Toy Model for Time Evolution of Black String/Hole Transition.

Takeshi Morita

Tata Institute of Fundamental Research

based on collaboration with M. Mahato, G. Mandal and S. Wadia

(work in progress)

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Introduction and Motivation

- Dynamical time evolutions of gravity and naked singularities.

  Big ban singularity, Black hole evaporation, Gregory-Laflamme transition, $\cdots$

  General relativity cannot describe the process beyond the singularity.

  cf. Cauchy problem, Initial value problem

- Conjecture:

  Quantum effects of gravity will make smooth these singularities.

  Q. How can string theory answer this problem?

In this study, we considered this problem

in the Gregory-Laflamme transition

by using the gauge/gravity correspondence.

Gregory-Laflamme transition

Gregory-Laflamme (1993), Kol (2005)

In IIA super gravity on $R^{1,8} \times S^1$, we have two solutions.

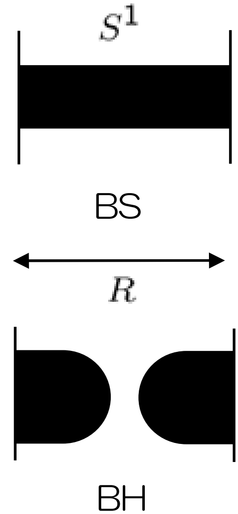
Black string

$$ds^2 = ds^2_{9dBH} + dz^2$$

z : S^1 coordinate
$z \sim z + 2\pi R$

Black hole

$$ds^2 = ds^2_{10dBH} \qquad (r_H \ll R)$$



・Stability of the solutions

$R > R_*(M) \qquad S_{BH}(M,R) > S_{BS}(M,R)$

$R < R_*(M) \qquad S_{BH}(M,R) < S_{BS}(M,R)$

Gregory-Laflamme transition

(We are considering the near extremal limit.)

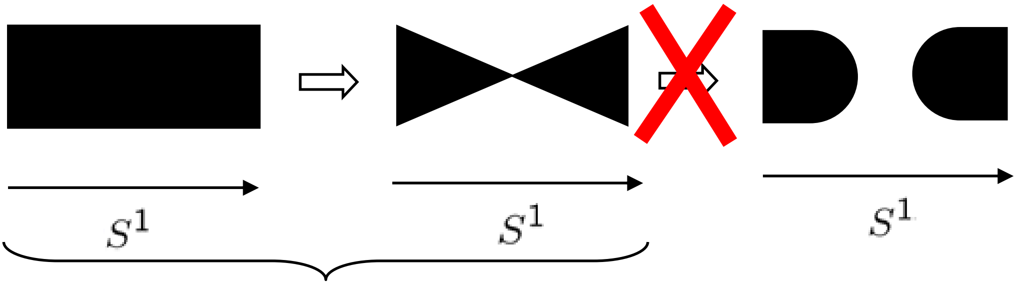
Gregory-Laflamme transition

Gregory-Laflamme (1993), Kol (2005)

· Horowitz-Maeda conjecture (2001)

If we assume that there are no singularities outside the horizon, the classical event horizon cannot pinch off at any finite affine time.

Start unstable BS ($R > R_*(M)$)



Infinite affine time

Infinite affine time $\xrightarrow{?}$ Infinite asymptotic time

(natural time for the gauge theory)

Marolf (2005)

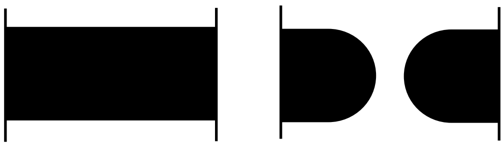
In asymptotically flat case, it is unclear whether the infinite affine time means the infinite asymptotic time. However, the near extremal case, it must be true.

Gauge/Gravity correspondence

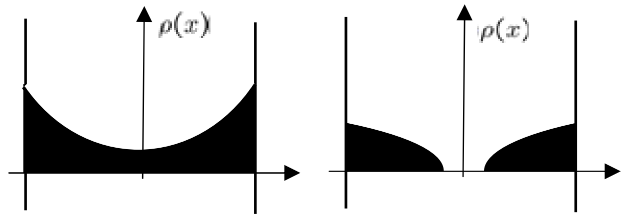
Gravity | **Gauge theory**

Gregory-Laflamme transition ⟷ Gross-Witten-Wadia transition

Configuration of the horizon | Configuration of Wilson line along S^1



BS | BH



ungapped phase | gapped phase

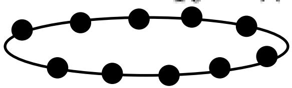
quantum effect ⟷ $1/N$ effect

By considering the time evolution of the gauge theory,
we evaluate the time evolution of the Gregory-Laflamme transition
and show how quantum effects resolve the naked singularity.

