The Covariant Approach to Dark Matter an Energy

Qasem Exirifard
IPM, Tehran

Fifth regional meeting of string theory,
OAC, July 2009

Quantum Gravity?

Pure Quantum Gravity

The first natural toy model to reality.

Path Integral for pure Gravity

$$Z[q_{ij}] = \sum \frac{1}{\text{vol G}} \int Dg_{ij} e^{-i\int d^4x\sqrt{-\det g}R - i\int d^4x\sqrt{-\det g}g^{ij}q_{ij}}$$

Effective action for pure gravity

$$\Gamma[g^c_{ij}] = \ln Z[q_{ij}] - q^{ij} g^c_{ij}$$

$$g^c_{ij} = \frac{\partial}{\partial q_{ij}} \ln Z[q_{ij}]$$

Computation

We do not know how to systematically compute the effective action.

$$\Gamma[g_{ij}, R_{ijkl}, \nabla_i, l_p]$$

Planck's length

perturbative series and its break

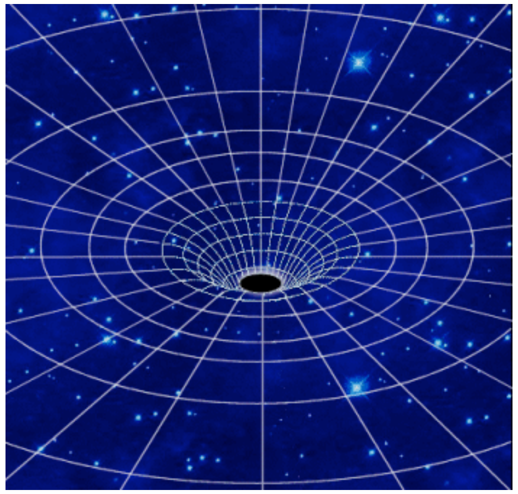
$$\Gamma[g_{ij}] = R + c_1 l_p^2 \boldsymbol{R}^2 + c_2 l_p^4 \boldsymbol{R}^3 + \cdots$$

If $|\text{Riemann Tensor}| \propto \frac{1}{l_p^2}$

then the sum over quantum corrections diverges

When $|\text{Riemann Tensor}| \propto \frac{1}{l_p^2}$ occurs?

Let us consider a black hole solution

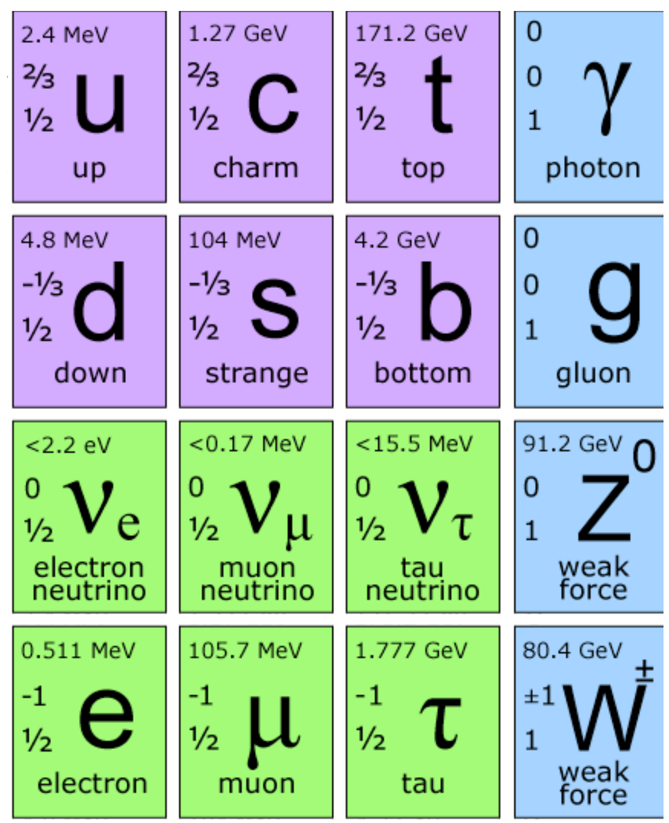


$$|\text{Riemann Tensor}| \propto (R_{ijkl}R^{ijkl})^{\frac{1}{2}}$$

$M_{BH} > M_p$ otherwise quantum corrections are not convergent

Realistic Quantum Gravity

Nature is not all gravity.
Include other fields too.



