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Non-singular String Vacua in the Early Universe with Massive Boson–Fermion Degeneracy

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Work Based on: C. Kounnas; arXiv:0808.1340 Fortsch.Phys.
I. Florakis and C. Kounnas; arXiv:0901.3055 Nucl.Phys.
also on:

C. Angelantonj, H. Partouche, N. Toumbas; arXiv:0808.1357 Nucl.Phys.
F. Bourliot, C. Kounnas, H. Partouche; arXiv:0902.1892 Nucl.Phys.
T. Catelin-J, C. K., H. P., N.T.; arXiv:0710.3895, arXiv:0901.0259 Nucl.Phys.
C. Kounnas, N. Toumbas, J. Troost; arXiv:0704.1996 JHEP

1. Introduction

In stringy gravity and cosmology new interesting phenomena occur.
Conventional notions from general relativity like :

Geometry and Topology

are well defined only as *low energy and/or small curvature approximations* of the stringy setup.

• At very small distances and at strong curvature scales, purely stringy phenomena imply that the physics can be quite different from what one might expect from the "naive" field theory approximation.

• New possibilities in the context of quantum cosmology and especially in the context of the "Stringy Big-Bang" picture

versus

"the initial singularity of the Big-Bang picture in General Relativity" .

Assuming for instance a compact space and sufficiently close to the singularity, the typical scale of the universe reaches at these early times the gravitational scale M_{string}.

At this early epoch classical gravity is no longer valid and has to be replaced by a more fundamental singularity-free theory such as (super-)string theory.

- The main obstruction in the stringy cosmological framework is
  the Hagedorn temperature limitation $T < T_H$.

It is well known that for high temperatures, $T > T_H$,
the string partition function diverges

A thermal winding state becomes tachyonic.
However, this is not a pathology in string theory.
It is a signal of a phase transition towards to a new vacuum.

Many proposals were made about the *"High Temperature Phase of the Universe"*.

The Hagedorn-like singularities have to be resolved :

• by a stringy phase transition

OR

• by choosing Hagedorn-free string vacua in the early stage of the universe.

It is of fundamental importance to show that :

• the space of Hagedorn-free vacua is not empty

and that

• their existence is *at least equally natural* as the Hagedorn - singular ones.

• A noticeable progress has been made in constructing Hagedorn-free string vacua which are characterized by the presence of non-trivial magnetic fluxes.
(see the talk given by N. Toumbas in this meeting).

• Stringy vacua with a "Massive boson-fermion Spectrum-Degeneracy Symmetry, $MSDS$" are proposed to describe the early *"Stringy non-geometric era"* of the universe.

2. The Maximally Symmetric $MSDS$ -vacua

The proposed $MSDS$-vacua have at least 8 extremely small compact dimensions, close to the string scale. $\longrightarrow$ $d \leq 2$ target space

Their connection to the "higher-dimensional universes " in late cosmological times is achieved via large marginal deformations of current-current type : $M_{IJ}\ J_L^I \times J_R^J$

The large M_{IJ}-deformation limit *"induces an effective higher-dimensional space"*

In this limit one recovers a geometric field theory description in terms of an effective "higher-dimensional" conventional superstring theory

- The space-time supersymmetry appears to be *spontaneously broken via*

"Geometrical" and "Thermal" fluxes.

In the most symmetric $MSDS$-vacua all compact space coordinates are expressed in terms of free 2d-world-sheet fermions rather than the conventional compact bosonic coordinates.

The advantage of this fermionization lies in the consistent separation of left- and right-moving world-sheet degrees of freedom into terms of left- and right-moving 2d-fermions that permit easier manipulations of the left-right asymmetric (and even non-geometrical) constructions of vacua in string theory.

Type II degrees of freedom

In the "critical" Type II theories the left- and right- moving degrees of freedom are:

- The light-cone degrees: $(\partial X^0,\ \Psi^0)$, $(\partial X^L,\ \Psi^L)$
- The super-reparametrization ghosts: (b, c), (β, γ)
- The transverse super-coordinates: $(\partial X^I \equiv i y^I w^I,\ \ \Psi^I),\ I = 1, ...8$

The transverse super-coordinates $(\partial X^I,\ \Psi^I)$ are replaced by (y^I, w^I, Ψ_I) so that for every $I = 1, ..., 8$, $\{y^I, w^I, \Psi^I\}$ define the adjoint representation of a $SU(2)_{k=2}$.