· Dual 2d SYM

Aharony, Marsano, Minwalla, Wiseman (2004) + Papadodimas, Raamsdonk (2005)

IIA gravity on $R^{1,8} \times S^1$ $\longleftrightarrow$ 1d SYM (Matrix theory) with S^1



D-particles on S^1

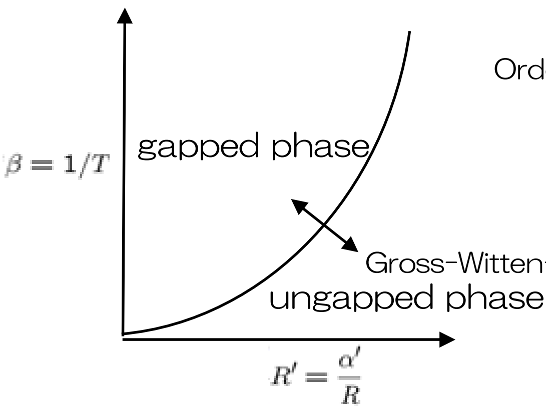
S^1 radius: R

$\updownarrow$ T-dual on the S^1 direction

2d SYM(D1-branes) on S^1

S^1 radius: $R' \equiv \dfrac{\alpha'}{R}$

· Phases of 2d SYM



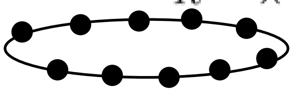
Order parameter: Eigen value density of the Wilson loop along the S^1.

$$U = \exp\left(i \int_0^{R'} A_z dz\right)$$

· Dual 2d SYM

Aharony, Marsano, Minwalla, Wiseman (2004) + Papadodimas, Raamsdonk (2005)

IIA gravity on $R^{1,8} \times S^1$ ⟷ 1d SYM (Matrix theory) with S^1



T-dual on the S^1 direction

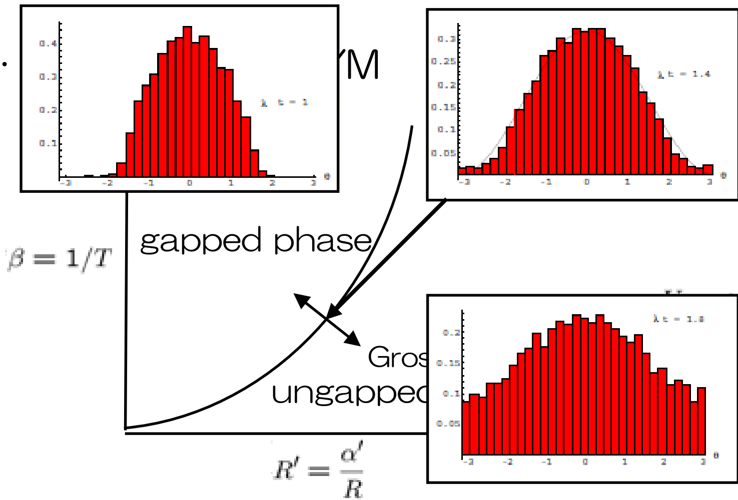
D-particles on S^1

S^1 radius: R

2d SYM(D1-branes) on S^1

S^1 radius: $R' \equiv \dfrac{\alpha'}{R}$

· ...YM



Eigen value density of the Wilson loop along the S^1.

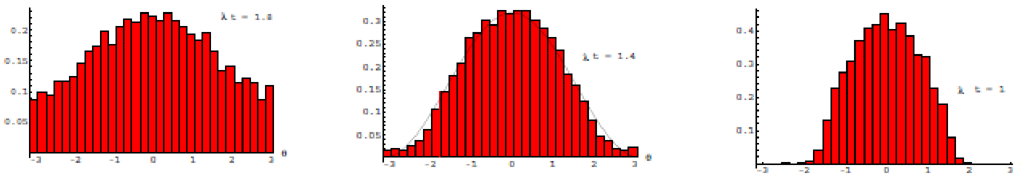
$W \ldots \exp\left(i \int_0^{R'} A_z dz\right)$

