Holographic realization of general gauge mediation

Marika Taylor

University of Amsterdam
Gauge mediated scenarios of supersymmetry breaking combine many attractive features (flavor blindness) with unresolved problems (satisfactory realization of electroweak symmetry breaking).

It is important to understand which features are generic and which are model dependent: the recently proposed general gauge mediation of Seiberg et al provides such a framework.

As we will see, the framework of general gauge mediation includes strongly coupled hidden sectors cf most earlier discussions of gauge mediation.
Of course, strongly coupled hidden sectors have been excluded from most earlier discussions through lack of tractability but new **holographic tools** exist.

The aim here will thus be to use holography to realize general gauge mediation with a strongly coupled hidden sector.

Such holographic models potentially provide novel mechanisms for overcoming problems such as naturally realizing electroweak symmetry breaking.
"Holographic realization of gauge mediated supersymmetry breaking",
K. Skenderis and M. Taylor, 0907.xxxx

"Holographic realization of general gauge mediation",
Plan

1. Review of general gauge mediation
2. Assembling holographic ingredients
3. Holographic realization of general gauge mediation
Gauge mediation is one of the oldest, simplest and most robust ways of transmitting supersymmetry breaking to the (M)SSM.

The basic idea is to couple the MSSM to a separate hidden sector that breaks susy via couplings to the visible sector gauge fields.

This coupling communicates the supersymmetry breaking to the MSSM and generates soft breaking terms.
Traditionally in gauge mediation: give a hidden sector model, and its direct or messenger couplings to the visible sector.

Recently Meade, Seiberg and Shih introduced a more systematic framework, called general gauge mediation (GGM).
In GGM one considers models in which the hidden sector has **global symmetry** which is weakly gauged by the coupling to the visible sector.

I.e. it includes a linear superfield $\mathcal{J}$ defined by conservation conditions:

$$\bar{D}^2 \mathcal{J} = D^2 \mathcal{J} = 0,$$

so in components:

$$\mathcal{J} = J + i\theta j - i\bar{\theta} \bar{j} - \theta \sigma^\mu \bar{\theta} j_\mu + \cdots$$

with $\partial_\mu j^\mu = 0$. 

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**General gauge mediation**

Marika Taylor  Holographic gauge mediation
The current superfield is then coupled to the vector superfield of the visible sector via:

\[ L_{int} = 2g \int d^4 \theta J \mathcal{V} + \cdots; \]

\[ = g(JD - \lambda j - \bar{\lambda} \bar{j} - j^\mu V_\mu) + \cdots \]

where the ellipses denote terms of order \( g^2 \) needed for gauge invariance.

Hidden sector current is needed for all of the visible sector gauge group, \( U(1) \times SU(2) \times SU(3) \) or GUT group, \( \mathcal{J} \rightarrow \mathcal{J}^{(r)}. \)
Soft masses in visible sector determined by two point functions of hidden sector currents:

\[
\langle J(x)J(0) \rangle = \frac{1}{x^4} C_0(x^2 M^2); \\
\langle j_\alpha(x)\bar{j}_\dot{\alpha}(0) \rangle = -i\sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu \left( \frac{C_{1/2}(x^2 M^2)}{x^4} \right); \\
\langle j_\mu(x)j_\nu(0) \rangle = (\eta_{\mu\nu} \partial^2 - \partial_\mu \partial_\nu) \left( \frac{C_1(x^2 M^2)}{x^4} \right); \\
\langle j_\alpha(x)j_\beta(0) \rangle = \epsilon_{\alpha\beta} \frac{1}{x^5} B_{1/2}(x^2 M^2).
\]

Here \( M \) is a characteristic mass scale; \( B_{1/2} \) is complex whilst the \( C_a \) are real.
Broken susy

- **Unbroken susy** $\rightarrow C_0 = C_{1/2} = C_1 > 0, B_{1/2} = 0$.

