Lessons from black holes --
Equations of motion, scales & effective coupling of quantum gravity

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- The Einstein equations for generalized theories of gravity and the thermodynamic relation $\delta Q = T\,\delta S$ are equivalent
- The gravitational analogue of 't Hooft's coupling $\lambda_G(l) = N\, G_N\, l^{-2}$ is bounded $\lambda_G(l) < 1$ for $l > l_{UV}$

The Einstein equations & $\delta Q = T\,\delta S$ are equivalent

Idea: (Einstein-Hilbert, Jacobson '95)

- Equivalence principle ➔ free falling observer can define a local Rindler (acceleration) horizon
- Rindler horizons are associated with thermodynamics δQ, T, δS
- T - Unruh temp., δQ – energy flow across the horizon, δS - entropy (entanglement)

Extension to generalized metric theories of gravity

Idea: (R.B+Hadad)

- Use semiclassical BHs to define δQ, T, δS

for accelration horizons in generalized theories

$$\mathscr{L} = \mathscr{L}_m(g_{ab},\phi) + \mathscr{L}_G(R_{abcd}) + \mathscr{L}_{int}(g_{ab}, R_{abcd}, \phi)$$

$\{\phi\}$- matter

Temperature T

$$\chi_b \nabla^b \chi_a = \kappa \chi_a$$

χ- Rindler horizon killing vector
κ – Surface gravity

Define temperature as for BHs (limiting procedure)

$$\kappa = 2\pi T$$

Heat δQ

$$E = \int_{\mathscr{H}} T_{ab} \tilde{\chi}^a \epsilon^b$$

Energy measure by an observer

For constant ε (fixed horizon)

$$\delta E = \int_{\mathscr{H}} \chi^c \nabla_c \left(T_{ab} \chi^a \right) \epsilon^b$$

$$\delta E = \cancel{\int_{\mathscr{H}} \chi^c \nabla_c T_{ab} \tilde{\chi}^a \epsilon^b} + \int_{\mathscr{H}} T_{ab} \chi^a \epsilon^b$$

Energy variation due to causal boundary

$$\delta Q = \int_{\mathscr{H}} T_{ab} \chi^a \epsilon^d$$

$$\chi^a = \kappa \tilde{\chi}^a$$

$$\epsilon^b = \tilde{\chi}^b \Sigma$$

Σ – volume element

$\mathscr{H}$ Rindler horizon

Entropy δS

$$S = -\frac{1}{T}\oint_{\partial\mathscr{H}} \frac{\partial\mathscr{L}}{\partial R_{abcd}}\hat{\epsilon}_{ab}\epsilon_{cd}$$

$$\hat{\epsilon}^{cd} = \nabla^c\tilde{\chi}^d$$

$$\epsilon^{cd} = \hat{\epsilon}^{cd}\bar{\epsilon}$$

$\bar{\epsilon}$ is the area element

$$S = -\frac{2}{T}\int_{\mathscr{H}} \nabla_c\left(\frac{\partial\mathscr{L}}{\partial R_{abcd}}\hat{\epsilon}_{ab}\right)\epsilon_d$$

For constant ε (fixed horizon)

$$\begin{aligned}\delta S &= -\frac{2}{T}\int \chi_m\nabla^m\nabla_c\left(\frac{\partial\mathscr{L}}{\partial R_{abcd}}\hat{\epsilon}_{ab}\right)\epsilon_d \\ &= -4\pi\int \tilde{\chi}_m\nabla^m\nabla_c\left(\frac{\partial\mathscr{L}}{\partial R_{abcd}}\hat{\epsilon}_{ab}\right)\epsilon_d\end{aligned}$$

Equations of motion for generalized theories of gravity

$$\sqrt{-g}\left(\frac{\partial \mathscr{L}}{\partial g^{ab}} + 2\nabla_p\nabla_q \frac{\partial \mathscr{L}}{\partial R_{pabq}} + \frac{\partial \mathscr{L}}{\partial R_{pqr}{}^{a}} R_{pqrb}\right) - \frac{1}{2} g_{ab}\mathscr{L} = 0$$

$$\mathscr{L} \;=\; \mathscr{L}_m\left(g_{ab},\phi\right) + \mathscr{L}_G\left(R_{abcd}\right) + \mathscr{L}_{int}\left(g_{ab}, R_{abcd}, \phi\right)$$

