Fine-Tuning Problem(s) in Gauge Theory/Inflation – Higher-Dimensional Gauge Theory

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1. Introduction – Inflation model in Higher-Dim Gauge Theory

A. Summary of our work

Fine-tuning problems in particle physics and cosmology

i) Gauge hierarchy problem

ii) Parameters of the *inflaton potential*

We argue: The two may be solved by the same mechanism – higher-dimensional (SUSY) gauge theory.

$A_M = (A_\mu, A_y)$ $\quad$ $(4+n)D$ gauge field

$A_y^{(0)}$=Higgs h $\quad$ gauge-Higgs unification

$A_{y'}^{(0)}$=*inflaton* ϕ $\quad$ gauge-inflaton unification

Nice points:

⋆ Higgs/inflaton potential arises at 1-loop, hence,

Higgs mass m_h^2 is small. $\quad$ Inflaton potential $V(\phi)$ is "slow-roll".

$\star$ Our toy model reproduces the inflation parameters without tuning (to our surprise).

$$g^2/4\pi \simeq 0.05 \ \longleftrightarrow \ g^2_{\rm GUT} \simeq 0.02 - 0.05$$

$\not\star$ 1-loop potential is not affected by

other quantum effects, quantum gravity effects

$\because$ *gauge symmetry* in extra dim

B. Summary of Higgs/inflaton

Scalar fields play intriguingly important roles.

Higgs (makes mass), *inflaton* (makes universe)

Fine-tuning problem in Higgs potential

$V_H(h) = m^2 h^\dagger h + \lambda (h^\dagger h)^2$

$(\delta m^2)_{1-\mathrm{loop}} \sim O(\Lambda^2) \;\rightarrow\; m_h^2$ How?

A few alternative solutions

technicolor (PNGB), supersymmetry

higher-dim gauge theory

An analogus *fine-tuning* problem in inflaton potential is observed.

$V(\phi) = M_P^4 \Sigma_n \lambda_n (\phi/M_P)^n$, e.g., $\lambda_4 \sim 10^{-12}$, extremely small.

Question: Construct an inflation model which is free from fine-tuning.

A few attempts: slow-roll (new inflation) – Linde (81);

$V = a\phi^n$ (chaotic); $V(\phi_1, \phi_2)$ (hybrid)

PNGB (technicolor) – Freese et al (90)

SUSY (SUGRA)

extra-natural – Arkani-Hamed et al (03)

We propose: Inflaton ϕ arises from extra dim of gauge fields,

$A_y^{(0)} = \phi$.

2. Basics of inflaton

$\cancel{A}$. Why inflation?

$\cancel{\star}$ Historical: Problems remain in big-bang scenario, among others,
horizon, flatness problems

They are solved if inflation occurred.

$\star$ Recent context: Inflation may generate
irregularities (fluctuation in CMB) $\leftrightarrow$ various models of inflation

$\cancel{B}$. What is inflation?

During the inflation epoch,

$\rho \simeq \rho_{vac}$ vaccuum energy dominates

$a(t) \sim \mathrm{e}^{Ht}$ universe expands exponentially

$$H = \dot{a}/a \sim \sqrt{\rho_{vac}}$$

C. Inflaton potential

A small causally connected region of universe grows for a "long" time to solve the horizon/flatness problems.