In a more general fermionization the transverse super-coordinates are replaced by a set of free fermions in the adjoint representation of a semi-simple gauge group H :

$$\{\chi^a\},\ a = 1, ...n,\ \ n = \dim[\,H\,] = 24$$

The simplest choice of H is: $\quad H = SU(2)^8$

Other non-trivial choices of fermionization are also possible:

$$H = SU(5),\ \ H = SO(7) \times SU(2),\ \ H = G_2 \times Sp(4),$$
$$H = SU(4) \times SU(2)^3,\ \ H = SU(3)^3.$$

For simplicity I will restrict to the choice $H = SU(2)^8$ for both left- and right- moving transverse degrees of freedom.

Heterotic degrees of freedom

- The left-moving sector is identical to that of Type II theories.

The right-moving degrees of freedom are:

- The light-cone degrees: $(\partial X_0,\ \partial X_L)$
- The reparametrization ghosts: (b, c)
- The transverse coordinates: $(\partial X^I,\ I = 1, ...8)$
- The extra 32 right-moving fermions $(\psi^A,\ A = 1, 2, ...32)$

In total there are 48 free fermions in the right moving sector $\{\bar{\chi}^a,\ a = 1, 2, ...48\}$

i) 16 $(y^I, w^I,\ I = 1, 2, ..., 8)$ from coordinate fermionization $i\partial X^I = y^I w^I$

ii) extra 32 right-moving fermions $(\psi^A,\ A = 1, 2, ...32)$, for the anomaly cancelation.

The basic left- and right-moving chiral operators and partition functions

In both Type II and Heterotic theories the left-moving T_B and T_F have the same form:

$$T_B = -\frac{1}{2}(\partial X_0)^2 - \frac{1}{2}\Psi_0\partial\Psi_0 + \frac{1}{2}(\partial X_L)^2 + \frac{1}{2}\Psi_L\partial\Psi_L \; + \; \sum_{a=1}^{24} \frac{1}{2}\, \chi^a \partial \chi^a$$

$$T_F = i\partial X_0 \Psi_0 \; + \; i\partial X_1 \Psi_1 \; + \; \sum_{a,b,c} f_{abc}\, \chi^a \chi^b \chi^c \; ,$$

f_{abc} are the structure constants of the group $H_L = SU(2)^8$ and $\{\chi^a\}$ $(a = 1, 2, ..., 24)$

The heterotic right-moving $\bar{T}_B(\bar{z})$:

$$\bar{T}_B = -\frac{1}{2}(\bar{\partial} X_0)^2 + \frac{1}{2}(\bar{\partial} X_L)^2 + \sum_{a=1}^{48} \frac{1}{2}\, \bar{\chi}^a \bar{\partial} \bar{\chi}^a \, .$$

Following the rules of the fermionic construction

i) $H_L \times H_R = SU(2)^8 \times SU(2)^8$ in type II

ii) $H_L \times H_R = SU(2)^8 \times SO(48)$ in the heterotic

iff the choice of boundary conditions respects the global existence of the $H_L \times H_R$ symmetry, then the latter is promoted to a local gauge symmetry on the target space-time, both in Type II and the Heterotic

- We can construct very special tachyon free vacua, with left–right holomorphic factorization of the partition function.

