Tachyon Free Thermal String Models

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Start with a weakly coupled supersymmetric string theory, on an initially flat background:

$$R^D \times T^{10-D}, \qquad R_i \sim l_s$$

When $g_s \ll 1, \qquad l_s \gg l_p$

(In 4D $l_p^2 = g_s^2 l_s^2$)

At finite temperature, thermal fluctuations produce an energy density

$\longrightarrow$ backreaction induces a cosmological evolution.

At low temperature

$$\rho = (D-1)P \sim T^D$$

leading to $a \sim \frac{1}{T}$, a is the scale factor of the universe.

If we extrapolate the cosmological evolution back in time, the temperature increases

so that more and more massive string states get thermally excited until we get a phase transition at the Hagedorn temperature:

$$T_H \sim \frac{1}{l_s}$$

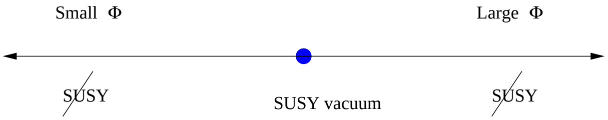
... before the big bang.

Clearly to understand the very early history of a large class of string gas cosmologies $\rightarrow$

we need to be able to handle the Hagedorn instabilities of string theory at high temperature.

Consider now a different but closely related problem concerning tree level supersymmetry breaking in perturbative superstrings.

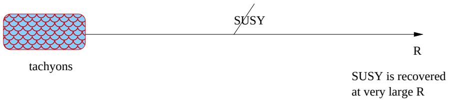
We would like to start with a susy string compactification and turn on a modulus that breaks susy



... then vacua in the neighborhood of the supersymmetric point would have arbitrarily small breaking.

Unfortunately if such a vacuum exists, it has to be at the boundary of moduli space.

E.g. susy breaking via geometrical fluxes (stringy Scherk-Schwarz mechanism) along a spatial cycle of radius R:



$$M \sim \frac{1}{R}, \quad \text{decompactification problem}$$

On the other end we often find tachyonic modes whenever $R \leq l_s$.

We may also consider such a spontaneously broken susy model at finite temperature T.

Then in many situations the string partition function satisfies

$$Z(T, M) = Z(M, T)$$

gravitino mass/temperature duality

The existence of tachyons at small R is related to the Hagedorn instabilities at high temperature.

In this talk I will present *asymmetric* type II orbifolds which exhibit susy breaking, but are free of tachyonic instabilities.

The models admit an interesting thermal interpretation and have a temperature duality symmetry:

$$T \to \frac{T_H^2}{T}$$

In string theory there is an exponential growth in the density of single particle states as a function of the mass.

E.g. in in type II theories, for large mass

$$\rho(m) \sim m^{-10}\ e^{\frac{m}{m_0}}, \qquad m_0 = (2\pi\sqrt{2\alpha'})^{-1}$$

As a result the canonical ensemble

$$Z = \mathrm{Tr} e^{-\beta H}, \quad \beta = \frac{1}{T}$$

converges only for temperatures below the Hagedorn temperature:

$$T_H \sim \frac{1}{l_s}$$

$$2\pi T_H = \frac{1}{\sqrt{2\alpha'}}$$

in type II theory

At $T = T_H$, the free energy

$$\frac{F}{T} = -\ln Z$$

exhibits non-analytic behavior in $t = T_H - T$;

for a large enough number of non-compact dimensions, it is finite as $T \to T_H$ from below.

It has been argued by many authors that at $T \sim T_H$, the system undergoes a phase transition

similar in fact to the deconfining phase transition of large N QCD at high temperature.

The partition function can be computed via a Euclidean path integral on $S^1 \times \mathcal{M}$

(S^1 is the Euclidean time circle with period β)

In field theory:

- periodic BC for bosonic fields
- anti-periodic BC for fermionic fields

... Or sum over particle paths in a first quantized approach.

If a path winds $\tilde{m}$ times the Euclidean time circle, it must be weighted by a phase $(-1)^{\tilde{m}F}$
(F is the space-time fermion number).

In string theory we have both momentum and winding numbers: $(\tilde{m}, n)$.

At $T > T_H$ a certain stringy winding mode ($n \neq 0$) becomes tachyonic.

$\rightarrow$ divergence can be thought of as an IR instability and the phase transition is driven by tachyon condensation.