- **Broken susy** $\rightarrow$ functions asymptote to these values in the UV.

- **Key observation**: soft masses expressible in terms of these functions.
To leading order the effective Lagrangian for the gauge supermultiplet is:

$$\delta L = -\frac{1}{2}g^2(M\tilde{B}_{1/2}(0)\lambda\lambda + c.c.) + \cdots$$

where $\tilde{B}_{1/2}(p)$ is the two point function coefficient in momentum space and ellipses denote terms responsible for wavefunction renormalization.

Susy broken $\rightarrow$ **gaugino masses** generated at tree level in effective theory:

$$m_r = g_r^2 M^2 \tilde{B}_{1/2}^{(r)}(0)$$

for different gauge groups $(r)$. 
Soft masses

@ Meade et al
Soft masses

- Susy breaking is communicated to squarks by 1-loop diagrams with intermediate gluinos.
- Resulting squark masses are

\[ m^2_{\tilde{f}} = \sum_r g^4_r c_2(f; r) A_r; \]

\[ A_r = -\frac{M^2}{16\pi^2} \int dy \left( 3\tilde{C}_1^{(r)}(y) - 4C_{1/2}^{(r)}(y) + C_0^{(r)}(y) \right); \]

where \( c_2(\tilde{f}; r) \) is Casimir of \( \tilde{f} \) under \( (r) \) gauge group.
- Mass manifestly vanishes in susy limit.
1. General definition covers many/most earlier models (Giudice and Ratazzi review) but
   - does not assume weakly coupled hidden sector;
   - does not need identifiable messenger fields.

2. Formulation in terms of current correlators: holography naturally computes operator correlation functions.

3. This framework can be extended to include the couplings to the Higgs sector; see later.
Since all sfermion masses are given in terms of $A_r$, sfermion mass sum rules are generic in gauge mediation:

$$\text{Tr}(Y m^2) = \text{Tr}((B - L)m^2) = 0,$$

for hypercharge $U(1)_Y$ and $U(1)_{B-L}$, and trace over given generations.

Gaugino mass and sfermion masses are a priori unrelated (cf most known models) and realizing specific relations constrains the hidden sector theory.
1. Review of general gauge mediation
2. Assembling holographic ingredients
3. Holographic realization of general gauge mediation
Aim: Explore strongly coupled hidden sector holographically.

Ie: Find holographic dual to 4d (gauge) theory with global symmetry and spontaneously broken susy.

Then use standard holographic techniques to compute current correlators in this theory.
Other holographic realizations

- Our approach holographically realizes only hidden sector.
- Compliments, and is more precise than, previous attempts to holographically engineer gauge mediated models which include visible sector Franco et al.
But:

1. How do we find holographic backgrounds with spontaneously broken susy (which are at least metastable)?

2. Isn’t this hard? For example, we would need to solve 2nd order sugra equations, cf 1st order susy equations.

Here we will exploit fake supersymmetry (Freedman et al, 2003) to find such (meta)stable non-supersymmetric solutions.
Non-supersymmetric domain walls

A large class of holographic backgrounds dual to $d$-dimensional QFTs with spontaneously broken susy can be obtained by considering $(d+1)$-dimensional non-susy domain wall solutions to gravity/scalar theories, embedded into $(d+1)$-dimensional gauged sugra theories, which in turn uplift to 10d or 11d sugra.
These domain wall solutions arise as solutions of actions involving gravity coupled to scalars:

\[
S = \int d^{d+1}x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - \frac{1}{2} g_{ab}(\phi^c) \partial_m \phi^a \partial^m \phi^b - V(\phi^a) \right],
\]

and have the form

\[
ds^2 = dr^2 + e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu; \quad \phi^a = \phi^a(r),
\]

with the scalars depending only on radial coordinate (\(d\)-dimensional Poincaré invariance).
Equations of motion:

\[
\ddot{A} = -\frac{\kappa^2}{(d-1)} \dot{\phi}^2; \quad \dot{A}^2 = \frac{\kappa^2}{d(d-1)} \dot{\phi}^2 - \frac{2\kappa^2}{d(d-1)} V(\phi)
\]

\[
\ddot{\phi} + d\dot{A} \dot{\phi} = \frac{\partial V}{\partial \phi},
\]

where \(\dot{f} = \frac{\partial f}{\partial r}\) and \(f' = \frac{\partial f}{\partial \phi}\).

