^4x d^2\theta d^2\lambda_b\} e^{-S_\varepsilon} y_j^u \epsilon_{mn} \epsilon_{rl} \langle \lambda_b^m Q^n u_j^c H_u^r \lambda_{b^*}^s \rangle e^{Z'}$$

(and similarly for down quarks) leads to the tree level superpotential couplings

$$\mathcal{W}_{n.p.} = \lambda_j^u Q u_j^c H_u + \lambda_{pj}^d Q'_p d_j^c H_d$$

Final forms of quark matrices

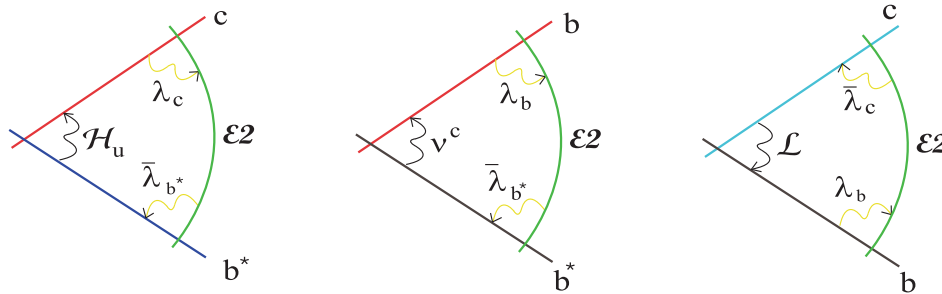
$$m_Q = \begin{pmatrix} \lambda_{11}^u & \lambda_{12}^u & \lambda_{13}^u \\ \lambda_{21}^u & \lambda_{22}^u & \lambda_{23}^u \\ \lambda_{31}^u & \lambda_{32}^u & \lambda_{33}^u \end{pmatrix}, \quad m_d = \begin{pmatrix} \lambda_{11}^d & \lambda_{12}^d & \lambda_{13}^d \\ \lambda_{21}^d & \lambda_{22}^d & \lambda_{23}^d \\ \lambda_{31}^d & \lambda_{32}^d & \lambda_{33}^d \end{pmatrix}$$

$$\lambda_{ji}^{u,d} \ll \lambda_{ji}^{u,d} \quad (\text{Instanton or NR contributions})$$

$$\lambda_{ji}^{u,d} \sim \lambda_{ji}^{u,d} \quad (\text{Additional Higgs})$$

Neutrinos: Absent at tree level (minimal model)

Instanton induced masses through the following $D6 - E2$ intersections



Remarks:

- 1.) Exponential suppression: $m_\nu \propto e^{-S_\epsilon} \frac{m_D^2}{M_S}$
- 2.) Factorizable couplings: $y_{ij} = e^{i\phi_{ij}} \lambda_i^\nu \lambda_j^\nu$

Results: Instantons induce a new hierarchical structure of mass matrices

Convenient parametrization

$$m_D = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ \zeta_{21} & \zeta_{22} & \zeta_{23} \\ \kappa_{31} & \kappa_{32} & \kappa_{33} \end{pmatrix}, |\vec{x}| \gg, |\vec{\kappa}|, |\vec{\zeta}|$$

Since m_D is non-symmetric, we form

$$m_D m_D^T = \begin{pmatrix} \vec{x} \cdot \vec{x} & \vec{x} \cdot \vec{\zeta} & \vec{x} \cdot \vec{\kappa} \\ \vec{x} \cdot \vec{\zeta} & \vec{\zeta} \cdot \vec{\zeta} & \vec{\zeta} \cdot \vec{\kappa} \\ \vec{x} \cdot \vec{\kappa} & \vec{\zeta} \cdot \vec{\kappa} & \vec{\kappa} \cdot \vec{\kappa} \end{pmatrix}$$

- A simple parametrization of m_D : $\vec{x} \cdot \vec{\zeta} = 0 \rightarrow$

$$m_D m_D^T = \begin{pmatrix} 1 & -r \cos(\theta) & r \sin(\theta) \\ -r \cos(\theta) & s^2 & 0 \\ r \sin(\theta) & 0 & s^2 \end{pmatrix} m_0^2 \quad (12)$$

with $r = \frac{\sqrt{(m_b^2 - m_s^2)(m_s^2 - m_d^2)}}{m_b^2 + m_d^2 - m_s^2}$, $s^2 = \frac{m_s^2}{m_b^2 + m_d^2 - m_s^2}$.