Realistic Quantum Gravity

A very bad news:
Particles are not natural in the gravity's scale!

$$l_{\text{compton of electron}} \approx 10^{22} l_p$$

Gravity and a massive field

$$S_g = -8\pi G \int d^4x \sqrt{-\det g} R$$

$$S_\xi = \int d^4x \, \sqrt{-detg} \, L(\xi, m_\xi)$$

$$m_p \gg m_\xi$$

Quantum Theory for Gravity and the massive field

Write the path integral.

Compute the effective action:

$$\Gamma[g_{ij}, R_{ijkl}, \nabla_i, l_p, l_\xi]$$

Surprise

The effective action by construction has two length scales:

$$\Gamma[g_{ij}, R_{ijkl}, \nabla_i, l_p, l_\xi]$$

Planck's length (l_p)

Compton's wavelength of the massive particle (l_ξ)

The effective action for small Riemann curvature

$$
\begin{aligned}
\Gamma[g_{ij}, R_{ijkl}, \nabla_i, l_p, l_\xi] = R \quad &+ \quad l_p^2(\boldsymbol{R}^2 + l_\xi^2 \boldsymbol{R}^3 + l_\xi^4 \boldsymbol{R}^4 + \cdots) \\
&+ \quad l_p^4(\boldsymbol{R}^3 + l_\xi^2 \boldsymbol{R}^4 + l_\xi^4 \boldsymbol{R}^5 + \cdots) \\
&+ \quad l_p^6(\boldsymbol{R}^4 + l_\xi^2 \boldsymbol{R}^5 + + l_\xi^4 \boldsymbol{R}^6 + \cdots) \\
&+ \quad \cdots
\end{aligned}
$$

Quantum Corrections

Validity of perturbative series

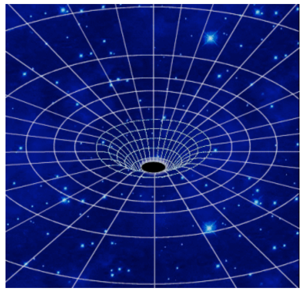
$$l_\xi \gg 10^{20} l_p$$

$$\begin{aligned}\Gamma[g_{ij}, R_{ijkl}, \nabla_i, l_p, l_\xi] = R \quad &+ \quad l_p^2(\boldsymbol{R}^2 + l_\xi^2 \boldsymbol{R}^3 + l_\xi^4 \boldsymbol{R}^4 + \cdots) \\ &+ \quad l_p^4(\boldsymbol{R}^3 + l_\xi^2 \boldsymbol{R}^4 + l_\xi^4 \boldsymbol{R}^5 + \cdots) \\ &+ \quad l_p^6(\boldsymbol{R}^4 + l_\xi^2 \boldsymbol{R}^5 + + l_\xi^4 \boldsymbol{R}^6 + \cdots) \\ &+ \quad \cdots\end{aligned}$$

It is not convergent when $|\boldsymbol{R}| \propto \dfrac{1}{l_\xi^2}$

When $|\boldsymbol{R}| \propto \dfrac{1}{l_\xi^2}$ occurs?

Consider a black hole solution



$M_{BH} > \dfrac{M_p}{M_\xi} M_p$ otherwise quantum corrections are not necessarily convergent.

Minimum mass of a black hole

$$M_{BH} > \frac{M_P}{M_e} M_P$$

$$M_p \propto 10^{19} GeV \longrightarrow M_{BH} > 10^{15} Kg$$

If the gravity's scale is lower

$$M_{BH} > \frac{M_P}{M_e} M_P$$

$$M_P \propto 10 TeV \longrightarrow M_{BH} > 10^7 TeV$$

We can not say that LHC will see black holes.