...ion

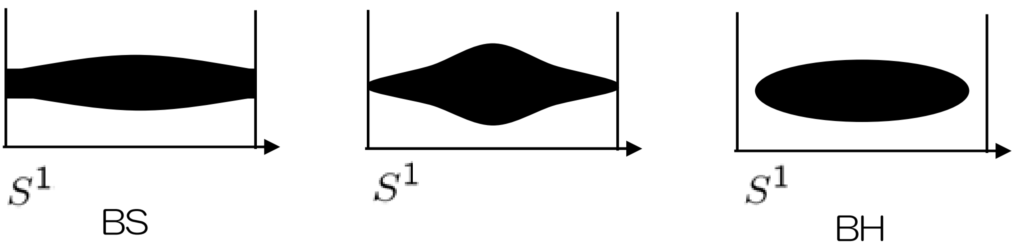
· Dual 2d SYM

· Correspondence between BS/BH and GWW transition

GWW type transition

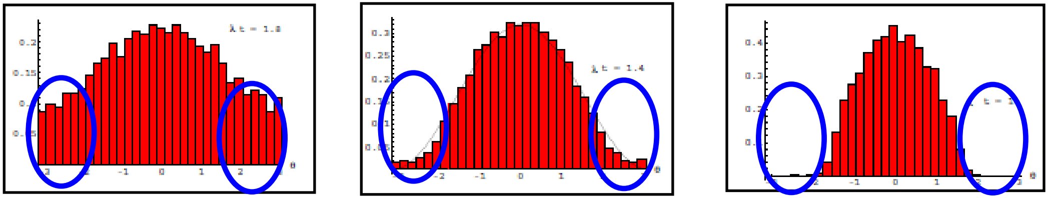


BS/BH transition



Matrix Model description

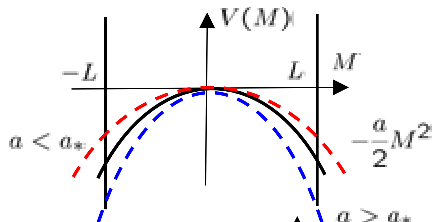
・ c=1 matrix model



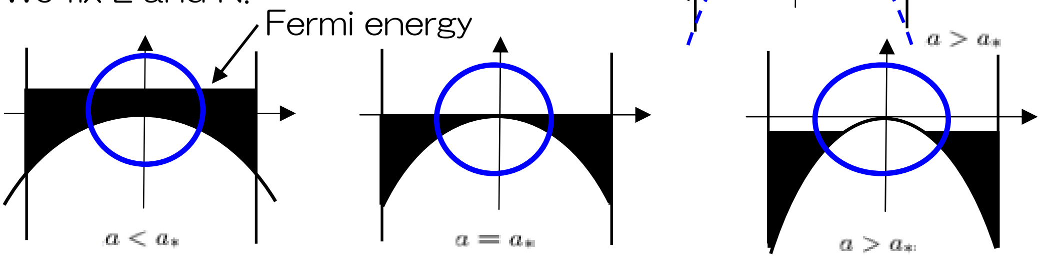
According to the universality, the effective theory near the critical point will be described by c=1 matrix model with the inverse harmonic potential.

$$S = \int dt Tr_{U(N)} \frac{1}{2}\left(\dot{M}^2 + aM^2\right)$$

$$|M| < L$$

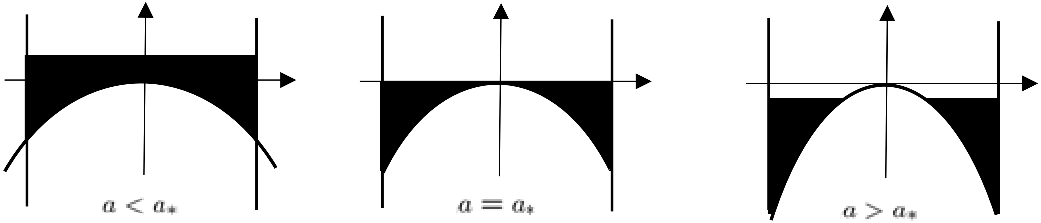


: We fix L and N.