In terms of the $SO(2n)$ characters ($n = 12$ or $n = 24$) :

$$V_{2n} = \frac{\theta_3^n - \theta_4^n}{2\eta^n}, \qquad O_{2n} = \frac{\theta_3^n + \theta_4^n}{2\eta^n}, \qquad S_{2n} = \frac{\theta_2^n - \theta_1^n}{2\eta^n}, \qquad C_{2n} = \frac{\theta_2^n + \theta_1^n}{2\eta^n}\,,$$

Type II and Heterotic partition functions :

$$Z_{II} = \int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}\tau)^2} \; \left(V_{24} \; - \; S_{24}\right) \left(\; \overline{V}_{24} - \overline{S}_{24} \; \right)$$

$$Z_{Het} = \int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}\tau)^2} \; \left(V_{24} \; - \; S_{24}\right) \left(\; \overline{O}_{48} + \overline{C}_{48} \; \right) .$$

• The expression for Z_{II} remains the same for any choice of left- and right-moving H-group H_L, H_R, since the dimension of each is always equal to 24. In this respect, Z_{II} is a unique tachyon-free partition function (modulo the chirality of the left- and right-spinors) *that respects the* $H_L \times H_R$ *gauge symmetry*.

• The expression of the left-moving part in Z_{Het} remains the same as well. The right-moving part, depends on the choice of H_R:

$$H_R \;\; \longrightarrow \;\; SO(48), \qquad E_8 \times SO(32), \qquad E_8^3$$

• Both Z_{II} and Z_{Het} show a Massive Spectrum Degeneracy Symmetry.
This spectacular property reflects the relations between the characters of the $SO(24)$:

$$V_{24} \; - \; S_{24} = \text{constant} = 24 \; .$$

This follows from the well-known Jacobi identities between theta functions:

$$\theta_3^4 - \theta_4^4 - \theta_2^4 = 0, \qquad \theta_1^4 = 0, \qquad \theta_2\theta_3\theta_4 = 2\eta^3,$$

$$\longrightarrow \qquad \frac{\theta_3^{12} - \theta_4^{12}}{2\eta^{12}} - \frac{\theta_2^{12} - \theta_1^{12}}{2\eta^{12}} = 24$$

• The spectrum of massive bosons and fermions is identical to all string mass levels!
This is similar to the analogous property of supersymmetic theories.

• In the massless level, however, although there are 24 left-moving bosonic degrees of freedom there are no massless fermionic states.

• In Type II there are 24 right-moving bosonic states as well, so in total there are 24×24 scalar bosons at the massless level transforming under the adjoint of $H_L \times H_R$.

• The integrated type II partition function :

$$Z_{II} = \frac{\pi^2}{3}\, d(H_L) \times d(H_R), \qquad \mathcal{I} = \int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}\tau)^2} = \frac{\pi^2}{3}$$

• In the Heterotic the left-moving sector gives constant contribution as in the Type II ($d(H_L) = 24$). The right-moving massive states are expressed in terms of the unique holomorphic modular invariant function $j(\tau)$:

$$Z_{Het} = \int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}\tau)^2}\, d(H_L) \times \{d(H_R) + [j(\bar{\tau}) - 744]\} = \frac{\pi^2}{3}\, d(H_L) \times d(H_R)$$

• The contribution of the anti-holomorphic function $[j(\bar{\tau}) - 744]$ vanishes when integrated over the fundamental domain.

• The final integrated expression for both Z_{Het} and Z_{II} are proportional to the number of massless states of the models.

$$Z = \frac{\pi^2}{3}\, d(H_L) \times d(H_R).$$

Depending on the choice of H_R in the Heterotic, the number of the massless states is :

$$d(H_L) \times d[SO(48)] = 24 \times 1128$$

$$d(H_L) \times d[E_8 \times SO(32)] = 24 \times 744$$

$$d(H_L) \times d[E_8^3] = 24 \times 744$$

• The massive boson-fermion degeneracy symmetry of the $MSDS$-vacua is not an accidental property of the above constructions.
It follows from the existence of a new superconformal symmetry.

3. Chiral superconformal algebra and spectral flow in $MSDS$

The symmetry operators of the $MSDS$ vacuum are the usual holomorphic (anti-holomorphic) operators $T_B,\ T_F\ (\bar{T}_B,\ \bar{T}_F\ )$ giving rise to the standard $\mathcal{N} = (1,1)$ world-sheet superconformal symmetry in type II and the $\mathcal{N} = (1,0)$ in the heterotic

The extra symmetry operators are the currents of conformal weight $h_J = 1$, associated with the H_L- and H_R-affine algebras:

$$J^a \equiv f^a_{bc}\,\chi^b\chi^c \qquad \text{and} \qquad \bar{J}^a \equiv \bar{f}^a_{bc}\,\bar{\chi}^b\bar{\chi}^c$$

Furthermore, there are two $SO(24)$ spin-field operators with conformal weight $\frac{3}{2}$ and opposite chirality :

$$C = Sp\{\chi^a\}_+ \qquad \text{and} \qquad S = Sp\{\chi^a\}_-$$

The existence of the chiral operator C, of conformal weight $h_C = \frac{3}{2}$, together with T_B, T_F, J^a, χ^a, form a *new chiral superconformal algebra* implying the massive boson-fermion degeneracy of the spectrum.