Corrections in the Realistic Quantum Gravity

$$\begin{aligned}\Gamma[g_{ij}, R_{ijkl}, \nabla_i, l_p, l_\xi] = R &+ l_p^2(\boldsymbol{R}^2 + l_\xi^2\boldsymbol{R}^3 + l_\xi^4\boldsymbol{R}^4 + \cdots) \\ &+ l_p^4(\boldsymbol{R}^3 + l_\xi^2\boldsymbol{R}^4 + l_\xi^4\boldsymbol{R}^5 + \cdots) \\ &+ l_p^6(\boldsymbol{R}^4 + l_\xi^2\boldsymbol{R}^5 + + l_\xi^4\boldsymbol{R}^6 + \cdots) \\ &+ \cdots\end{aligned}$$

Corrections in the Realistic Quantum Gravity

$$\begin{aligned}\Gamma[g_{ij}, R_{ijkl}, \nabla_i, l_p, l_\xi] = R \quad &+ \quad \frac{l_p^2}{l_\xi^4}\left(l_\xi^4 \boldsymbol{R^2} + l_\xi^6 \boldsymbol{R^3} + \cdots\right) \\ &+ \quad \frac{l_p^4}{l_\xi^6}\left(l_\xi^6 \boldsymbol{R^3} + l_\xi^6 \boldsymbol{R^4} + \cdots\right) \\ &+ \quad \cdots\end{aligned}$$

Corrections in the Realistic Quantum Gravity

$$\Gamma[g_{ij}, R_{ijkl}, \nabla_i, l_p, l_\xi] = R + \frac{l_p^2}{l_\xi^4}\, f_1(l_\xi^2 R_{ijkl}, l_\xi \nabla, g_{ij}) + \frac{l_p^4}{l_\xi^6}\, f_2(l_\xi^2 R_{ijkl}, l_\xi \nabla, g_{ij}) + \cdots$$

Approximated Realistic Quantum Gravity

$$\Gamma[g_{ij}, R_{ijkl}, \nabla_i, l_p, l_\xi] = R \quad + \quad \frac{l_p^2}{l_\xi^4} \, f_1(l_\xi^2 R_{ijkl}, l_\xi \nabla, g_{ij})$$

Relax a bit our imagination:
No assumption on

$$f_1(l_\xi^2 R_{ijkl}, l_\xi \nabla, g_{ij})$$

A trivial example of Approximated Realistic Quantum Gravity

$$\Gamma[g_{ij}, R_{ijkl}, \nabla_i, l_p, l_\xi] = R \quad + \quad \frac{l_p^2}{l_\xi^4}\, f_1(l_\xi^2 R_{ijkl}, l_\xi \nabla, g_{ij})$$