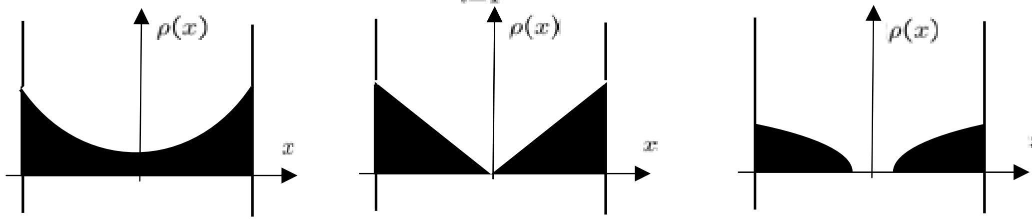


Matrix Model description

・ c=1 matrix model



Eigen value density: $\rho(x) = \frac{1}{N}\sum_{i=1}^{N}\delta(x-\lambda_i)$ λ_i : i-th Eigen value of M:



Matrix Model description

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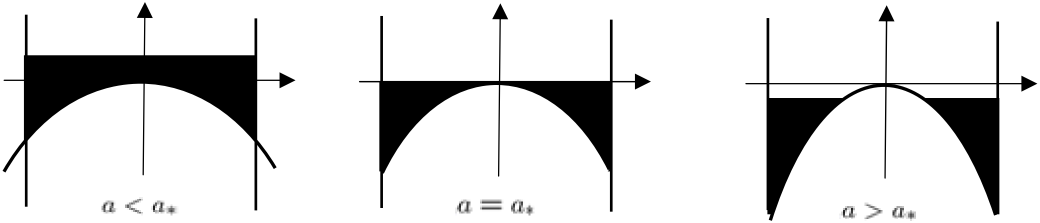

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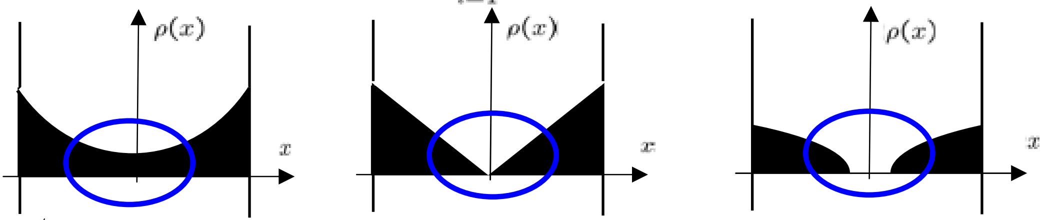
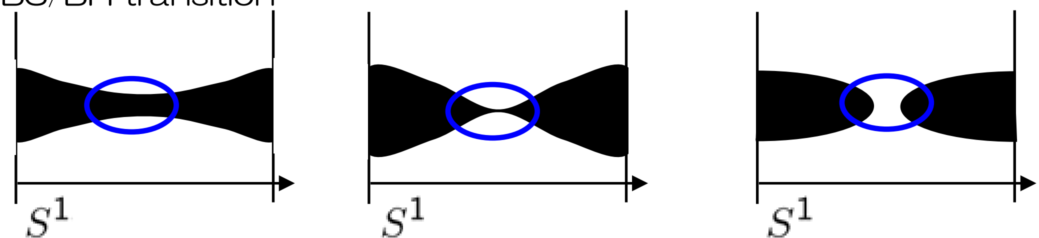
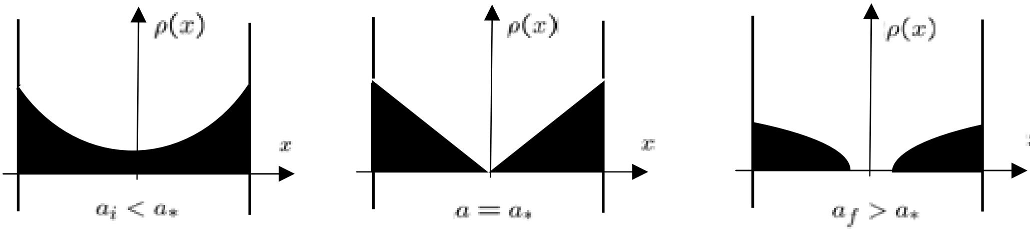


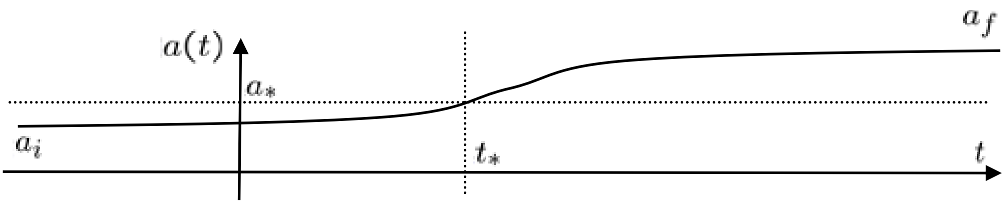
BS/BH transition



Forcing and time evolution



$$a \to a(t) = a_i + F(t) \quad \left(L = L_0 + \frac{1}{2} F(t) tr M^2 : \ \text{external force} \right)$$



