

2D Gravity on AdS_2 with Chern-Simons Correction

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based on

M. Alishahiha, R. F and A. E. Mosaffa, JHEP
0901:069,2009(arXiv:0812.0453)

Fifth Crete Regional Meeting on String Theory
Crete 2009

Why 2D Gravity on AdS_2 ?!

- AdS_2 geometry is the factor which appears in the near horizon geometry of the extremal black holes in any dimension.
- Understanding quantum gravity on AdS_2 might ultimately help us understand the origin of the black hole entropy in other dimension.

- Being an AdS background it is natural to define the quantum gravity in terms of the dual CFT via AdS/CFT correspondence.
- Although AdS_{d+1}/CFT_d correspondence has been understood for $d > 1$ mainly due to explicit examples, little has been known for $d = 1$.

- What does this one dimensional CFT mean?
- Using asymptotic symmetries of AdS_2 , it has been proposed that it is a chiral half of the 2D *CFT* [Hartman, Strominger, 08].

2D Gravity with Chern-Simons correction

- We study the theory of gravity with the action

$$S = \frac{1}{8G} \int d^2x \sqrt{-g} e^\phi \left(R + 2\partial_\mu \phi \partial^\mu \phi + \frac{2}{l^2} e^{2\phi} - \frac{l^2}{4} F_{\mu\nu} F^{\mu\nu} \right) + \frac{1}{32G_\mu} \int d^2x \left(l R \epsilon^{\mu\nu} F_{\mu\nu} + l^3 \epsilon^{\mu\nu} F_{\mu\rho} F^{\rho\delta} F_{\delta\nu} \right) \quad (1)$$

- It is obtained by reducing 3D *topologically massive gravity (TMG)* along an S^1 .

- The action of TMG:

$$\begin{aligned}
 S = & \frac{1}{16\pi G} \int d^3x \sqrt{-G} \left(R + \frac{2}{l^2} \right) \\
 & + \frac{1}{32\pi G \mu} \int d^3x \sqrt{-G} \epsilon^{\lambda\mu\nu} \Gamma_{\lambda\sigma}^{\rho} \left(\partial_{\mu} \Gamma_{\rho\nu}^{\sigma} + \frac{2}{3} \Gamma_{\mu\tau}^{\sigma} \Gamma_{\nu\rho}^{\tau} \right)
 \end{aligned}
 \tag{2}$$

- The second term is gravitational Chern-Simons term.

- We are looking for AdS_2 solutions of (1). It is simply done by utilizing the entropy function formalism [Sen, 05].
- A generic solution preserving $SO(1, 2)$ isometry of the AdS_2 is given by

$$ds^2 = v\left(-r^2 dt^2 + \frac{dr^2}{r^2}\right), \quad e^\phi = u, \quad F_{01} = \frac{e}{l^2}. \quad (3)$$

- The parameters e , v and u can be obtained by extremizing the entropy function defined by

$$\mathcal{E} = 2\pi[qe - f(e, v, u)] \quad (4)$$

- q is the charge of the gauge field and $f(e, v, u)$ is the lagrangian density evaluated for the above ansatz.
- The entropy is given by the value of the entropy function evaluated at the extremum.
- For generic μ and l we find three different solutions:

$$1 : v = \frac{1 + 1/\mu l}{-16Gq}, \quad e^{2\phi} = \frac{-4Gql^2}{1 + 1/\mu l}, \quad \frac{e}{l} = -\sqrt{\frac{1 + 1/\mu l}{-16qG}}, \quad q < 0,$$

$$2 : v = \frac{1 - 1/\mu l}{16Gq}, \quad e^{2\phi} = \frac{4Gql^2}{1 - 1/\mu l}, \quad \frac{e}{l} = \sqrt{\frac{1 - 1/\mu l}{16qG}}, \quad q > 0,$$

$$3 : v = \frac{1}{8Gq\mu l}, \quad e^{2\phi} = \frac{72Gq\mu l^3}{\mu^2 l^2 + 27}, \quad \frac{e}{l} = \sqrt{\frac{\mu l}{2Gq(\mu^2 l^2 + 27)}}, \quad q > 0.$$

- The entropy of the corresponding solutions written in a suggestive form is

$$\begin{aligned}
 1 : \quad S &= 2\pi \sqrt{\frac{-ql^2}{6} \frac{3}{2G} \left(1 + \frac{1}{\mu l}\right)}, \\
 2 : \quad S &= 2\pi \sqrt{\frac{ql^2}{6} \frac{3}{2G} \left(1 - \frac{1}{\mu l}\right)}, \\
 3 : \quad S &= 2\pi \sqrt{\frac{ql^2}{6} \frac{12\mu l}{G(\mu^2 l^2 + 27)}},
 \end{aligned} \tag{6}$$

which may be compared with Cardy formula for the entropy $S = 2\pi \sqrt{\frac{L_0}{6} c}$.