$$f_1(l_\xi^2 R_{ijkl}, l_\xi \nabla, g_{ij}) \approx cte$$

$$\Lambda \approx \frac{l_p^2}{l_\xi^4}$$

$$m_\xi \propto 0.01\,\mathrm{eV}$$

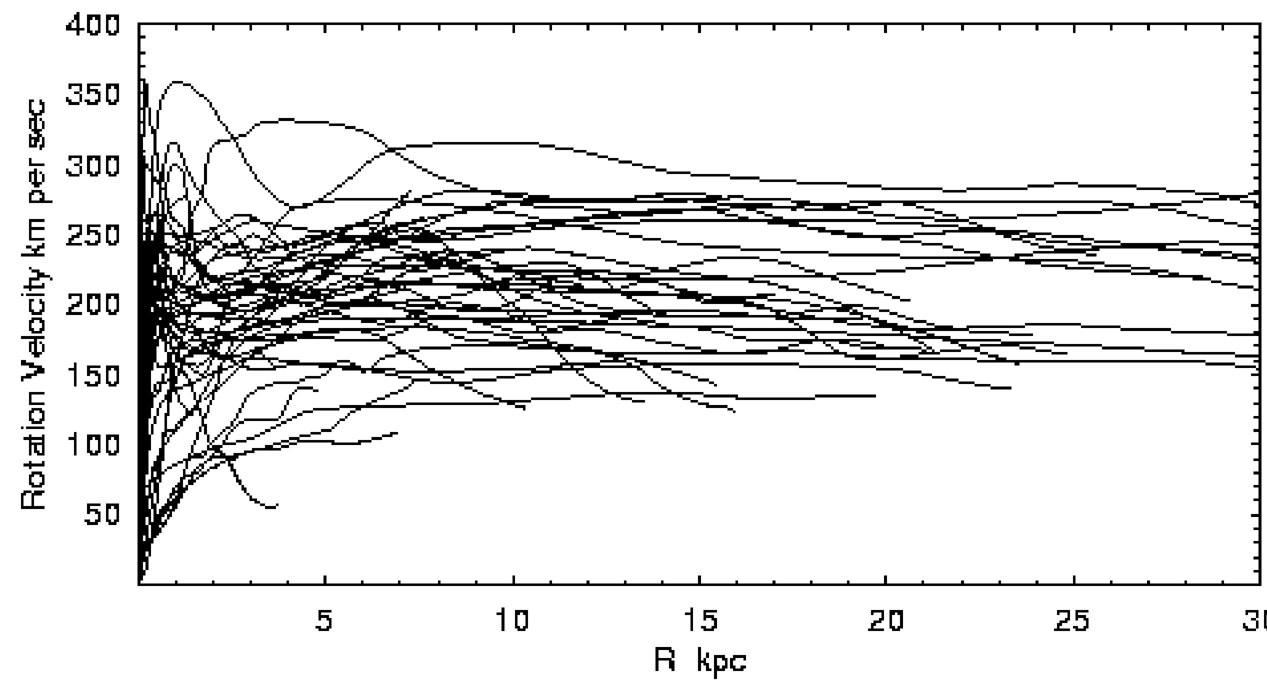
The plan of the rest of the talk

$$\Gamma[g_{ij}, R_{ijkl}, \nabla_i, l_p, l_\xi] = R \quad + \quad \frac{l_p^2}{l_\xi^4} \; f_1(l_\xi^2 R_{ijkl}, l_\xi \nabla, g_{ij})$$

Can ARQG account for dark matter and dark energy?

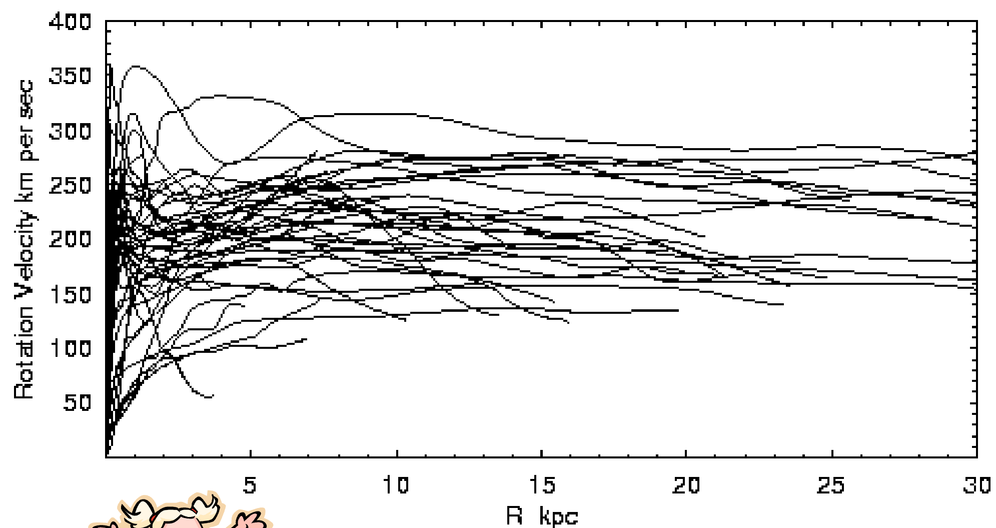
Covariant Resolution of Dark Matter: the missing mass problems in the galaxies

Photo, uploaded from Internet, Galaxy zoo [?]