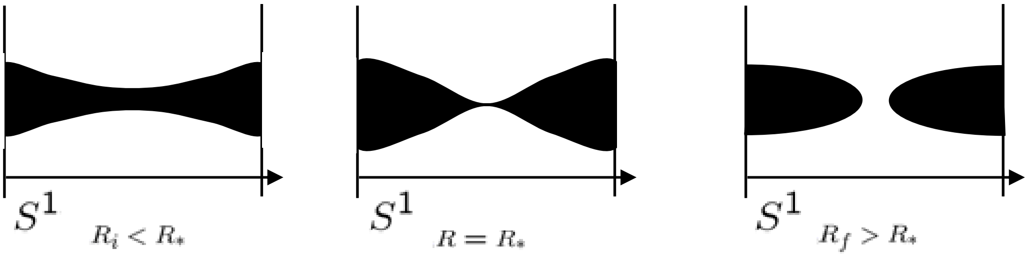
If we replace the constant a as a time dependent increasing function a(t),

then we can naively expect the transition happen when t=t*.

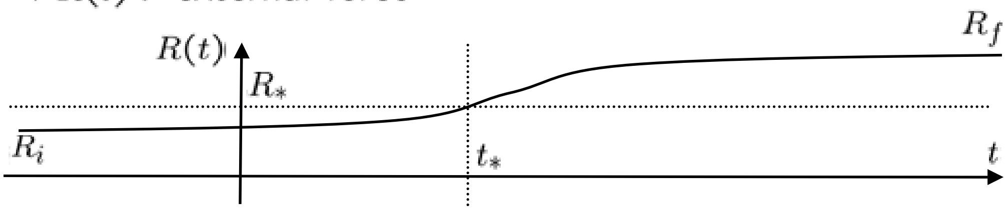
Forcing and time evolution

- Forcing and time evolution

It is natural to guess that this gauge theory process corresponds to the following gravity process



$R \to R(t)$: external force



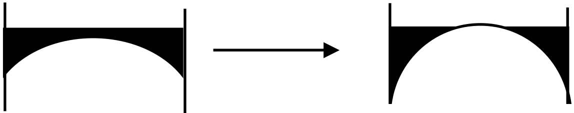
However, through the argument in the Horowitz-Maeda conjecture:,

the transition doesn't happen in the classical gravity, because of the naked singularity.

On the other hand, if we consider quantum effects, something will happen around t=t*.

Brief Summary of our Result

・Gauge theory



1. If N is infinite, infinite time is necessary for the transition.
2. If N is finite, the transition happens in finite time.
3. If we consider the opposite process (gapped to ungapped), the transition happen in finite time.

⇕ Quantum effect ⟷ 1/N effect

・Known facts in gravity

1. Horowitz-Maeda conjecture:

   In classical general relativity, infinite time is necessary for the GL transition to avoid the naked singularity.
2. Quantum effect would make smooth the naked singularity.
3. The transition from BH to BS happens without any singularity.

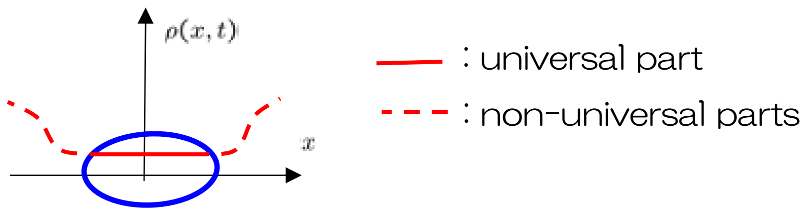
Classical time evolution of the matrix

Classical time evolution of the matrix

Universal late time behavior

$$\rho(x,t) = 2\beta e^{-\sqrt{a_f}t} \quad (x \sim 0,\ t \gg 1, a(t) \sim a_f)$$

β : model dependent constant.



We found that in case the transition happens, this behavior is universal!!

Independent of the detail of a(t), forcing (for example $b(t) tr M^{2n}$)

and even unitary matrix models behave like this.