Any solution of 1st order equations:

\[
\dot{A} = -\frac{\kappa^2}{(d-1)} W; \quad \dot{\phi} = W'
\]

also solves the second order equations.
Here $W$ is related to the potential $V$ via:

$$V = \frac{1}{2} \left[ (W')^2 - \frac{d\kappa^2}{d - 1} W^2 \right],$$

and $W$ is a (fake) superpotential.

Note that $W$ does not necessarily coincide with the superpotential $\mathcal{W}$ of the supergravity theory into which this is embedded.

Here we are interested in cases where $W \neq \mathcal{W}$: then the domain wall has fake supersymmetry, implying stability properties\(^1\), but breaks the (real) supersymmetry.

\(^1\)Freedman et al, Skenderis and Townsend
Search strategy for non-susy holographic duals:

1. Scan scalar potentials $V$ which arise in consistent subsectors of (gauged) sugra theories.

2. Find cases for which $W \neq \mathcal{W} \rightarrow$ broken susy $^2$.

$^2$Many 4d examples found by Papadimitriou.
Holography best understood in AdS cases: restrict first to potentials $V$ which admit susy critical points at $\phi^a = \phi^a_o$ with $V_0 \equiv V(\phi^a_o) < 0$.

At the AdS critical point:

$$ds^2 = dr^2 + e^{2r} \eta_{\mu\nu} dx^\mu dx^\nu,$$

i.e. $A = r$ and

$$V_0 = -\frac{d(d-1)}{2L^2}.$$

Asymptotically AdS domain walls will have $A \to r$ and $\phi^a = \phi^a_o$ as $r \to \infty$. 
Vevs v. deformation parameters

According to the standard holographic dictionary, asymptotically AdS domain walls can correspond to states in the CFT or states in a deformed theory.

Canonical susy examples would be the Coulomb branch of $\mathcal{N} = 4$ SYM versus the GPPZ (deformation driven) flow.

Here we are interested in susy breaking states in the CFT → restrict to domain walls for which the holographic analysis indicates vevs rather than deformation parameters which explicitly break susy.
In 2006 ISS found metastable susy breaking vacua of certain $\mathcal{N} = 1$ theories, and argued that such vacua were generic.

At the same time, susy breaking domain walls are also generic; fake susy ensures that these are also (meta)stable.

So why was it so difficult to find string realizations of ISS?
Holography currently not applicable for original ISS examples, $SU(N_c)$ SQCD with $N_f > N_c$ massive flavors; brane constructions resulted in theories which in the UV were deformations of the original SQCD theory.

Here a key difference is that we do not insist on a specific susy theory in the UV; we allow asymptotically conformal and large $N$ theories.

In this framework, metastable susy breaking domain walls are likely to be generic, holographically realizing the ISS scenario.
Outline

1. Review of general gauge mediation
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An example: dilaton domain wall

- An illustrative example which captures almost all features of interest is the dilaton domain wall.
- Starting from an action with constant potential, $V = V_0$, the defining equation for $W$ is

$$W' = \pm \frac{d\kappa^2}{d - 1} \left[ W^2 - \frac{(d - 1)^2}{\kappa^4} \right]^{1/2}.$$  

One solution is $W = (d - 1)/\kappa^2$ which leads to the $AdS_{d+1}$ solution, but the general solution for $W$ is

$$W = \frac{(d - 1)}{\kappa^2} \cosh \left( \kappa \sqrt{\frac{d}{d - 1}} (\phi - \phi_o) \right),$$

where $\phi_o$ is an integration constant.
The first order equations then admit a solution:

\[ \phi = \phi_o + \sqrt{\frac{(d-1)}{d\kappa^2}} \ln \left[ \frac{1 - e^{-d(r-r^*)}}{1 + e^{-d(r-r^*)}} \right] ; \]
\[ A = r + \frac{1}{d} \ln(1 - e^{-2d(r-r^*)}) , \]

Here \( r^* \) is an integration constant, such that \( M = e^{r^*} \) will characterize the susy breaking.