One needs to utilize the OPE relations between C and S:

$$C(z)\ C(w) \sim \frac{1}{(z-w)} \left\{ \frac{\mathbf{1}}{(z-w)^2} + \frac{\hat{\chi}\hat{\chi}}{(z-w)} + \ \ldots \right\},$$

$$S(z)\ S(w) \sim \frac{1}{(z-w)} \left\{ \frac{\mathbf{1}}{(z-w)^2} + \frac{\hat{\chi}\hat{\chi}}{(z-w)} + \ \ldots \right\},$$

$$C(z)\ S(w) \sim \frac{1}{(z-w)^{\frac{1}{2}}} \ \left\{ \frac{\hat{\chi}}{(z-w)^2} + \frac{\partial\hat{\chi} + \hat{\chi}\hat{\chi}\hat{\chi}}{(z-w)} + \ \ldots \right\},$$

$\hat{\chi} \equiv \gamma^a \chi_a \ \ , \ \ \gamma^a \rightarrow \gamma$-matrices of $SO(24)$

$C(z)S(w)$ OPE implies a boson-fermion Spectral Flow which guaranties the massive boson-fermion degeneracy of the Vacuum.

Spectral flow and the $MSDS$ operator-relations

The vertex operators are dressed by the super-reparametrization ghost Φ:

$$e^{q\Phi} \quad \longrightarrow \quad \text{with} \qquad h_q = -\frac{1}{2}\, q\, (q+2)$$

- Space-time boson vertices are expressed either in the 0 or the (-1) ghost picture.

$$\mathbf{V}_{(\mathbf{0})} \; = \; e^{-\Phi}\, \hat{\chi}, \qquad\qquad \mathbf{V}_{(\mathbf{1})} \equiv e^{-\Phi}\, (\, \partial\hat{\chi} \; + \; \hat{\chi}\, \hat{\chi}\, \hat{\chi}\,)$$

$\mathbf{V}_{(\mathbf{0})}, \mathbf{V}_{(\mathbf{1})}$ have conformal weight $h_0 = 1, h_1 = 2$.

The string spectrum of bosons starts from a massless sector which is described by $\mathbf{V}_{(\mathbf{0})}$.

$\mathbf{V}_{(\mathbf{1})}$ define a massive bosonic vertex at mass level 1.

• Space-time fermions are in the $(-\frac{1}{2})$ or $(-\frac{3}{2})$ pictures.

$$\mathbf{S} \;=\; e^{-\frac{1}{2}\Phi-\frac{1}{2}iH_0}\, S \qquad \text{or} \qquad \mathbf{S} \;=\; e^{-\frac{3}{2}\Phi+\frac{1}{2}iH_0}\, S$$

H_0 is the usual helicity field defined via bosonization $i\partial H_0 = \Psi_0 \Psi_L$.

$\mathbf{S}$ has weight $h_S = 2$ in both the $(-\frac{1}{2})$ and $(-\frac{3}{2})$ pictures
$\longrightarrow$ all space-time fermions are massive, starting from mass level 1.

The flow of $\mathbf{V}_{(\mathbf{0}),(\mathbf{1})}$ states to $\mathbf{S}$-states is expressed by the action of a "Spectral-flow operator" $\mathbf{C}$:

$$\mathbf{C} \;\equiv\; e^{\frac{1}{2}(\Phi - iH_0)}\, C \ .$$

$\mathbf{C}$ is written in the $(+\frac{1}{2})$ ghost picture. It has conformal dimension $h_{\mathbf{C}} = 1$ and $(-1/2)$ helicity charge.