Rotational curves of the spiral galaxies obtained by combining CO data for the central regions, optical for disks, and HI for outer disk and halo, Y. Sofue and V. Rubin, Ann. Rev. Astron. Astrophy. 39(2001)137.

Gravity is not a force, it is the side effect of the deformation of the space-time geometry



Where anomaly occurs the dynamics of the space-time is not given by the Einstein-Hilbert action

The space-time geometry around a galaxy

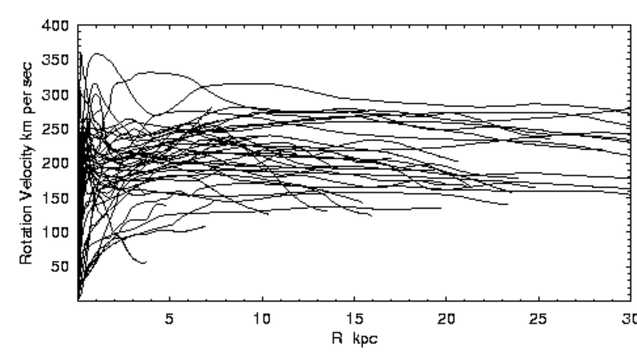
$$ds^2 = \text{Holy smoke}$$

Geometry around a galaxy

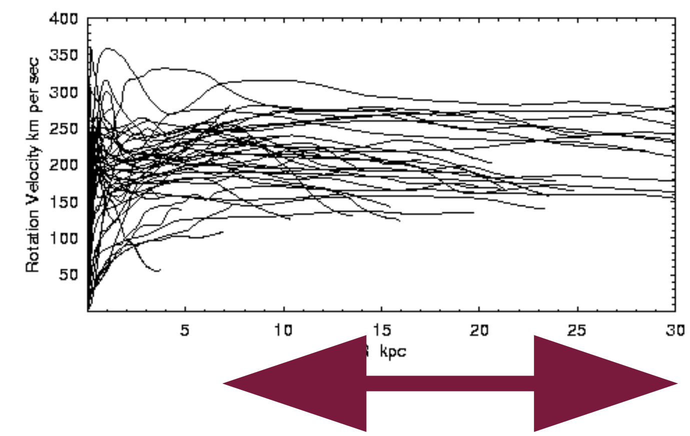
- $$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\Omega^2$$

Close to the center of a galaxy, Enistein-Hilbert Lagrangian governs the dynamics

$$\begin{aligned} A(r) &= 1 - \frac{2GM_g}{c^2 r} + \cdots \\ B(r) &= \frac{1}{A(r)} + \cdots \end{aligned}$$



Space-Time geometry at the edge of a galaxy



$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\Omega^2$$

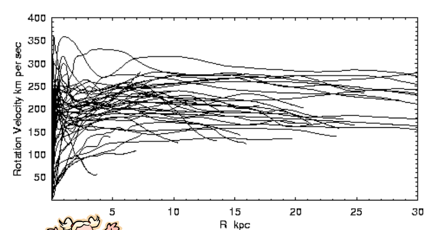
$$\left.\begin{array}{r} F = -\dfrac{c^2 g'_{tt}}{2} = \dfrac{c^2}{2} A'(r) \\[2em] a_{\text{radial}} = \dfrac{v^2}{r} \end{array}\right\} \quad \begin{array}{l} A(r) = 1 + a_0 \ln(r) + \cdots \\[2em] B(r) = 1 + b_0 \ln(r) + \cdots \end{array}$$

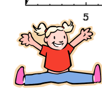
Tully-Fisher relation $\quad \dfrac{v^4}{M} \propto cte \;\rightarrow a_0 \propto M_g^{\frac{1}{2}}$