$$T_m^{ab} = -2/\sqrt{-g}\,\partial\left(\sqrt{-g}\mathscr{L}_m\right)/\partial g_{ab}$$

$$T_{int}^{ab} = -2/\sqrt{-g}\,\partial\left(\sqrt{-g}\mathscr{L}_{int}\right)/\partial g_{ab}$$

$$T^{ab} = T_m^{ab} + T_{int}^{ab}$$

$$T^{ab} = 2\left[2\nabla_p\nabla_q \frac{\partial \mathscr{L}}{\partial R_{pabq}} + \frac{\partial \mathscr{L}}{\partial R_{pqra}} R_{pqr}{}^{b}\right] - g^{ab}\mathscr{L}_G$$

The Einstein equations & $\delta Q = T\,\delta S$ are equivalent

δQ $\qquad$ $T\,\delta S$

$$\int T_{ab}\tilde{\chi}^a \epsilon^b = -2 \int \tilde{\chi}_m \nabla^m \nabla_c \left(\frac{\partial \mathscr{L}}{\partial R_{abcd}} \hat{\epsilon}_{ab} \right) \epsilon_d$$

Einstein equations!

$$T^{ab} = 2\left[2\nabla_p \nabla_q \frac{\partial \mathscr{L}}{\partial R_{pabq}} + \frac{\partial \mathscr{L}}{\partial R_{pqra}} R_{pqr}{}^{b} \right] + g^{ab} f$$

Conservation ➔ $f = -\mathscr{L}_G + \Lambda$

Bonus

The NCE obeys the 2nd law

$$\int T_{ab}\tilde{\chi}^a \epsilon^b = -2\int \tilde{\chi}_m \nabla^m \nabla_c \left(\frac{\partial \mathscr{L}}{\partial R_{abcd}} \hat{\epsilon}_{ab} \right) \epsilon_d$$

$$\delta S = \frac{1}{2\pi}\int T_{ab}\tilde{\chi}^a \tilde{\chi}^b \Sigma$$

$$\chi^a = \kappa \tilde{\chi}^a$$

$$\epsilon^b = \tilde{\chi}^b \Sigma$$

$$T_{ab}\tilde{\chi}^a \tilde{\chi}^b \geq 0 \Longrightarrow \delta S \geq 0$$

Null Energy Condition

The Einstein equations & $\delta Q = T\,\delta S$ are equivalent

- **Assumption:** causal barrier entropy behaves in a similar way to BH entropy.
- **Fact:** causal barrier entropy is associated with entanglement with hidden d.o.f
- **Speculation:** turn the logic around ➔ BH entropy results entanglement with hidden d.o.f

The Einstein equations & $\delta Q = T\,\delta S$ are equivalent

- **Speculation:** quantum gravity is not fundamental *only* thermodynamic/macroscopic description→ at some scale a microscopic description without gravity should exists (Sakharov's induced gravity ?, gauge-gravity duality?, …?)

A bound on the effective gravitational coupling from semiclassical black holes

- $\lambda_G(l) = N\, G_N\, l^{-2}$ is bounded $\lambda_G(l) < 1$ for $l > l_{UV}$
- N light species $m < \Lambda_{UV}$, $\Gamma < m$, weak coupling
- Metric theories ➔ the equivalence principle
- l_{UV} : scale above which exchanges of metric perturbations processes become strong

* The previous parametrization is not very useful ➔ need another path to prove bound for generalized theories of gravity

Definition: $l_{SCBH} \equiv l_P \sqrt{N}$

$\lambda_G(l) = N\, G_N\, l^{-2}$

Proof of the bound $\lambda_G(l) < 1$ for $l > l_{UV}$

1. $\lambda_G(l) < 1 \text{ for } l > l_{SCBH}$
2. l_{SCBH} is an absolute lower bound on the size of semiclassical BHs in *any* consistent theory of gravity.
3. In any consistent theory of gravity $l_{SCBH} < l_{UV}$