- **Question:** Is there a compatible up-quark texture?

We can check it using the

Cabbibo-Kobayashi-Maskawa (CKM) Matrix.

If $V_u^{L,R}$ and $V_d^{L,R}$ the diagonalizing matrices:

$$m_U^{diag.} = V_u^L m_U V_u^{R\dagger} \quad (13)$$

$$m_D^{diag.} = V_d^L m_D V_d^{R\dagger} \quad (14)$$

and V_d is known, we can use now the CKM matrix and the relation

$$V_u = V_{CKM} V_d \quad (15)$$

to construct the diagonalizing matrix of the up quarks. Then,

$$\mathcal{M}_U^2 \equiv m_U m_U^\dagger = V_U^\dagger (m_U^2)_{diag.} V_U \quad (16)$$

Wolfenstein parametrization of CKM:

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \quad (17)$$

with $\lambda \sim 0.2257$ and $A, \rho, \eta = \mathcal{O}(1)$, while putting $(\xi \sim s \sim r)$

Resulting up-quark mass texture:

see *G.K.L., Stringy Instantons and Fermion Masses: (in Annual Meeting of HEP Society, Demokritos, Athens, 23/05/2009, Greece*
<http://indico.cern.ch/conferenceDisplay.py?confId=55216>

$$\epsilon = A \sin(\theta) \lambda^2 + \xi \cos(\theta)$$

$$\epsilon' = \xi \sin(\theta) - A \lambda^2 \cos(\theta)$$

$$m_U m_U^T \sim \begin{pmatrix} 1 & -\epsilon & \epsilon' \\ -\epsilon & \epsilon^2 & \epsilon\epsilon' \\ \epsilon' & \epsilon\epsilon' & \epsilon'^2 \end{pmatrix} m_t^2$$

$\Rightarrow m_U, m_D$ are **Aligned**

Neutrino sector

Since the couplings are factorizable,

$$y_{ij} = e^{i\phi_{ij}} \lambda_i^\nu \lambda_j^\nu \quad (18)$$

the Dirac neutrino mass matrix can be written

$$m_D^\nu = \begin{pmatrix} 1 & r \cos(\theta) & r \sin(\theta) \\ r \cos(\theta) & e^{i\phi_1} r^2 \cos^2(\theta) & e^{i\phi_2} \frac{r^2}{2} \sin(2\theta) \\ r \sin(\theta) & e^{i\phi_3} \frac{r^2}{2} \sin(2\theta) & e^{i\phi_4} r^2 \sin^2(\theta) \end{pmatrix} \quad (19)$$

Simple choice: $\chi_1 = 0$ and $\phi_i = \chi_{2,3} = \pi$, $r = 1 + \epsilon + O(\epsilon^2)$,

$$m_{\nu_1} \sim (-1 - \epsilon)m_0, \quad m_{\nu_2} \sim (1 - \epsilon)m_0, \quad m_{\nu_3} = 0$$

Inverse mass hierarchy!

A FEW COMMENTS

Simple D -brane configurations imply a variety of interesting SM-successors with desirable features. Among them:

- i) Extra $U(1)$'s \Rightarrow **hierarchy**, and **proton stability**
- ii) $6d$ internal space \Rightarrow Calculability
- **Quark mass ratios** determined in terms of the $6d$ compact space structure
- Factorizable neutrino mass texture, easy to handle
- **Instanton corrections** imply new mass textures not previously explored
- **A new parametrization of mass textures and mixing is needed....**
(*with N.D. Vlachos, to appear...*)