$N \simeq H\Delta t \simeq 60$ e-foldings

Inflation occurs if universe is filled with a scalar field ϕ. $-$ *inflaton*

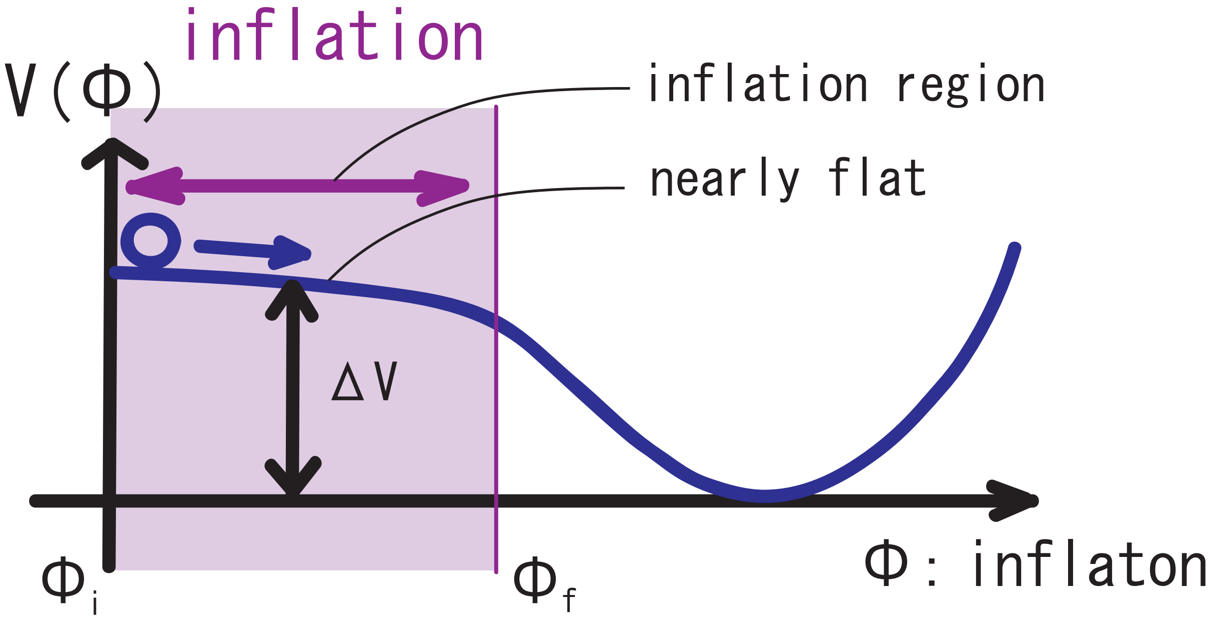
$\ddot{a}/a = -(\rho + 3P)/6M_P^2 > 0$ $\therefore$ $-$tive pressure

$\rho,\ P = \frac{1}{2}\dot{\phi}^2 \pm V(\phi), \quad \therefore V(\phi) > 0$

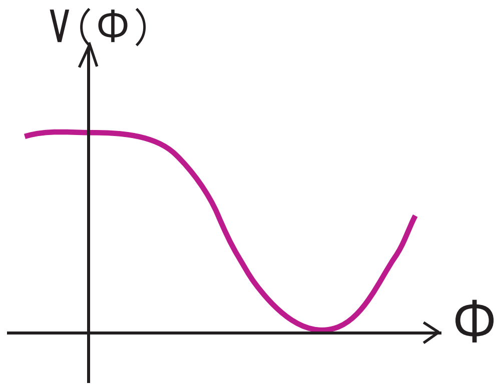
Inflaton potential $V(\phi)$ determines how inflation proceeds. It should be nearly flat to meet the data.$-$ "slow-roll" potential [Fig.1]

sufficiently long inflation period, $\Delta t \equiv t_f - t_i \simeq 60/H$

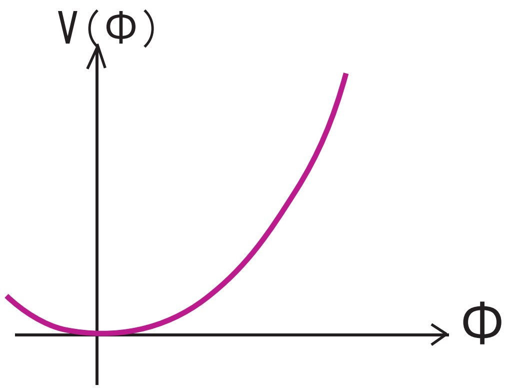
CMB anisotropy limit on density fluctuation, $\Delta V/(\Delta\phi)^4 \leq 10^{-6}$



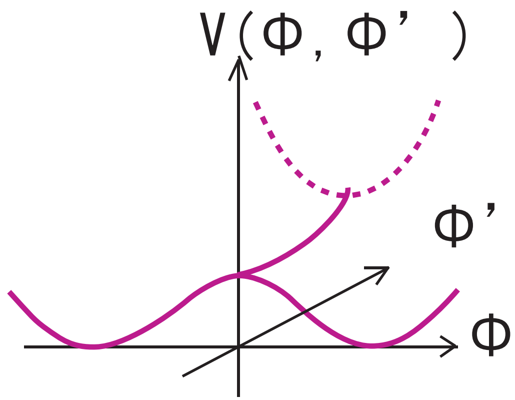
ii) new inflation



iii) chaotic



iv) hybrid



C. Fine-tuning problem

Astrophysics data demand

$V(\phi)$ slow-roll

$$\epsilon = [(V'/V)M_P]^2 \simeq 0.01, \quad \eta = (V''/V)M_P^2 \simeq 0.01$$

$$n_s = 1 - 6\epsilon + 2\eta, \quad 0.95 \le n_s \le 0.97$$

Past models fail to solve this fine-tuning problem.