Glueing center and edge of the galaxy

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\Omega^2$$

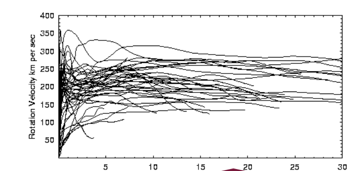
$$\begin{aligned} A(r) &= 1 - \frac{2GM_g}{c^2 r} + \cdots \\ B(r) &= \frac{1}{A(r)} + \cdots \end{aligned}$$





$$A(r) = 1 + \alpha M_g^{\frac{1}{2}} ln(r) + \cdots$$

$$B(r) = 1 + b_0 \ln(r) + \cdots$$



$$A(r) = 1 - \frac{m}{r} + \epsilon \alpha m^{\frac{1}{2}} \ln(r) + O(\epsilon^2)$$

Also done by Sobouti, Y. 0810.2198 [gr-qc], arXiv:0812.4127 [gr-qc].

What action has the following solution?

$$S = \int d^4x \sqrt{-\det g} L(R_{ijkl}, \nabla_i, g_{ij})$$

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\Omega^2$$

$$A(r) = 1 - \frac{m}{r} + \epsilon \alpha m^{\frac{1}{2}} \ln(r) + O(\epsilon^2)$$

$$B(r) = 1 + \frac{m}{r} + \epsilon b(r) + O(\epsilon^2)$$

$$L = R + ???$$

$$A(r) = 1 - \frac{2GM_g}{c^2 r} + \cdots$$
$$B(r) = \frac{1}{A(r)} + \cdots$$

$$A(r) = 1 + \alpha M_g^{\frac{1}{2}} ln(r) + \cdots$$

$$B(r) = 1 + b_0 \ln(r) + \cdots$$

Common Gravitational force on boundary of the above two regions , for every mass, requires:

$$\frac{M_g}{r^2} \approx cte$$

$$\frac{M_g}{r^2} \propto \frac{\text{Order of magnitude of } R_{ijkl}}{\text{Order of magnitude of } \nabla_r}$$

Lagrangian should have some covariant derivatives of the Riemann Tensor

The Lagrangian

$$S = \int d^4x \sqrt{-\det g} L(g_{ab}, R_{abcd}, \nabla_{a_1} R_{abcd}, \cdots, \nabla_{(a_1} \dots \nabla_{a_m)} R_{abcd})$$

$$L = R + \epsilon R \, \tilde{g}(G, |\nabla_a G|^2, \Box G) + O(\epsilon^2)$$

$$G = R_{ijkl}R^{ijkl} - 4R_{ij}R^{ij} + R^2$$

Ansatz for the Lagrangian: $$L = R + \epsilon R \tilde{g}(G, |\nabla_a G|^2, \Box G) + O(\epsilon^2)$$

This Ansatz has the following solution

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\Omega^2$$

$$A(r) = 1 - \frac{m}{r} + \epsilon\alpha m^{\frac{1}{2}} \ln(r) + O(\epsilon^2)$$

$$B(r) = 1 + \frac{m}{r} + \epsilon b(r) + O(\epsilon^2)$$

provided that

$$\tilde{g}(G, |\nabla_a G|^2, \Box G)|_{\epsilon=0} \approx -\alpha m^{\frac{1}{2}} \ln r$$

The correction to the action

$$\tilde{g}(G, |\nabla_a G|^2, \Box G)|_{\epsilon=0} \approx -\alpha m^{\frac{1}{2}} \ln r$$

$$\begin{aligned} G|_{\epsilon=0} &= \frac{12m^2}{r^6}\,, \\ (\nabla G)^2|_{\epsilon=0} &= \frac{5184m^4(r-m)}{r^{15}} \approx 5184\frac{m^4}{r^{14}}\,, \\ (\Box G)|_{\epsilon=0} &= \frac{4(90m^2r - 108m^3)}{r^9} \approx 360\frac{m^2}{r^8}\,. \end{aligned}$$