The solution is manifestly asymptotically \( AdS_{d+1} \) as \( r \to \infty \) but there is a curvature singularity at \( r = r^* \).

Fortunately the correlators we compute are well-defined despite this curvature singularity (just as for Coulomb branch flow, GPPZ).
We now need to demonstrate that:

1. The gravity and scalar action can be embedded into a (gauged) supergravity action.
2. The dilaton domain wall is a non-supersymmetric metastable solution of the (gauged) supergravity theory, i.e. $W \neq \mathcal{W}$.
3. The domain wall corresponds to a susy breaking state in the dual CFT.
4. The gauged supergravity theory contains fields dual to current superfields in the CFT; recall that it is correlators of the latter which we wish to compute.
The dilaton domain wall is easily embedded into type IIB supergravity, compactified on a Sasaki-Einstein $X_5$:

$$ds^2 = (dr^2 + e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu) + ds^2_{SE};$$

$$F_5 = \frac{N \sqrt{\pi}}{2V} (\eta_{SE} + *\eta_{SE}),$$

with $\phi = \phi(r)$.

With $(A(r), \phi(r))$ as before, this is a solution for all Sasaki-Einstein; note that the isometries of the Sasaki-Einstein are unbroken.

This solution was one of the earliest holographic models for QCD (Gubser,... but has too many undesirable features).
Susy breaking

- The solution is not supersymmetric: this follows eg from the dilatino supersymmetry transformation which is:

\[ \delta \chi = \frac{i}{2} \gamma^M \partial_M \phi \epsilon^* + \cdots \]

where the ellipses denote terms vanishing in this background. Clearly \( \delta \chi \neq 0 \).

- Holographic dictionary \( \rightarrow \) susy breaking state in QFT:

\[ \langle O_\phi \rangle = \frac{4}{\kappa} \sqrt{3} M^4; \quad \langle T_{\mu\nu} \rangle = \frac{2}{\kappa} \sqrt{3} M^4 \eta_{\mu\nu}, \]

so as anticipated \( M \) parameterizes susy breaking.
Recall the goal is to compute correlators of the global currents $j^{(r)}_\mu$ in a strongly coupled hidden sector.

At the conformal point, the global currents $j^{(r)}_\mu$ are dual to the KK gauge fields on $AdS_5 \times X_5$; the $\Delta = 2$ operators $J^{(r)}$ and $\Delta = 5/2$ operators $j^{(r)}_{\alpha}$ are dual to certain KK scalars and fermions respectively.

The dual sugra fields are contained in 5d gauged sugra → convenient to truncate to this theory, though not necessary\(^3\).

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\(^3\)Correlators can be computed directly from 10 dimensions using Kaluza-Klein holography KS + MT, 2006.
Compactification on a generic $X_5 \to \mathcal{N} = 2$ gauged sugra theory with $N_v$ vector multiplets and $N_h$ hypermultiplets.

Here we are interested in $X_5$ such that $N_v \geq 1$, so that the dual CFT has at least a $U(1)$ flavor symmetry, which will be weakly gauged.
Fluctuation equations

- Fields dual to currents are contained in the vector multiplets, whilst the dilaton is a zero mode of the hypermultiplet space.