Notice that the winding tachyon does not correspond to any physical, propagating state of the original theory, and its “rolling down” the potential is not a real time process.

But we can compare the free energy for different values of the condensate $\phi^*\phi$ so as to characterize the phases of the system.

Unfortunately, as the phase transition is first order, it involves large values of the tachyon condensate $|\phi|^2$

and so it cannot be controlled in perturbation theory ...

If a high temperature saddle point exists,

$$F \sim \frac{1}{g_s^2}$$

This is a genus zero contribution and it is a signal of "deconfinement" as in the case of large N QCD.

The genus zero contribution leads to large backreaction, since

$$G\rho \sim 1$$

in string units. $\rightarrow$ thermal equilibrium ceases to exist.

- type II: tachyons for $\beta < 2\pi\sqrt{2}$. ($l_s = 1$).
- Heterotic: tachyons for $\frac{4\pi^2}{\beta_H} < \beta < \beta_H$.

The heterotic case is more intriguing, since the partition function has a thermal duality symmetry. ($X_R^0 \to -X_R^0$ is a symmetry of the worldsheet CFT.)

$$\beta \to \frac{4\pi^2}{\beta}$$

Due to this symmetry, it has been argued by Antoniadis, Derendinger and Kounnas,

$\longrightarrow$ that the exact tachyon potential has a stable minimum

The potential was derived using properties of $N = 4$ gauged supergravity . . .

Can we deform the thermal ensemble so as to avoid tachyonic instabilities?

The answer is yes if we introduce gauge field condensates with zero field strength but with a non-zero value of the Wilson line

$$U = P \exp(i \int_0^\beta A_0 dX^0)$$

At finite temperature, (abelian) vacuum potentials in the range $0 \leq \frac{A_0}{T} \leq \pi$ are gauge inequivalent.

The Model

Consider type IIB on $T^2 \times T^8$.

* T^8 is a very large eight-torus

* T^2 is a rectangular torus $S^1_T \times S^1$, the first circle is the Euclidean time circle of radius R_0.

Initially the model is supersymmetric

$$\mathcal{Z} = \frac{1}{4} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \frac{1}{(\eta\bar{\eta})^{12}} \Gamma_{(1,1)}(R_0) \Gamma_{(1,1)}(R_1) \Gamma_{(8,8)}$$

$$\times \sum_{a,b=0,1} (-1)^{a+b+ab} \theta^4 \begin{bmatrix} a \\ b \end{bmatrix} \sum_{\bar{a},\bar{b}=0,1} (-1)^{\bar{a}+\bar{b}+\bar{a}\bar{b}} \bar{\theta}^4 \begin{bmatrix} \bar{a} \\ \bar{b} \end{bmatrix}$$

Or in terms of the $SO(8)$ characters

$$O_8 = \frac{\theta_3^4 + \theta_4^4}{2\,\eta^4}\,, \qquad V_8 = \frac{\theta_3^4 - \theta_4^4}{2\,\eta^4}\,,$$

$$S_8 = \frac{\theta_2^4 - \theta_1^4}{2\,\eta^4}\,, \qquad C_8 = \frac{\theta_2^4 + \theta_1^4}{2\,\eta^4}\,,$$

$$\mathcal{Z} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \frac{1}{(\eta\bar{\eta})^8} \left| V_8 - S_8 \right|^2 \Gamma_{(1,1)}(R_0)\, \Gamma_{(1,1)}(R_1)\, \Gamma_{(8,8)}$$

- All oscillators along the X^0, X^1 directions can be gauged away
- When we decompactify the T^8 torus, we get an $SO(8)$ symmetry

Spacetime fermion number receives contributions from both the left and right worldsheet movers

$F = F_L + F_R$

Under $(-1)^{F_L}$ the left moving R sector changes sign, similarly for F_R.

Conventional thermal deformation: Insert the phase

$$(-1)^{\tilde{m}_0(a+\bar{a})+n_0(b+\bar{b})}$$

Asymmetric deformation:

$$\frac{R_0}{\sqrt{\tau_2}} \sum_{\tilde{m}_0, n_0} e^{-\frac{\pi R_0^2}{\tau_2}|\tilde{m}_0+n_0\tau|^2} (-)^{\tilde{m}_0 a+n_0 b+\tilde{m}_0 n_0}$$

$$\frac{R_1}{\sqrt{\tau_2}} \sum_{\tilde{m}_1, n_1} e^{-\frac{\pi R_1^2}{\tau_2}|\tilde{m}_1+n_1\tau|^2} (-)^{\tilde{m}_1 \bar{a}+n_1 \bar{b}+\tilde{m}_1 n_1}$$

In this way, the X^0 lattice is "thermally" coupled to the left-moving world-sheet degrees of freedom, while the X^1 lattice is "thermally" coupled to the right-moving world-sheet degrees of freedom.