Assumptions about SCBH

M , R_S , β =1/T

$$(a) \quad -\frac{dR_S}{dt} < 1$$

$$(b) \quad -\frac{d\beta}{dt} < 1$$

$$(c) \quad -\frac{R_S}{M}\frac{dM}{dt} < 1$$

$$(d) \quad -\frac{\beta}{M}\frac{dM}{dt} < 1$$

$$(e) \quad \frac{\Gamma}{M} < 1$$

Assumptions about SCBH

I $$-\frac{dM}{dt} = N(T)T^4R_S^2$$

II $$TR_S = 1$$

$$-dM/dt = \Gamma T$$

$$\frac{\beta}{} \frac{dM}{} < 1$$

$$(e)\ \frac{\Gamma}{M} < 1$$

$$R_S M \quad > \quad N\frac{d\ln R_S}{d\ln M}$$

$$\boxed{R_S M \quad > \quad N}$$

A bound on the effective gravitational coupling for Einstein gravity

Einstein gravity $M = M_P^2 R_S$

$$R_S M > N$$

$$R_S M > N \frac{d \ln R_S}{d \ln M}$$

$$\Longrightarrow \quad R_S > l_P \sqrt{N} = l_{SCBH}$$

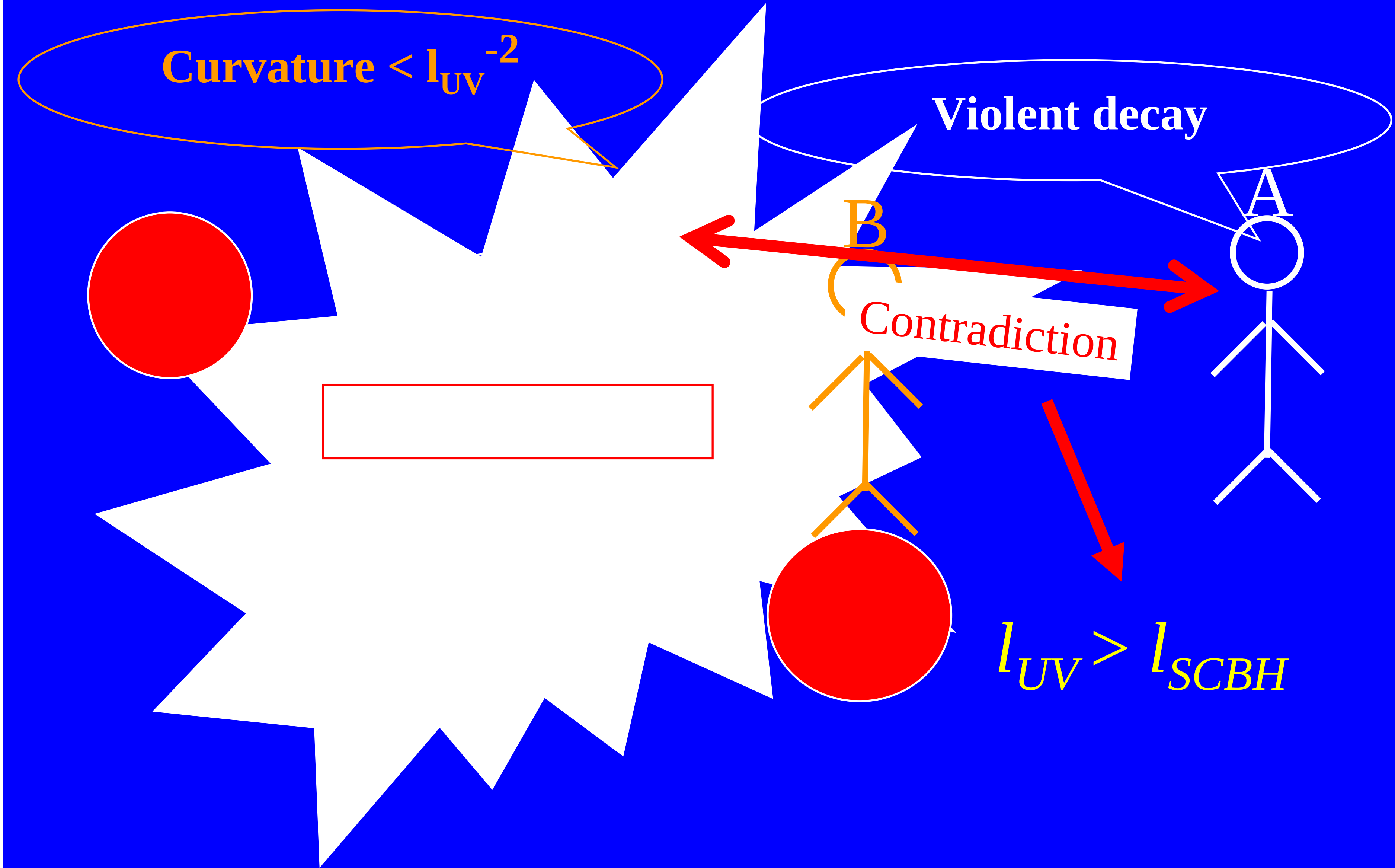
$$\lambda_G(l) = N \frac{l_P^2}{l^2}$$

$$l_{UV} \geq l_{SCBH}^{\,*}$$

$$\Longrightarrow \quad \lambda_G(l) < 1 \text{ for } l > l_{UV}$$

* Prove next

A though experiment: Assume $l_{UV}<l_{SCBH}$



Generalized theories

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

The one particle exchange amplitude $G \equiv t^{\mu\nu}\langle h_{\mu\nu}h_{\alpha\beta}\rangle T^{\alpha\beta}$

$$G = \frac{1}{M_P^2}\frac{t_{\mu\nu}T^{\mu\nu} - \frac{1}{2}t^{\mu}_{\mu}T^{\nu}_{\nu}}{\Box} + \sum_i \frac{1}{M_i^2}\frac{t_{\mu\nu}T^{\mu\nu} - \frac{1}{3}t^{\mu}_{\mu}T^{\nu}_{\nu}}{\Box - m_i^2} + \sum_j \frac{1}{(\overline{M}_j)^2}\frac{t^{\mu}_{\mu}T^{\nu}_{\nu}}{\Box - (\overline{m}_j)^2}$$

All coefficients are +ve for ghost/tachyon free theories, mass screening that reduces the acceleration of the probe not possible!

Vectors irrelevant for conserved sources

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

I(r)>0