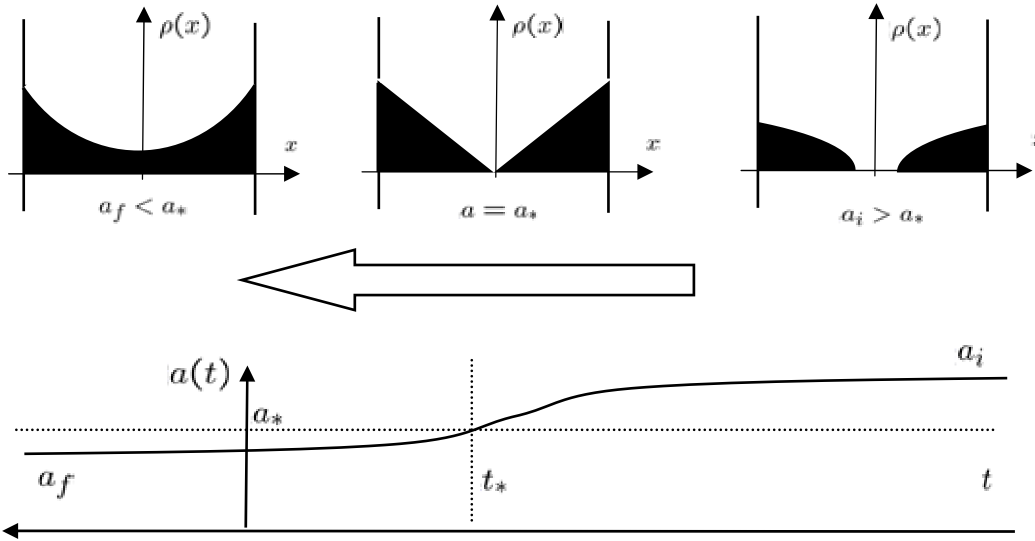
The transition (the gap arises) happens at $t = \infty$

$\updownarrow$ Consistent!!

Horowitz-Maeda conjecture

Classical time evolution of the matrix

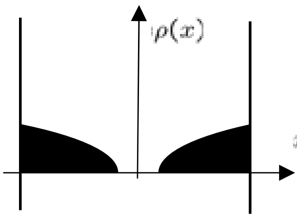
- gapped to ungapped transition

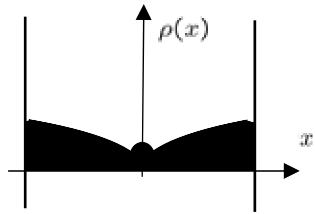


Classical time evolution of the matrix

- gapped to ungapped transition

$$\rho(0,t)=0 \xrightarrow{\text{finite time}} \rho(0,t)\neq 0$$





The transition happens always in finite time.

But we have not found the equation describing the transition.

It seems that this result is consistent with BH to BS transition.

Quantum time evolution of the matrix

Quantum time evolution of the matrix

- a=constant

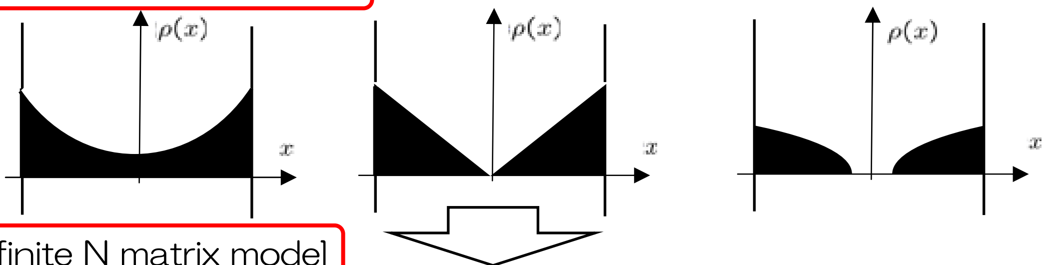
$$h_i = \frac{1}{2}\left(p_i^2 - a x_i^2\right) \quad , i = 1 \ldots N < \infty, \;\; h_i \psi_i(x) = \epsilon_i \psi_i(x), \;\; \psi_i(x = \pm L) = 0$$

We can solve Schrodinger equations by using parabolic cylinder function. (Moore 1992)

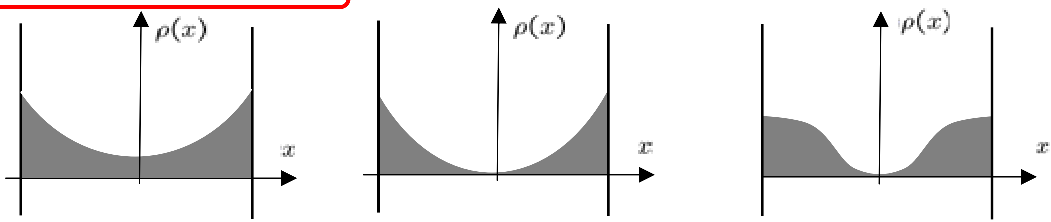
$$\psi_i(x) = F_i(x, a, L)$$

- Eigen value density: $\rho(x) = \sum_i^N |\psi_i(x)|^2$

infinite N matrix model



finite N matrix model



The densities are made smooth through the 1/N effects.