In the n_0 (n_1) odd winding sector the left (right) GSO projection is reversed

The string partition function takes the form

$$\mathcal{Z} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \frac{\Gamma_{(8,8)}}{(\eta\bar{\eta})^8}$$

$$\times \sum_{m_0,n_0} \left(V_8 \, \Gamma_{m_0,2n_0} + O_8 \, \Gamma_{m_0+\frac{1}{2},2n_0+1} - S_8 \, \Gamma_{m_0+\frac{1}{2},2n_0} - C_8 \, \Gamma_{m_0,2n_0+1} \right)$$

$$\times \sum_{m_1,n_1} \left(\bar{V}_8 \, \Gamma_{m_1,2n_1} + \bar{O}_8 \, \Gamma_{m_1+\frac{1}{2},2n_1+1} - \bar{S}_8 \, \Gamma_{m_1+\frac{1}{2},2n_1} - \bar{C}_8 \, \Gamma_{m_1,2n_1+1} \right) .$$

$\rightarrow$ The $O\bar{O}$ sector appears in the spectrum, which typically becomes tachyonic in some regions of moduli space.

Here, however it carries non-zero momentum and winding charges and so

$$2\, m^2_{O\bar{O}} = \left(\frac{1}{\sqrt{2}R_0} - \sqrt{2}R_0\right)^2 + \left(\frac{1}{\sqrt{2}R_1} - \sqrt{2}R_1\right)^2$$

It is never tachyonic. Massless when $R_0 = R_1 = \frac{1}{\sqrt{2}}$, dual fermionic point.

Spectrum

Only the $V\bar{V}$ ($G_{\mu\nu}$, $B_{\mu\nu}$, Φ) sector is massless.

Fermions

$$2\, m^2_{V\bar{S}} = \frac{1}{(\sqrt{2}R_1)^2}\,, \qquad 2\, m^2_{S\bar{V}} = \frac{1}{(\sqrt{2}R_0)^2}$$

and from the odd winding sectors

$$2\, m^2_{V\bar{C}} = (\sqrt{2}R_1)^2\,, \qquad 2\, m^2_{C\bar{V}} = (\sqrt{2}R_0)^2$$

- At large radii the spinors S, $\bar{S}$ are light.
- At small radii the conjugate spinors C, $\bar{C}$ are light.

All RR fields are massive since these are charged under $(-1)^{F_L}$ and $(-1)^{F_R}$.

In fact the spectrum is T-duality invariant under

$$R_0,\ R_1 \rightarrow \frac{1}{2R_0},\ \frac{1}{2R_1}$$

$$S,\ \bar{S} \rightarrow C, \bar{C}$$

Supersymmetry is restored at both ends of moduli space.
At large radii: chiral IIB model
At small radii: the equivalent chiral IIB' model.

At the self-dual point $R_0 = R_1 = \frac{1}{\sqrt{2}}$, we get additional massless states from the $O\bar{O}$, $V\bar{O}$ and $O\bar{V}$ sectors $\rightarrow$

$SU(2)_L \times SU(2)_R$ enhanced gauge symmetry

Observe also that when

$$R_0 >> 1, \qquad R_1 \sim 1$$

the light states arise in the $V\bar{V}$ and $S\bar{V}$ sectors with masses

$$m^2_{V\bar{V}} = 0\,, \qquad 2\,m^2_{S\bar{V}} = \frac{1}{(\sqrt{2}R_0)^2}$$

$\rightarrow$ the light spectrum is precisely thermal.