$\mathbf{C}$ acting on "physical" bosonic states produces "physical" fermionic states at the same string level and vice-versa.

Although the $\mathbf{C}$-action looks like a space-time supersymmetry transformation, the actual situation turns out to be drastically different.

The $\mathbf{C}$-action leaves the massless bosonic states of the theory invariant $\longrightarrow$ the boson-to-fermion mapping does not exist for the massless states.

$$\mathbf{C}(z)\ \mathbf{V}_0(w)\ \sim\ \mathbf{S}\,,\quad \text{finite as } z \rightarrow w.$$

The absence of singular terms in $(z-w)$ shows clearly that the massless states are invariant under the $\mathbf{C}$-transformation.

$\mathbf{C}$ acts non-trivially on the massive states:

$$\mathbf{C}(z)\ \mathbf{V}_{\mathbf{1}}(w)\ \sim\ \frac{\mathbf{S}(w)}{(z-w)}\ +\ \text{finite terms.}$$

$$\mathbf{C}(z)\ \mathbf{S}(w)\ \sim\ \frac{\mathbf{V}_{(\mathbf{1})}(w)}{(z-w)}\ +\ \text{finite terms}$$

massive bosonic states are mapped into the *massive fermionic states* and vise-versa.

4. Marginal deformation of the $MSDS$ vacua

The massless states of $MSDS$-vacua are $d_L \times d_R$ scalars parametrizing a manifold similar to that gauged supergravities with $G = H_L \times H_R$:

$$\mathcal{K} = \frac{SO(d_L,\ d_R)}{SO(d_L) \times SO(d_R)}.$$

The non-abelian structure of $H_L \times H_R$ implies that the only marginal deformations are those that correspond to the Cartan sub-algebra.

The moduli space of these deformations: $M_{IJ}\ J_L^I \times J_R^J$ is reduced to:

$$\mathcal{M} = \frac{SO(r_L,\ r_R)}{SO(r_L) \times SO(r_R)}\ .$$

The maximal number of the moduli M_{IJ} is when:

$$H_L = H_R\ =\ SU(2)^8_{k=2}, \quad \text{with} \quad r_L = r_R = 8\,.$$

The deformed partition function factors out *a shifted lattice* $\Gamma_{8,8}(M)\left[{a,\bar a \atop b,\bar b}\right]$

The *shifted lattice* couples non-trivially to the "parafermion numbers" defined by the gauged WZW-cosets :

$$\prod_{I_L=1,\dots 8}\left(\frac{SU(2)_{k=2}}{U(1)}\right)_{I_L}\times\prod_{I_R=1,\dots 8}\left(\frac{SU(2)_{k=2}}{U(1)}\right)_{I_R}$$

For $k=2$, the above coset structure is equivalent to 8 left-moving world sheet fermions, Ψ_{I_L} and 8 right-moving ones, Ψ_{I_R} in type II.

At the end, the shifted lattice couples nontrivially to the R-symmetry charges of the conventional type II superstings: $\{a_I, b_I\ ;\ \bar a_I, \bar b_I\}$ of $\{\Psi_{I_{L,R}}\}$!

$$Z_{II}=\frac{1}{2}\sum_{a,b}\ (-)^{a+b}\,\frac{\theta[^a_b]^4}{\eta^{12}}\ \times\Gamma_{8,8}(M)\left[{a,\bar a \atop b,\bar b}\right]\times\frac{1}{2}\sum_{\bar a,\bar b}\ (-)^{\bar a+\bar b}\,\frac{\bar\theta[^{\bar a}_{\bar b}]^4}{\bar\eta^{12}}$$

• In the large moduli limit (modulo S, T, U-dualities), the $\Gamma_{8,8}$ lattice decompactifies and the correlations with the R-symmetry charges become irrelevant.
One recovers the conventional ten dimensional type II supersymmetric vacua !

• For large but not infinitely large deformations, the obtained vacua are those of "spontaneously broken supersymmetric vacua in the presence of geometrical fluxes".