Quantum time evolution of the matrix

- $a \to a(t)$

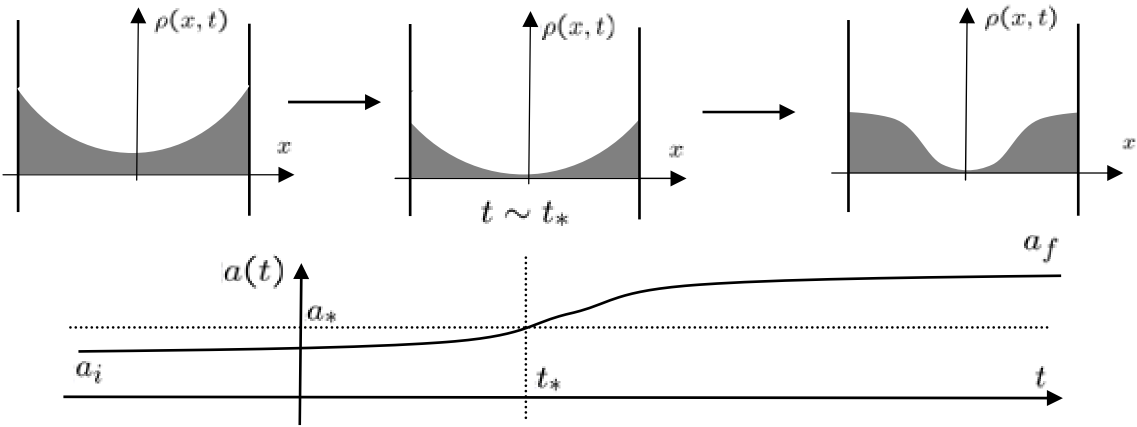
Since the energy spectrum is discrete,

we can apply adiabatic approximation if a(t) satisfies: $\partial_t a(t) \ll \frac{1}{N^2 \log N}$.