We argue: ϵ and η are *naturally small* in the model of higher-dim gauge theory, for the same reason as in the gauge hierarchy problem.

3. Gauge-Higgs unification from higher dim

A. Higer-dim gauge theory   Antoniadis (90), ..., Hatanaka, Lm & Inami (98)

d-dim space-time: $X^M = (x^\mu, x^m)$; $\mu = 0, ..., 3,\ m = 5, .., d$

Gauge field $A_M = (A_\mu, A_m)$

$A_\mu^{(0)}$ gauge field

$A_m^{(0)}$ scalar $\rightarrow$ Higgs

~~B~~. Gauge hierachy problem

Higgs mass term gets huge corrections from one-loop effects.

$M^2 \sim \Lambda^2 \gg m_h^2$

Solution in higher-dim gauge theory:

At tree level, $M^2 = 0$ $\because$ *gauge symmetry* in extra dim

At one-loop $M^2 = \text{finite} \ll \Lambda^2$, i.e., solved!

4. Models of inflation

A. Models of inflation   Two approaches:

a) Write a "good functional form" of inflaton potential $V(\phi)$.

ii) new, iii) chaotic, iv) hybrid, ... [Fig.2]

b) Construct a "good model" of inflation from GUT, SUGRA, string, ...

$\longrightarrow$ our approach

B. Inflation parameters

From recent astrophysical data:

Temperature of the universe (at it's age of 390,000 years)

$\star$ Same temperature from all directions

$\longrightarrow$ horizon problem $\longrightarrow$ solved by inflation

$\star$ Tiny anisotropy at scales of $10' - 5^\circ$ $\longrightarrow$ constrains inflaton models

Parameters: spectral index n_s, curvature pert δ_H, ...

$\cancel{C}$. Fine-tuning problem − an example

Chaotic inflation

$$V \propto \phi^n \rightarrow \lambda M_P^4(\phi/M_P)^4 \quad \text{(inflation occurs for } \phi \geq M_P\text{)}$$

Comparison with curvature pert data

$$\delta_H = ...(\sqrt{\lambda}/M_P^3)\phi_*^3 = 2 \times 10^{-5} \text{ (observed value)}$$

$$\rightarrow \lambda \sim 10^{-12}, \quad \text{i.e., extreme fine-tuning necessary}$$

5. Model of $5D$ SUSY gauge theory on $M_4 \times S^1$ $-$ inflaton model

Assumption: $A_5^{(0)} =$ Higgs $h =$ inflaton ϕ

Aims and Questions:

a) Are two fine-tuning problems solved simultaneously.

Higgs/inflaton potential

b) Does SUSY play a role?

B. Model

$5D$ N=1 SYM theory (Gauge group $SU(2) \times$ global $SU(2)_R$)

Gauge multiplet

Vector A_M, Real scalar Σ, Majorana spinor λ_L^i $(i = 1, 2)$

Matter multiplet (fundamental)

Scalar H_i, Dirac spinor $\Psi = (\Psi_L, \Psi_R)^T$

Action (in component fields) Pomarol & Quiros (98)

$$L_{5,gauge} = \mathrm{Tr}[\tfrac{1}{2}(F_{MN})^2 + \cdots]$$

$$L_{5,mat} = |D_M H_i|^2 - m^2|H_i|^2 + \bar{\Psi}(i\gamma^M D_M - m)\Psi + \cdots$$

Compactification to $\underline{M_4 \times S^1}$ $x^M = (x^\mu, y)$, $y \sim y + L$ $(L = 2\pi R)$

Set boundary conditions.

$\lambda^i(x^\mu, y + L) = (e^{i\beta^a\sigma^a})^i{}_j \lambda^j(x^\mu, y)$, same for H^i

other fields are periodic

SUSY is broken *à* la Scherk-Schwarz (79). $\beta = \sqrt{(\beta^a)^2}$

Kaluza-Klein expansion:

$A_M(x, y) = \Sigma_{n=-\infty}^{\infty} A_M^{(n)}(x)\, \mathrm{e}^{iny/R}$, same for other fields

$\lambda^i(x, y) = \Sigma_{n=-\infty}^{\infty} \lambda^{i(n)}(x)\, \mathrm{e}^{i(n+i\beta^a\sigma^a/2\pi)y/R}$, same for H^i

$A_M \to 4D$ gauge field $A_\mu^{(0)}$, $4D$ scalar $A_5^{(0)}$ = Higgs = inflaton ϕ

C. Inflaton potential

Purpose: Compute effective potential for ϕ.