- This implies that the linear equations of motion for the sugra fields of interest are free equations in the domain wall background, i.e.

\[
\begin{align*}
    j^{(r)}_{\mu} & \rightarrow D_{\mu} F^{\mu \nu (r)} = 0; \\
    j^{(r)}_{\alpha} & \rightarrow \gamma^\mu D_\mu \psi^{(r)} = \frac{1}{2} \psi^{(r)}; \\
    J^{(r)} & \rightarrow \square s^{(r)} = 4 s^{(r)}. \end{align*}
\]

where \( r \) denotes the flavor gauge group.
For example, for the scalar field $s$ dual to dimension two scalar operator $\mathcal{J}$, the bulk equation of motion is:

$$\left(\partial_y^2 + \coth(y) \partial_y - \frac{q^2}{\sqrt{\sinh(y)}} + \frac{1}{16}\right) \tilde{s}(y, q) = 0,$$

where $y = 4(r - r^*)$ and $q^2 \equiv k^2/M^2$ is momentum.

Equation has regular singular point at $y = 0$ (spacetime singularity), but scaling behavior indicates no analytic solution in terms of hypergeometrics → numerics, matched asymptotic expansions.

Exact regular solutions obtained analytically at $q = 0$ and $q \to \infty$. 
Expanding the regular solutions near $AdS$ boundary:

\begin{align*}
\tilde{s}(r, q) &= r e^{-2r} \tilde{s}_1(r, q) + e^{-2r} \tilde{s}_2(r, q); \\
\tilde{s}_1(r, q) &= \left( \tilde{s}_0(q) + e^{-2r} \tilde{s}_2(q) + \cdots \right); \\
\tilde{s}_2(r, q) &= \left( \tilde{S}_0(q) + \cdots \right),
\end{align*}

the holographic one point function of the operator $\mathcal{J}$ dual $\tilde{s}_0$ is

\[ \langle \mathcal{J} \rangle = -2\pi^2 \tilde{S}_0(q), \]

and thus two point function is obtained by differentiating again wrt source using regular solution of linearized equation.
Sugra action proportional to $N^2 \rightarrow$ usual holographic (BPS) operator normalization such that all correlation functions scale as:

$$\langle \mathcal{O}_h \mathcal{O}_h \cdots \rangle \propto N^2.$$

But appropriate normalization here is such that two point function in UV is unit normalized, i.e.

$$\langle \mathcal{I}(x) \mathcal{I}(0) \rangle_{x \to 0} = \mathcal{R} \left( \frac{1}{|x|^4} \right),$$

otherwise integrating out large $N$ hidden sector ill-defined.
Explicit forms of two point functions give:

\[ \tilde{B}_{1/2}(0) = \pi^2 \]
\[ \tilde{C}_0(0) = -\sqrt{2}\pi^2; \quad \tilde{C}_0(q^2)_{q\to\infty} = 2\pi^2(\gamma + \ln(q/2)); \]

etc with soft masses:

\[ m_r = \pi^2 g_r^2 \mathcal{M} \quad m_f^2 \approx 10^3 g_r^4 \mathcal{M}^2. \]

Recall \( g_r \) is the visible sector coupling.

The mass dependence is as expected for a one-scale (\( \mathcal{M} \)) hidden sector model, i.e. fixed by dimensional analysis.
One might worry that using a large $N$ hidden sector would induce large changes in visible sector beta function.

Wavefunction renormalization by hidden sector indeed induces

$$\Delta b^{(r)} = -\frac{N^{(r)}}{2\pi^2}; \quad \tilde{C}_0^{(r)}(q)_{q \to \infty} \to -N^{(r)} \ln(q)$$

but given unit normalization in UV $\Delta b^{(r)} = -1$.

Perturbativity up to GUT scale requires $|\Delta b| < 10$, satisfied here.
With "holographic" operator normalization, natural hierarchy of gaugino and squark masses:

\[ \frac{m_r^2}{m_f^2} \sim \frac{\langle O_h O_h \rangle^2}{\langle O_h O_h \rangle} \sim N^2 \]

but large change in visible sector beta function:

\[ \Delta b \sim -N^2, \]

so non-perturbative below GUT scale.