• Euclidian versions of the models, correspond to "thermal stringy vacua" with non-trivial left-right asymmetric "gravito-magnetic fluxes".
(see talk of N. Toumbas in this conference)

• The would be "initial" classical singularity of general relativity as well as the stringy Hagedorn-like singularities are both resolved by these fluxes !

The above generic properties of the deformed $MSDS$ vacua, strongly suggest the following *Cosmological Conjecture*.

Cosmological Conjecture

The $MSDS$ vacua, or even less symmetric orbifold versions
are potential candidates able to describe
the early non-singular phase of a stringy cosmological universe

- During the cosmological evolution $M_{IJ} \to M_{IJ}(t)$ evolves with the time.
Once $M_{IJ}(t)$ are sufficiently large (modulo S, T, U-dualities)
an effective field theory description emerges with an induced
"space-time geometry" of an *"effective higher dimensional space-time"* .

- The relevant degrees of freedom and interactions are well described by some
"no-scale" gauged supergravity theories of the conventional superstrings.

The effects of the initial $MSDS$ structure induces at the large moduli limit non-trivial
"geometrical" fluxes which in the language of the effective supergravity give rise to a
spontaneous breaking of supersymmetry and to finite temperature effects.

5. Orbifold reduction of the $MSDS$ structure

The originally proposed $MSDS$-vacua and in particular the ones with $H_L \equiv SU(2)^8$, are too symmetric to be phenomenologically viable.

• In the extreme large-M deformation limit (decompactification limit), the induced effective theory is that of *non-chiral* extended gauge supergravities, implemented with a well-defined set of geometrical fluxes.

• From our cosmological viewpoint, the strongly deformed $MSDS$-vacua should consistently represent our late time universe $\longrightarrow$ It should contain:

• A non-trivial net number of chiral families

• A reduced gauge group unifying in the most realistic manner the standard model interactions

In collaboration with I. Florakis, we able to classify all possible $\mathbb{Z}_2^N$-(asymmetric) orbifolds with reduced $MSDS$ symmetry.

In all proposed models the massive boson and fermion degrees of freedom exhibit sector by sector (untwisted and twisted) Massive Spectrum Degeneracy Symmetry.

Sector by sector, the number of massless bosons $n_I(b)$ and massless fermions $n_I(f)$ are different; $n_I(b) \neq n_I(f)$.

These remarkable properties follow from shifted versions of θ^{12}-identity.

For instance in Z_2-orbifold:

$$\text{Untwisted sectors}: \qquad V_{16}O_8 - S_{16}C_8 = 16\,, \qquad O_{16}V_8 - C_{16}S_8 = 8$$

$$\text{Twisted sectors}: \qquad V_{16}C_8 - S_{16}O_8 = 0\,, \qquad O_{16}S_8 - C_{16}V_8 = 8$$

The spectral-flow operator C_{24}, is truncated by Z_2 :

$$Z_2 \;:\; C_{24} = C_{16}C_8 + S_{16}S_8 \;\longrightarrow\; C_{Z_2} = C_{16}C_8$$

The global existence of C_{Z_2}, along with the truncated chiral algebra, are sufficient to guarantee massive supersymmetry of the spectrum.

In heterotic orbifold $MSDS$-vacua the anti-holomorphic contributions to the partition function are also constant numbers modulo a part proportional to $\bar{j}(\bar{\tau})$.

$$Z_{het,A} = n_A + m_A\,[\,\bar{j}(\bar{\tau}) - 744\,]\,, \qquad n_A = n_A(b) - n_A(f)$$

- There is a plethora of reduced $MSDS$ orbifold vacua in the heterotic framework.

The classification rules are given in the work done in collaboration with I. Florakis.

• Among the heterotic $MSDS$ orbifolds are those which are connected via large moduli deformations to semi-realistic four-dimensional heterotic chiral models. For instance, those with: (see A. Faraggi, C. Kounnas and I. Rizos) $H_R = SO(10) \times U(1)^3 \times SO(16)$ gauge group and with non-zero chiral families.