It is a standard technique to use Wilson line phase.

$$g_5 \int_0^L dy < A_5 >= g_5 L < A_5^{(0)} >= \mathrm{diag}(\theta, -\theta)$$

1-loop contributions (gauge multiplet and matter), periodic

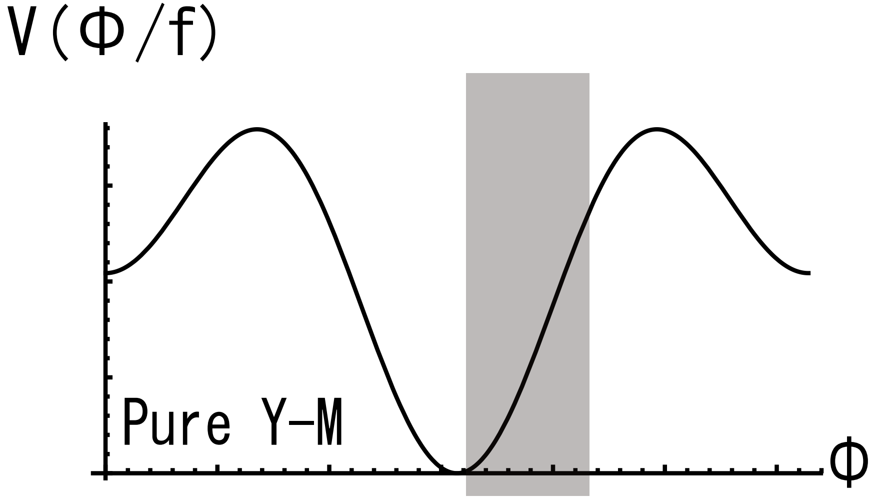
$$V^{gauge}(\theta) = - \cdots L^{-4} \Sigma_{n=1}^{\infty} n^{-5} [1 + \cos(2n\theta)](1 - \cos\beta) + \text{const}$$

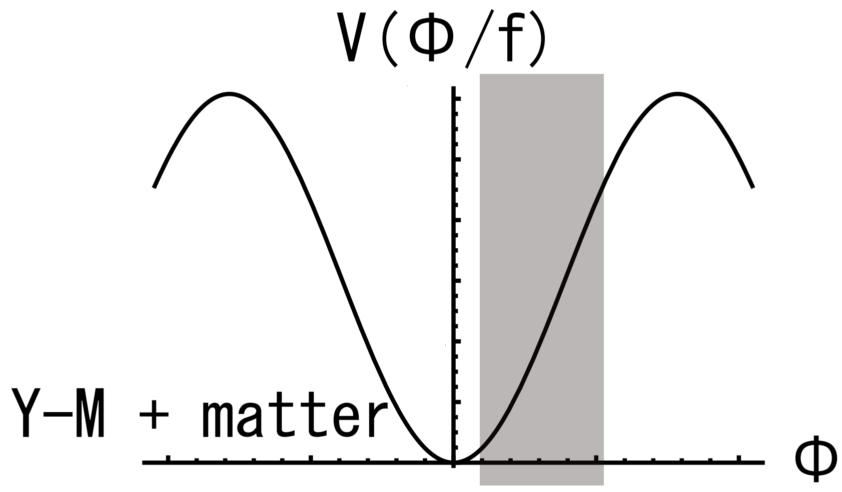
$$V^{mat}(\theta) = \cdots m^2 L^{-2} \cdots$$

Remarks: Divergent constant terms will be set to zero.