Worse: perturbatively integrating out hidden sector ill-defined.
More general hidden sectors

Next one could look for domain walls realizing:

1. **Multi-scale** hidden sectors, i.e. several distinct mass scales.

2. Hidden sectors with **broken R symmetry**.

Note that $AdS$ solutions could be used for strongly coupled hidden sectors in *Georgi’s* unparticle physics; also holographic realization of *Luty’s* squirks etc.
Complete model of gauge mediation requires the Higgs sector couplings, for which many models fall into the classes discussed by Komargodski and Seiberg:

\[
\int d^2 \theta (\lambda_u H_u \Phi_d + \lambda_d H_d \Phi_u); \\
\int d^2 \theta \lambda^2 S H_u H_d.
\]

The first class couples $SU(2)$ doublet hidden sector operators $\Phi$ to the Higgs superfields in the MSSM whilst the second class couples an $SU(2)$ singlet $S$ from the hidden sector.
Integrating out the hidden sector leads to effective Lagrangian containing:

\[
\int d^2 \theta (\mu \mathcal{H}_\mu \mathcal{H}_d) + \cdots
\]

\[
m_u^2 H_u H_u^\dagger + m_d^2 H_d H_d^\dagger + B_\mu H_u D_d + \cdots
\]

For satisfactory electroweak symmetry breaking one needs

\[
\mu^2 \sim B_\mu \sim m_{u,d}^2.
\]

and the difficulty in generating such terms from gauge mediation is the \(\mu/B_\mu\) problem; typically \(\mu^2 \ll B_\mu\).
To leading order when couplings $\lambda \ll 1$, these terms are given by two point functions of hidden sector operators, e.g. in doublet coupling

$$
\mu = \frac{i}{2} \lambda_u \lambda_d \langle \psi_{\phi_u} \psi_{\phi_d} \rangle (k = 0)
$$

$$
B_{\mu} = -i \lambda_u \lambda_d \langle F_{\phi_u} F_{\phi_d} \rangle (k = 0).
$$

and so can naturally be incorporated into our set up.
In our one scale holographic model however one necessarily finds

$$\langle \psi_{\phi_u} \psi_{\phi_d} \rangle (k = 0) \sim \mathcal{M}; \quad \langle F_{\phi_u} F_{\phi_d} \rangle (k = 0) \sim \mathcal{M}^2,$$

and hence (as in Komargodski, Seiberg)

$$\mu^2 \sim \lambda_u \lambda_d B_\mu \ll B_\mu,$$

so $\mu/B_\mu$ problem.

But can we perhaps use large $N$ to find novel resolutions of the $\mu/B_\mu$ problem? I.e. use large $N$ to enhance $\mu$?
In this talk we have discussed holographic modeling of hidden sectors in phenomenology, particularly in the context of gauge mediation.

Such strongly coupled hidden sectors widen the class of known models and may potentially lead to new mechanisms for realizing phenomenologically desirable features.

Note that these new mechanisms would not necessarily rely on holographic realization or strong coupling.
In this talk we have discussed holographic modeling of hidden sectors in phenomenology, particularly in the context of gauge mediation. Such strongly coupled hidden sectors widen the class of known models and may potentially lead to new mechanisms for realizing phenomenologically desirable features. Note that these new mechanisms would not necessarily rely on holographic realization or strong coupling.
Concluding remarks

- In this talk we have discussed holographic modeling of hidden sectors in phenomenology, particularly in the context of gauge mediation.
- Such strongly coupled hidden sectors widen the class of known models and may potentially lead to new mechanisms for realizing phenomenologically desirable features.
- Note that these new mechanisms would not necessarily rely on holographic realization or strong coupling.
Outlook

Systematically scan the possible holographic hidden sectors and use them to construct phenomenologically viable gauge mediated models.