A representative example of this class of $MSDS$-orbifolds is the one with:

i) Holomorphic partition function:

$$Z\left[{h_1 \atop g_1}\ {h_2 \atop g_2}\right] = \frac{1}{2}\sum_{a,b}(-)^{a+b}\,\frac{\theta\left[{a \atop b}\right]^6\,\theta\left[{a+h_1 \atop b+g_1}\right]^2\,\theta\left[{a+h_2 \atop b+g_2}\right]^2\,\theta\left[{a-h_1-h_2 \atop b-g_1-g_2}\right]^2}{\eta^{12}}\ ,$$

ii) Anti-holomorphic partition function:

$$\bar{Z}\left[{h_1 \atop g_1}\ {h_2 \atop g_2}\right] = \frac{1}{2^3\,\bar{\eta}^{24}}\sum_{\gamma,\delta}\bar{\theta}\left[{\gamma \atop \delta}\right]^5\,\bar{\theta}\left[{\gamma+h_1 \atop \delta+g_1}\right]\,\bar{\theta}\left[{\gamma+h_2 \atop \delta+g_2}\right]\,\bar{\theta}\left[{\gamma-h_1-h_2 \atop \delta-g_1-g_2}\right]\sum_{\epsilon,\zeta}\bar{\theta}\left[{\epsilon \atop \zeta}\right]^5\,\bar{\theta}\left[{\epsilon+h_1 \atop \zeta+g_1}\right]\,\bar{\theta}\left[{\epsilon+h_2 \atop \zeta+g_2}\right]\,\bar{\theta}\left[{\epsilon-h_1-h_2 \atop \zeta-g_1-g_2}\right]\sum_{\bar{a},\bar{b}}\bar{\theta}\left[{\bar{a} \atop \bar{b}}\right]^8$$

The full partition function can be written in a conventional *shifted and twisted* "$\Gamma_{8,8}$-lattice form" :

$$Z = \frac{1}{2^6\,\eta^{12}\bar{\eta}^{24}} \sum_{a,b,\gamma,\delta,h_i,g_i} (-)^{a+b}\,\theta[^a_b]\,\theta[^{a+h_1}_{b+g_1}]\,\theta[^{a+h_2}_{b+g_2}]\,\theta[^{a-h_1-h_2}_{b-g_1-g_2}] \times$$

$$\times\,\Gamma_{8,8}\left[\begin{matrix} a & , & \gamma \\ b & , & \delta \end{matrix}\,\middle|\,\begin{matrix} h_i \\ g_i \end{matrix}\right] \times \sum_{\gamma,\delta} \bar{\theta}[^\gamma_\delta]^5\,\bar{\theta}[^{\gamma+h_1}_{\delta+g_1}]\,\bar{\theta}[^{\gamma+h_2}_{\delta+g_2}]\,\bar{\theta}[^{\gamma-h_1-h_2}_{\delta-g_1-g_2}]\,\sum_{\bar{a},\bar{b}} \bar{\theta}[^{\bar{a}}_{\bar{b}}]^8\,,$$

$\Gamma_{8,8}\left[\begin{matrix} a & , & \gamma \\ b & , & \delta \end{matrix}\,\middle|\,\begin{matrix} h_i \\ g_i \end{matrix}\right]$ indicates the contribution of the eight fermionized coordinates

$$\{y^I, \omega^I \,|\, \bar{y}^I, \bar{\omega}^I\}$$

The $MSDS$-structure follows from the holomorphic side.
The full partition function for this representative example is :

$$Z = 12\bar{j}(\bar{\tau}) = 12\ \times 744 + 12\ [\,\bar{j}(\bar{\tau}) - 744\,]$$

Inserting in the representative model all possible discrete torsion coefficients permitted by the fermionic construction, a plethora of $MSDS$ Heterotic models can be obtained.

The resulting models will in general exhibit different bosonic and fermionic massless spectra in different representations of the chiral (right-moving) gauge group

$$H_R = SO(10) \times U(1)^3 \times SO(16)$$

The moduli space contains a subspace of would-be *geometrical M_{IJ}-deformations* associated with the conventional supersymmetric $Z_2 \times Z_2$ freely acting orbifolds.