Using the above relation we find that

$$\begin{aligned} \left(\frac{G}{(\Box G)^{\frac{3}{4}}}\right)_{\epsilon=0} &\approx \frac{12}{(360)^{\frac{3}{4}}}\, m^{\frac{1}{2}}\,, \\ \ln(\frac{G}{\Box G}) &\approx 2\ln r \end{aligned}$$

The correction to the action

$$\tilde{g}(G, |\nabla_a G|^2, \Box G)|_{\epsilon=0} = -\alpha_{_{DM}} \frac{G}{(\Box G)^{\frac{3}{4}}} \ln(\alpha^4_{_{DM}} \frac{G}{\Box G})$$

First Summary

$$S = \int d^4x \sqrt{-\det g}\, R \left(1 - \alpha_{_{DM}} \frac{G}{(\Box G)^{\frac{3}{4}}} \ln(\alpha^4_{_{DM}} \frac{G}{\Box G})\right)$$

$$ds^2 = -(1 - \frac{m}{r} + \frac{24\alpha_{_{DM}}}{(360)^{\frac{3}{4}}} m^{\frac{1}{2}} \ln r + O(\alpha^2_{DM}))dt^2 + (1 + \frac{m}{r} + O(\alpha^2_{DM}))dr^2 + r^2 d\Omega^2 \,.$$

This metric for $\alpha_{_{DM}} \propto 10^{-13}(\text{meters})^{-\frac{1}{2}}$ is capable of describing the flat rotational velocity curves of the galaxies with an arbitrary mass.

Action in the Approximated Realistic Quantum Gravity

$$R - \frac{l_p^2}{l_\xi^4} f(l_\xi^2 R_{ijkl}, l_\xi \nabla_\beta, g_{\mu\nu})$$

$$f \; = \; \frac{l_\xi^6 RG}{(l_\xi^6 \Box G)^{\frac{3}{4}}} \; \ln(\frac{G}{l_\xi^2 \Box G})$$

$l_\xi \propto 1.6 \times 10^{-23} (meters) \rightarrow$ Resolution of the missing mass problem

Lets look at the action

$$R - \frac{l_p^2}{l_\xi^4} f(l_\xi^2 R_{ijkl}, l_\xi \nabla_\beta, g_{\mu\nu})$$

$$f \; = \; \frac{l_\xi^6 RG}{(l_\xi^6 \Box G)^{\frac{3}{4}}} \; \ln(\frac{G}{l_\xi^2 \Box G})$$

The Correction diverges for the Homogeneous solution

Correcting the correction

$$R - \frac{l_p^2}{l_\xi^4} f(l_\xi^2 R_{ijkl}, l_\xi \nabla_\beta, g_{\mu\nu})$$

$$f = \frac{l_\xi^6 RG}{(l_\xi^6(\Box + R)G)^{\frac{3}{4}}} \ln(\frac{G}{l_\xi^2(\Box + R)G})$$