$$h_{00}(r) = -\frac{M}{M_P^2}\frac{1}{r}\left(1 + \int_0^\infty dm\,\rho(m)\,e^{-mr}\right)$$

Horizon @ $h_{00}(R_S) = -1 \Longrightarrow M = M_P^2 \dfrac{R_S}{1 + I(R_S)}$

$M(R_S)$ is maximal for $I(R_S)=0$ ⇔ Einstein

$T = dh_{00}/dr|_{r=R_S}$

$TR_S = 1 \rightarrow \dfrac{d\ln R_S}{d\ln M} = 1$

Examples

- Compactified D=4+n Einstein Gravity for $r < R_C$
- Weakly coupled string theory

$$l_{UV} = l_s \qquad l_P = l_s g_s \qquad \lambda_G|_{l_s} = N l_P^2 / l_s^2 = N g_s^2$$

$$g_s^2 N < 1$$

number of the energetically-available species in string theory seems to be exponentially large?! Yes, but

$$N \sim \left(\frac{T}{M_s}\right)^2$$

Non-rot.→ non-rot.

Consequences

- Triviality of QG: $G_N \to 0$ for $l_{UV} \to 0\,(\Lambda_{UV} \to \infty)$

$$\lambda_G(l_{UV}) = NG_N/l_{UV}^2 < 1 \Longrightarrow G_N < \frac{l_{UV}^2}{N}$$

Not possible to consistently renormalize any theory of QG with a finite fixed number of fields (N=8 SUGRA!)

Consequences

- The Sakharov induced gravity limit for a finite UV cutoff

The Tree-level $G_N \to \infty$ (the Tree-level E-H removed)

$$G_N < \frac{l_{UV}^2}{N}$$

The renormalized G_N remains finite and bounded

Consequences: String Theory

$$G_N < \frac{l_s^2}{N}$$

Weak coupling

$$\lambda_G = N l_P^2 / l_s^2 = N g_s^2$$

$$g_s^2 N < 1$$

Saturation

w/. N ~ 100s

$$G_N = \frac{l_s^2}{N}$$

$\lambda_G \sim \lambda_{GUT}$ @ GUT scale

$$\lambda_{GUT} = \alpha_{GUT} \widetilde{N}$$

N ~ #bosons $\gg$ *Ñ ~ group rank*

Consequences: Entropy bounds

Einstein gravity $$S_{BH}(R_S) = M_P^2 R_S^2$$

$$R_S M \quad > \quad N \quad \Longrightarrow \quad S_{BH} > N$$

more general? proof ?

Saturation is very interesting!

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