- If we identify ql^2 with the eigenvalue of L_0 of the dual CFT then the central charges of the corresponding CFTs read

$$1 : c_R = \frac{3}{2G} \left(1 + \frac{1}{\mu l}\right), \quad 2 : c_L = \frac{3}{2G} \left(1 - \frac{1}{\mu l}\right), \quad 3 : c_L = \frac{12\mu l}{G(\mu^2 l^2 + 27)}. \quad (7)$$

- If correct, this means that the 2D Maxwell-dilaton gravity on AdS_2 background is dual to chiral half of a 2D CFT characterized by the above central charges.
- The index L, R refer to the fact that whether the dual chiral CFT is left or right handed which in turn corresponds to the sign of the q .

Asymptotic symmetry and central charge

- Using Hartman-Strominger method (arXiv:0803.3621), we can do another calculation confirming our previous results.
- To proceed we choose a new coordinate $\sigma = \frac{1}{r}$, $t^\pm = t \pm \sigma$.
- Our AdS_2 solutions in this new coordinate can be recast to the form

$$ds^2 = -4v \frac{dt^+ dt^-}{(t^+ - t^-)^2}, \quad A_\pm = -\frac{e}{2\sigma l^2}, \quad u = \eta = \text{constant} \quad (8)$$

- We want to study the action of the 2D conformal group on this theory.
- To do so, we choose the conformal gauge for the metric and the Lorentz gauge for the gauge field:

$$ds^2 = -e^{2\rho} dt^+ dt^-, \quad \partial_+ A_- + \partial_- A_+ = 0 \quad (9)$$

- This gauge choice fixes the coordinate and $U(1)$ gauge transformation up to residual conformal and gauge transformation generated by

$$t^\pm \rightarrow t^\pm + \zeta^\pm(t^\pm), \quad a \rightarrow a + \theta(t^+) - \tilde{\theta}(t^-) \quad (10)$$

where $A_\pm = \pm \partial_\pm a$.

- On the other hand, in order to define the theory we must impose boundary conditions at $\sigma = 0$.
- Requiring no current flow out of the boundary imposes

$$j_\sigma|_{\sigma=0} = 0 \quad (11)$$

where $j_\sigma = j_+ - j_-$ and

$$j_\pm = \mp \frac{l}{16G\mu} \partial_\pm (8e^{-2\rho} \partial_+ \partial_- \rho + 3l^2 F^2) \quad (12)$$

- As a result, the boundary terms in the variation of the action will vanish if

$$A_\sigma|_{\sigma=0} = 0 \quad (13)$$

- In general the last condition is not preserved by the remaining allowed diffeomorphisms.
- In order to fix this, a diffeomorphism must be accompanied by a gauge transformation

$$\theta(t^+) = \frac{e}{2l^2} \partial_+ \zeta^+ , \quad \tilde{\theta}(t^-) = -\frac{e}{2l^2} \partial_- \zeta^- \quad (14)$$

- Therefore the improved conformal transformations are generated by the twisted energy momentum tensor

$$\tilde{T}_{\pm\pm} = T_{\pm\pm} \mp \frac{e}{2l^2} \partial_{\pm} \mathcal{G}_{\pm}, \quad (15)$$

where \mathcal{G}_{\pm} is the current generates the gauge transformations (14).

- Using this twisted energy momentum tensor, the central charge of the model reads

$$c = 3k \frac{e^2}{l^4} \quad (16)$$

where k is the level of $U(1)$ current which parameterizes the gauge anomaly.

- We have fixed k using the known solutions. For the case of $\mu \rightarrow \infty$ the central charge is found to be $3/(2G)$ [Alishahiha, Ardanan, 08],[Larsen et al, 08]. This determines k as

$$k = 8|q|l^2 \quad (17)$$

which results the same value (7) for the central charges.

Relation to 3D gravity

- Our two dimensional AdS_2 solutions can be uplifted to three dimension. The results are pure geometric with $SL(2, R) \times U(1)$ isometry.
- For our three solutions we get

$$1 : ds^2 = \frac{l^2}{4} \left(-r^2 dt^2 + \frac{dr^2}{r^2} + (dz - r dt)^2 \right),$$

$$2 : ds^2 = \frac{l^2}{4} \left(-r^2 dt^2 + \frac{dr^2}{r^2} + (dz + r dt)^2 \right),$$

$$3 : ds^2 = \frac{9l^2}{\mu^2 l^2 + 27} \left(-r^2 dt^2 + \frac{dr^2}{r^2} + \frac{4\mu^2 l^2}{\mu^2 l^2 + 27} (dz + r dt)^2 \right),$$

(18)

- In light of the recent terminology, the third metric is the *warped AdS_3* [Strominger et al, 08].
- It is notable that our third central charge is the same as left central charge proposed in Strominger's warped *AdS_3* conjecture.
- It could be interesting to see why we can not read the right central charge of the warped *AdS_3* gravity by this method.