The $Z_2 \times Z_2$ action reduces the deformation space:

$$Z_2{\times}Z_2 : \ \frac{SO(8,8)}{SO(8) \times SO(8)} \longrightarrow \frac{SO(4,4)}{SO(4) \times SO(4)} \times \frac{SO(2,2)}{SO(2) \times SO(2)} \times \frac{SO(2,2)}{SO(2) \times SO(2)}$$

Assuming very large deformations in the $(2,2)$ sub-space of $SO(4,4)$, a 4d flat space-time is generated, together with an internal 6-dimensional compact space described by

$$\frac{T^6}{Z_2 \times Z_2}$$

.

This class of models is connected with the 4d semi-realistic $N=1$ chiral vacua based on $SO(10)$. The $N=1$ supersymmetry appears broken spontaneously by very specific geometrical fluxes!

In the Euclidian version the deformed $MSDS$ correspond to "thermal stringy vacua" in the presence of non-trivial left-right asymmetric "gravito-magnetic fluxes".

The deformed integrated partition function becomes a non trivial function of the

- Temperature scale T
- SUSY breaking scale M
- All other moduli μ_I

6. Cosmological evolution in late times

i) Exit from $MSDS$ *era:*

Once the free energy is positive (negative pressure) the $MSDS$ vacuum is unstable
$\longrightarrow$ the moduli evolves towards larger values such that:

$$M, T << M_{string} \quad \longrightarrow \quad \text{Deformed } MSDS\text{-vacua at } t = t_{exit}.$$

• This transition will occur when $n(f) > n(b)$ so that the $MSDS$ partition function is negative.

ii) Intermediate cosmological era $t_{exit} \leq t \leq t_w$:

After the "$MSDS$ transition exit" $t \geq t_{exit}$
and before the electroweak symmetry breaking phase transition $t \leq t_w$

This cosmological phase was extensively studied in collaboration with:
F. Bourliot, and H. Partouche
T. Catelin-J, H. Partouche and N. Toumbas

We show that cosmological evolution is attracted to "radiation-like" evolution in an effective d-dimensional space-time :

$$RDS^d \; : \qquad M(t) \sim T(t) \sim 1/a(t) \sim t^{-2/d}, \quad \text{for} \quad t \geq t_{exit},$$

• This evolution is unique and stable at late times in certain physically relevant SUSY breaking schemes (structure of the fluxes).

Furthermore, for $T, M << M_{string}$ only the SUSY breaking moduli $M(t)$ and $T(t)$ can give a relevant contribution to the free energy $\mathcal{F}(T, M)$.

All other moduli, μ_I

• are attracted and stabilized to the extended gauge symmetry points, $\mu_I \sim M_{\text{string}}$

OR

• are effectively frozen to an arbitrary value such that $\mu_I \gg T, M$

In both cases, their contribution to $\mathcal{F}$ is exponentially suppressed.

$$\mathcal{F}(T, M, \mu_I) = \mathcal{F}(T, M) + \mathcal{O}\left[\exp(-\frac{\mu_I}{T}), \exp(-\frac{\mu_I}{M})\right]$$

Finally the limitation $t \leq t_w$ follows from the appearance of a new scale in low energies:
"The infrared renormalisation group invariant transmutation scale Q"
of the effective field theory.

At this scale the $(\text{mass})^2$ of the SUSY standard model Higgs becomes negative:

$\longrightarrow$ no-scale radiative breaking of $SU(2) \times U(1) \rightarrow U(1)_{\text{em}}$.

- Q is irrelevant when $M(t), T(t) \gg Q$

- Q becomes relevant and stops the evolution of $M(t)$ when $M \sim Q$ at $t \sim t_w$ and the electroweak breaking phase transition takes place.

The physics for $t \gg t_w$ is of main importance in particle physics and cosmology.

- Unfortunately the infrared phase at $t \gg t_w$, depends strongly on the specific initial $MSDS$-vacuum data as well as on the specific evolution of the deformation moduli.

$\longrightarrow$ A lot of work is necessary to select the initial $MSDS$-vacuum that leads to the late-time precise structure of our universe.

On the other hand, I would like to stress that the qualitative infrared behavior of the effective "no-scale" field theory, strongly suggests that we are definitely in an interesting "non-singular string evolutionary scenario connecting particle physics and cosmology"