Homogeneous solutions of the action

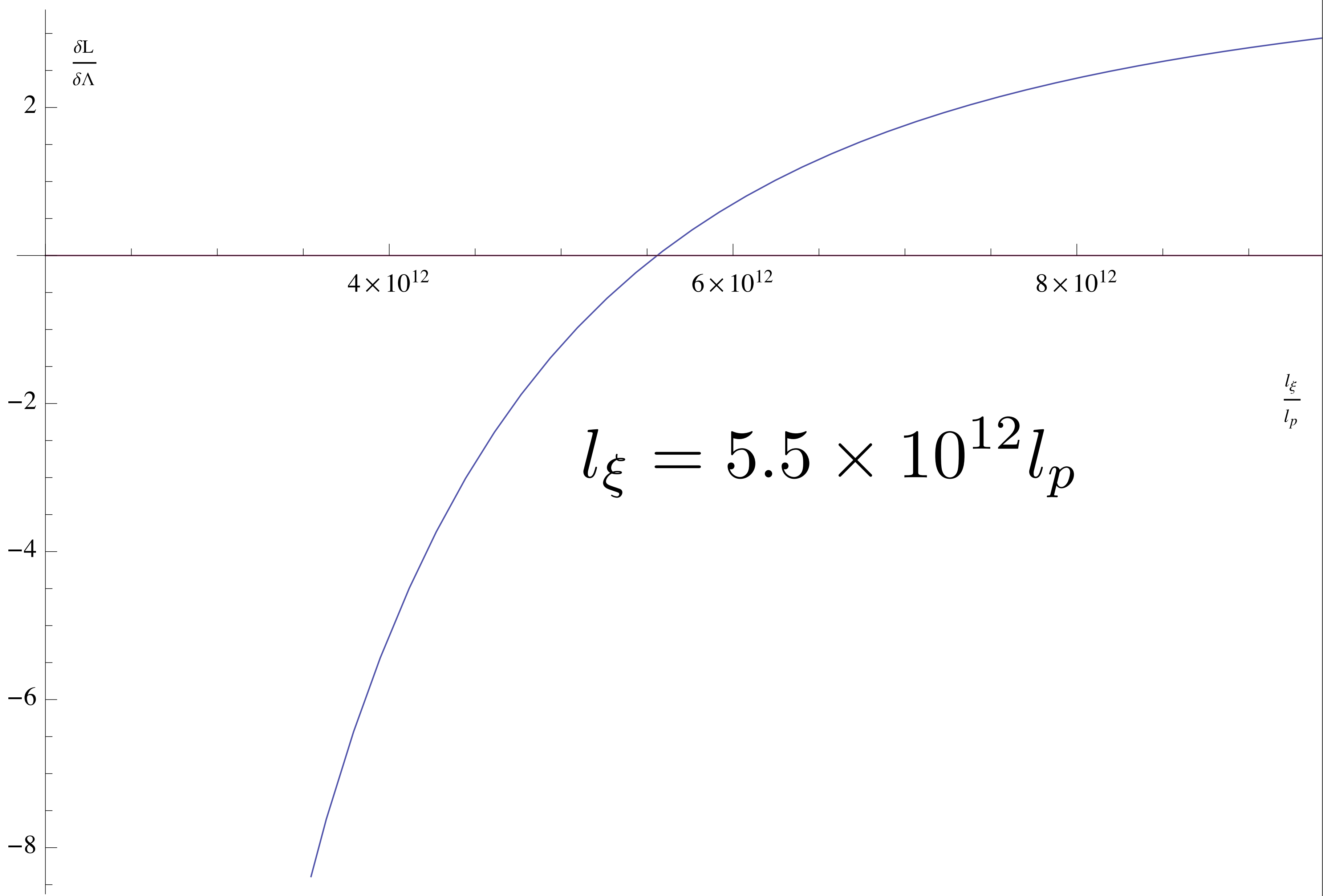
$$S = \int d^4x \sqrt{-\det g}(R - \frac{l_p^2}{l_\xi^4} f)$$

$$\begin{aligned} R^a{}_b &= \Lambda \delta^a{}_b \, , \\ R &= 4\,\Lambda \, , \\ G &= \frac{8}{3}\Lambda^2 \, , \\ \Box G &= 0 \end{aligned}$$

$$L_\Lambda = 4\Lambda - \frac{l_p^2}{l_\xi^4}\left(\frac{32}{3} l_\xi^6 \Lambda^3\right)^{\frac{1}{4}} \ln\left(\frac{1}{4 l_\xi^2 \Lambda}\right)$$

$$\frac{\delta L_\Lambda}{\delta \Lambda} = 0$$

$$\Lambda = 10^{-120} \frac{1}{l_p^2}$$



$$l_\xi = 5.5 \times 10^{12} l_p$$

Wowww

Resolution of Dark Energy requires $l_\xi = 5.5 \times 10^{12} l_p$

Resolution of Dark matter requires $l_\xi \propto 10^{12} l_p$

Conclusion:

- Dark Energy and dark matter problems can be unified with the framework of the Approximated Realistic Quantum Gravity
- The unification requires the existence of a massive particle with mass at order of 10^7 GeV
- ($m_p=10^{19}$ GeV)

Thanks

سپاس