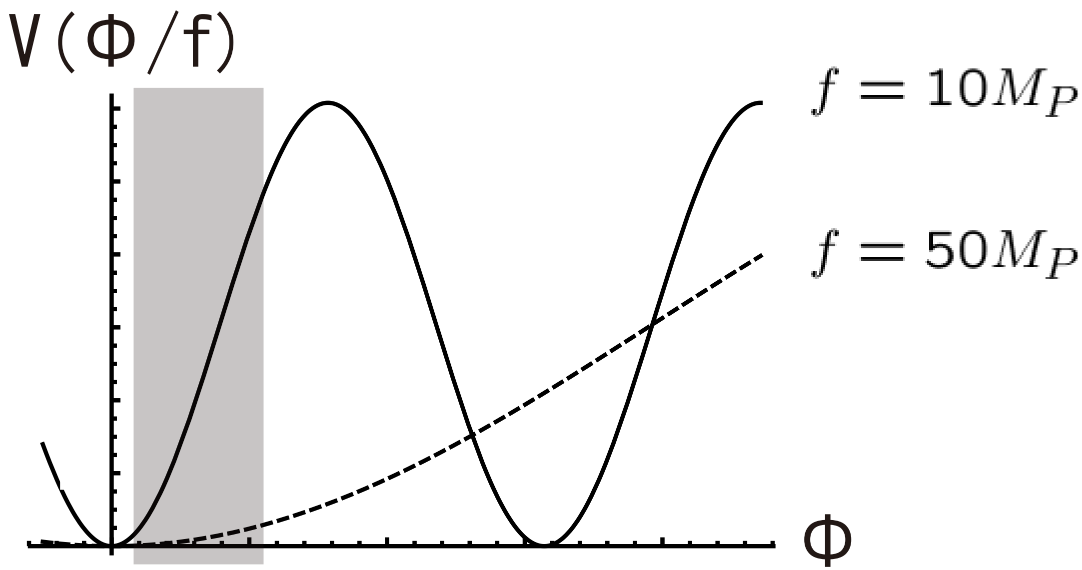
The $n = 1$ term is a good approximation. [Fig.3]

$$V^{gauge} = (6/\pi^2 L^4)(1 - \cos\beta)(1 - \cos(2\phi/f)) \quad (\propto \beta^2 \phi^2)$$





f dependence of potential



D. Constraints on potential from astrophysics data (and theory)

1) $\Lambda \simeq 0$

2) Slow-roll condition:

$$\epsilon \equiv \frac{1}{2}((V'/V)M_P)^2 \simeq 0.01$$
$$\eta \equiv (V''/V)M_P^2 \simeq 0.01$$

3) Spectral index: $n_s = 1 - 6\epsilon + 2\eta$

$$0.95 \leq n_s \leq 0.97$$

4) e-folds: $N \equiv \int_{t_i}^{t_f} H dt \sim M_P^{-2} |\int_{\phi_i}^{\phi_f} (V/V') d\phi| \simeq 50 - 60$

5) Curvature pert: $\delta_H = \cdots V^{1/2}/M_P^2 \epsilon^{1/2} = 1.91 \times 10^{-5}$

6) Quantum gravity effects may be ignored safely.

$G_4/L_5^2 \ll 1 \to R \geq 0.8 \times 10^{-20}\,\mathrm{GeV}^{-1}$ Appelquist & Chodos (82)

6. Application

It is curious to see whether our toy model of inflaton has any realistic meaning in the context of GUT or other unified models.

1) The scale parameter of the model is

$$f \equiv 1/(2\pi g R), \;\; g = g_5/\sqrt{L}$$

$$\theta \rightarrow \phi = f\theta$$

2) The potential differs a bit depending on the value of f.

I) For $f \gg 10 M_P$, V is quadratic, i.e., chaotic.

II) For $f \simeq 10 M_P$, V is cosine function, deviates from chaotic.

3) We recall that m_ϕ is constrained by the value of δ_H,

I) $m_\phi = 1.8 \times 10^{13}\,\mathrm{GeV}$ (chaotic)

II) $m_\phi \simeq 1.3 \times 10^{13}\,\mathrm{GeV}$

4) We have applied our *gauge-Higgs-inflaton* unification model to gauge theories for a typical value of f in both cases.

I) $f = 100\, M_P$. $\;g \simeq 0.09 - 0.0002,\;\; \beta/R \simeq 10^{14} - 10^{17}\,\mathrm{GeV}$

II) $f = 10\, M_P$. $\;g \simeq 0.9 - 0.0005,\;\; \beta/R \simeq 10^{15} - 10^{18}\,\mathrm{GeV}$

It is curious that the 4D gauge coupling constant g agrees with the realistic value. In case II) (non-chaotic),

$g^2/4\pi \simeq 0.05$ (or smaller) $\longleftrightarrow$ $g^2_{\mathrm{GUT}} \simeq 0.02 - 0.05$

7. Summary and outlook

♡ The inflation parameters are reproduced without fine tuning by our *gauge-Higgs-inflaton* unification picture. This good accord may be by chance, or encouraging?

♡ Inflaton mass is $m_\phi \sim 10^{13}\,\mathrm{GeV}$. What does this value mean in GUT is unclear to us.

♠ We may extend the (susy) gauge theory on $M_4 \times S^1$ to that on $M_4 \times T^2$. Then what roles the A_5 and A_6 may play?

♠ We may embed higher-dim gauge theory in string theory, and compute *string-loop* effective action in open string field theory. – work in progress

♠ The possibility that inflaton is a scalar field from higher-dim gravity.