Then we obtain the wave function is obtained by

$$\psi_i(x) = F_i(x, a, L) \to \psi_i(x) = F_i(x, a(t), L)$$

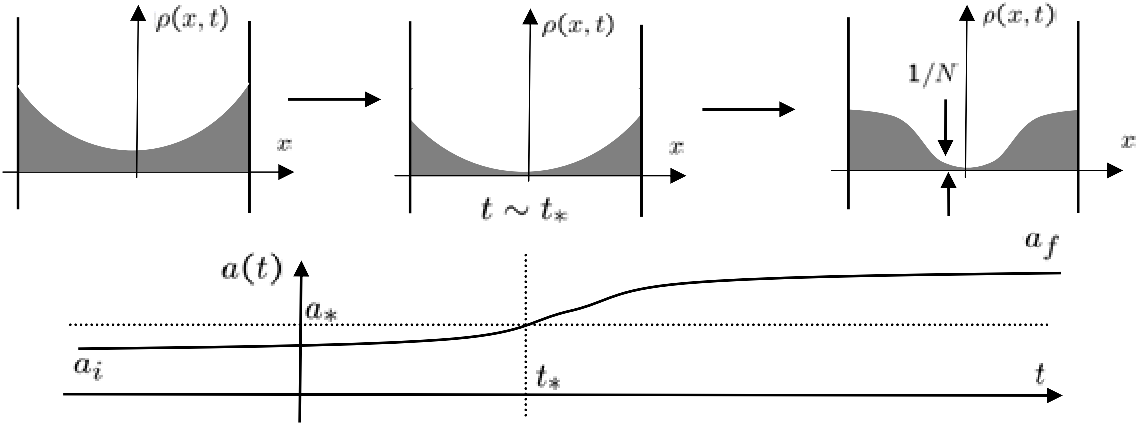
Time evolution of eigen value density



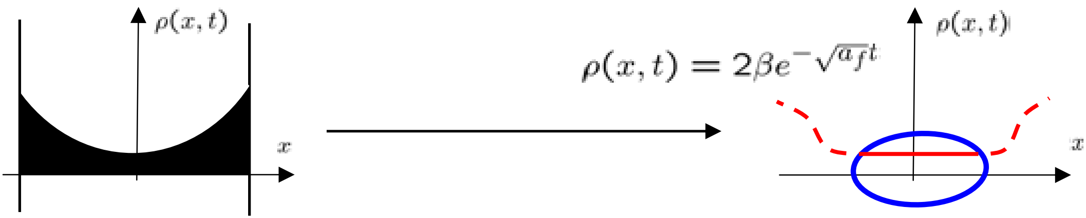
Quantum time evolution of the matrix

- Comparison to the infinite N

Time evolution of finite N matrix model

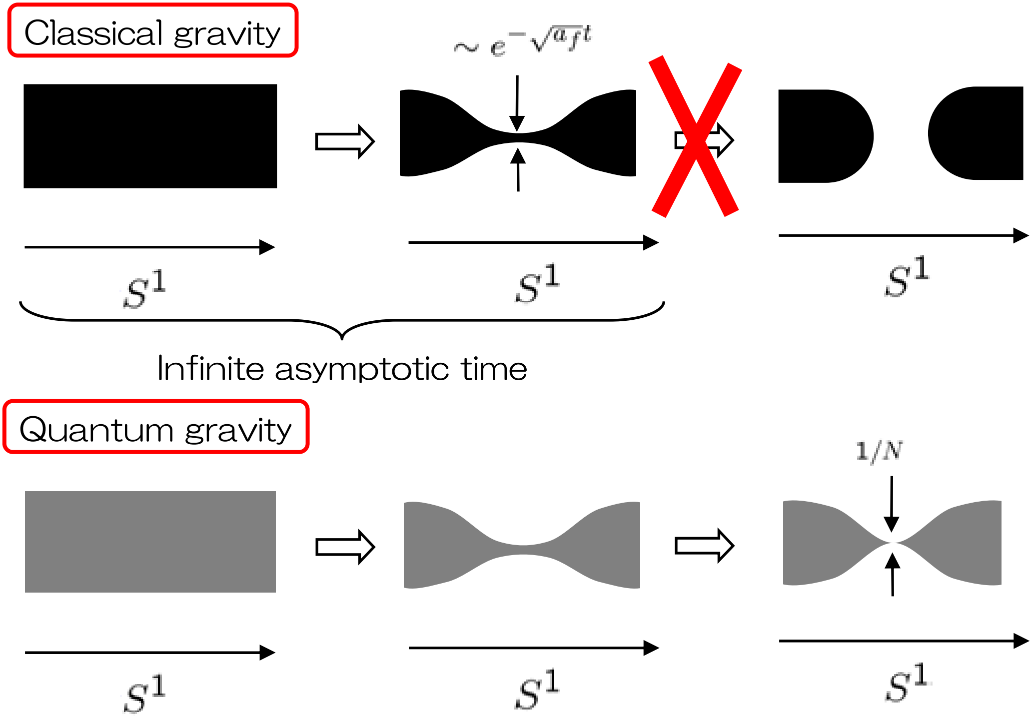


Time evolution of infinite N matrix model



Quantum time evolution of the matrix

- Conjecture on the gravity



Toward the derivation of the effective matrix model from 2d SYM

Toward the derivation of the effective matrix model from 2d SYM

Our matrix model

$$S = \int dt Tr \frac{1}{2}\left(\dot{M}^2 + aM^2\right)$$

Fundamental theory: 2d SYM on S1

$$S = \int_0^{R'} dtdz Tr\left(\frac{1}{g_{YM}^2}F_{\mu\nu}^2 + \frac{1}{2}(D\phi^i)^2 - \frac{g_{YM}^2}{4}[\phi^i, \phi^j]^2 + fermions\right)$$

- Is it possible to derive the matrix model from 2d SYM?
- What is 'a'?

→ We attempt to solve this problem in weak coupling analysis and mean field approximation.

→ Condensation of the adjoint scalars may give the potential.

(Work in progress...)

Conclusion

- We found that the one matrix model can reproduce the time evolution of the classical gravity qualitatively in the large N limit.
- The singular behavior is resolved by 1/N effect.
- The matrix model may be derived from 2d SYM through the condensation.

Future Work

- Complete the derivation of the matrix model from 2d SYM.
- Construction of more realistic model (including interaction, adjoint scalar)
- Calculation in gravity side and comparison to matrix model result
- Quantum evolution without the adiabatic approximation.
- Including Hawking Radiation.
- Application to other singular system. black hole evaporation.