Thermal Interpretation

Shift

$$\tilde{m}_1 \longrightarrow \tilde{m}_1 + \tilde{m}_0\,, \qquad n_1 \longrightarrow n_1 + n_0$$

We obtain a thermally coupled $\Gamma_{(2,2)}$ torus lattice, where there is a non-trivial B field background

$$B_{01} = -B_{10} = \frac{1}{2}$$

and a non-diagonal metric

$$ds^2 = R_0^2 (dx^0)^2 + R_1^2 (dx^1 + G\, dx^0)^2$$

$$\frac{G_{01}}{R_1^2} = G = 1$$

$$\frac{R_0\, R_1}{\tau_2} \sum_{\tilde{m},n} e^{-\frac{\pi}{\tau_2}\left[R_0^2|\tilde{m}_0+\tau n_0|^2+R_1^2|\tilde{m}_1+G\tilde{m}_0+\tau(n_1+Gn_0)|^2\right]}$$

$$\times e^{2i\pi B(\tilde{m}_1 n_0-\tilde{m}_0 n_1)}$$

$$\times(-1)^{\tilde{m}_0(a+\bar{a})+n_0(b+\bar{b})}\,(-1)^{\tilde{m}_1\bar{a}+n_1\bar{b}+\tilde{m}_1 n_1}$$

X^0-cycle: the deformation acts as a standard thermal deformation.

X^1-cycle: couples only to the right-moving fermion number F_R.

Special point: $2B = G = 1$

We think of the model as follows:

Start with the $10D$ type IIB theory and compactify the X^1 direction on a circle.

Coupling this circle to F_R, breaks the initial $(4,4)$ susy to $(4,0)$.

In addition we get two $U(1)$ gauge fields:

- The graviphoton field: $A_\mu = G_{1\mu}$
- The axial gauge field: $\tilde{A}_\mu = B_{1\mu}$

We then heat the system, giving vevs to $A_0 \rightarrow G = 1$ and to $\tilde{A}_0 \rightarrow B = 1/2$

Or turn on Polyakov loops for these gauge fields.

G and B appear in the statistical ensemble as chemical potentials for the charges

$$Q_+ = m_1 - \frac{1}{2}(\bar{a} + n_1) \equiv \frac{1}{2} R_1 (p_L + p_R)$$

$$Q_- = n_1 \equiv \frac{1}{2} R_1^{-1} (p_L - p_R)$$

The complete space-time partition function is then given by

$$Z(\beta, G, B) = \mathrm{Tr}\, e^{-\beta H}\, e^{2i\pi(GQ_+ - BQ_-)}$$

the trace is over the full Hilbert space of the $(4,0)$ theory.

When $G = 1$, $B = 1/2$ we get

$$Z = \mathrm{Tr}\, e^{-\beta H} (-1)^{F_R}$$

the right-moving fermion index, corresponding to the tachyon free model.

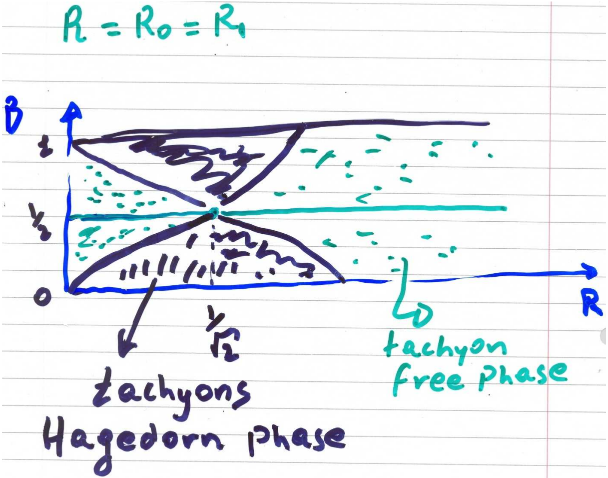
When $G = 0$, $B = 0$ we get

$$Z = \mathrm{Tr}\, e^{-\beta H}$$

the canonical ensemble, with thermal instabilities.

G_{10}, B_{10} are non-fluctuating backgrounds $\longrightarrow$ chemical potentials.

Phase diagram



The 1-loop partition function is finite and is characterized by a formal "temperature duality" symmetry: $R_0 \to 1/2R_0$.

However, in terms of the T-dual variables, the system at small R_0 is effectively cold.

The line in moduli space, $R = R_0 = R_1$, is interesting.

As we decrease R, the system contracts and heats up, until we reach the fermionic point. Then it expands and cools.

One can also find models, where at points in moduli space susy is broken but the massive spectrum is characterized by a boson/fermion degeneracy symmetry. Kounnas 0808.1340[hep-th]; Florakis, Kounnas 0901.3055[hep-th]

It would be interesting to see if we can use these models to reverse contraction to expansion in a cosmological setting.