A Phenomenological Model of Inflation from Quantum Gravity

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Quant. Gravitational Inflation

- Fund. IR gravity: $G_{\mu\nu} = -\Lambda g_{\mu\nu}$
- $\Lambda \sim [10^{12}$ GeV$]^2$ starts inflation
  - $ds^2 = -dt^2 + a^2(t)\, dx^2$ with $a(t) = e^{Ht}$
- QG "friction" stops inflation
  - $\rho_1 \sim +\Lambda^2$
  - $\rho_2 \sim -G\Lambda^3 \ln[a(t)]$
  - $\rho_L \sim -\Lambda^2 [G\Lambda \ln(a)]^{L-1}$
- Hence $p \sim -\rho \sim \Lambda^2 f[G\Lambda \ln(a)]$

Only Causality Stops Collapse!

- IR gravitons ➔ $\rho_1 \sim +\Lambda^2$
- w/o causality ➔ $\rho_2 \sim -G\Lambda^3\, a^2(t)$
  - $R(t) \sim a(t)/H$ and $M(t) \sim H\, a^3(t)$
  - $\Delta E(t) = -GM^2/R \sim -GH^3\, a^5(t)$
- Causality changes powers of a(t) to powers of ln[a(t)]
- But grav. Int. E. still grows w/o bound

Need Phenomenological Model

- Advantages of QG Inflation
  - Natural initial conditions
  - No fine tuning
  - Unique predictions
- But tough to USE!
- Try guessing most cosmologically significant part of effective field eqns

$G_{\mu\nu} = -\Lambda g_{\mu\nu} + 8\pi G T_{\mu\nu}[g]$

- $T_{\mu\nu}[g] = p\, g_{\mu\nu} + (\rho+p)\, u_\mu u_\nu$
  - Posit p[g]
  - Infer ρ and u_μ from conservation
- Getting p[de Sitter] = Λ^2 f[GΛ ln(a)]
  - […] must be nonlocal because

$$R_{\mu\nu\rho\sigma} = \Lambda/3\, [g_{\mu\rho}\, g_{\nu\sigma} - g_{\mu\sigma}\, g_{\nu\rho}]$$

  - Simplest is X = 1/□ R

R & $\Box \equiv (-g)^{-1/2}\partial_\mu[(-g)^{1/2}g^{\mu\nu}\partial_\nu]$

- $R = 6\, dH/dt + 12\, H^2$ for flat FRW
- $\Box f(t) = -a^{-3}\, d/dt\, [a^3\, df/dt]$
  - Hence $1/\Box\, f = -\int^t du\, a^{-3} \int^u dv\, a^3 f(v)$
- For de Sitter $a(t) = e^{Ht}$ and $dH/dt = 0$
  - $1/\Box\, R = -4\, Ht + 4/3\, [1 - e^{-3Ht}] \sim -4 \ln(a)$

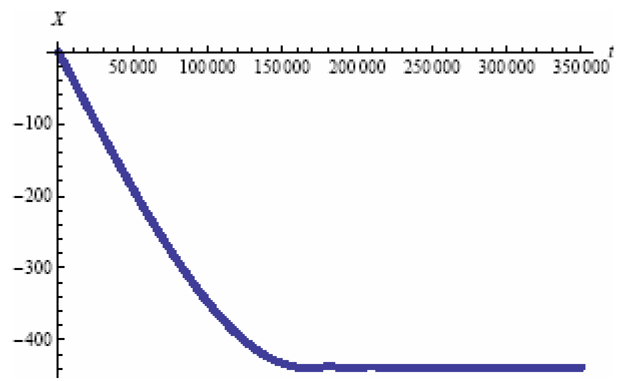
Spatially Homogeneous Case

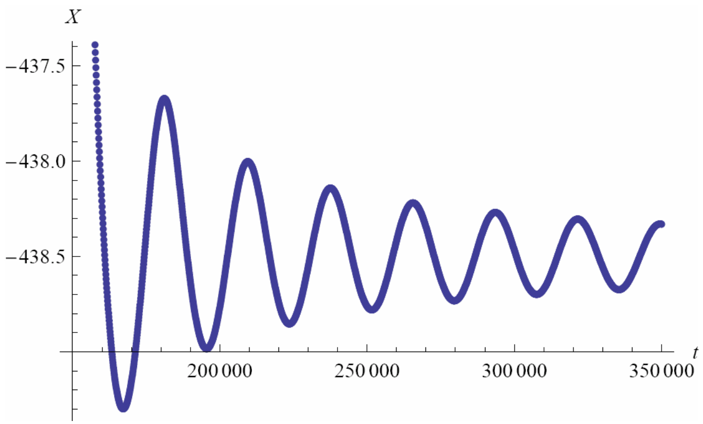
- $G_{\mu\nu} = (p-\Lambda)g_{\mu\nu} + (\rho+p)\, u_\mu u_\nu$
  - $X = 1/\Box\, R = -\int^t du\, a^{-3} \int^u dv\, a^3\, [12H^2 + 6dH/dv]$
  - $p = \Lambda^2 f(-G\Lambda\, X)$
  - $\rho+p = a^{-3}\int^t du\, a^3\, dp/du$ and $u^\mu = \delta^\mu{}_0$
- Two Eqns
  - $3H^2 = \Lambda + 8\pi G\, \rho$
  - $-2dH/dt - 3H^2 = -\Lambda + 8\pi G\, p$ (easier)
- Parameters
  - 1 Number: $G\Lambda$ (nominally $\sim 10^{-12}$)
  - 1 Function: $f(x)$ (needs to grow w/o bound)

Numerical Results for $G\Lambda = 1/300$ and $f(x) = e^x - 1$

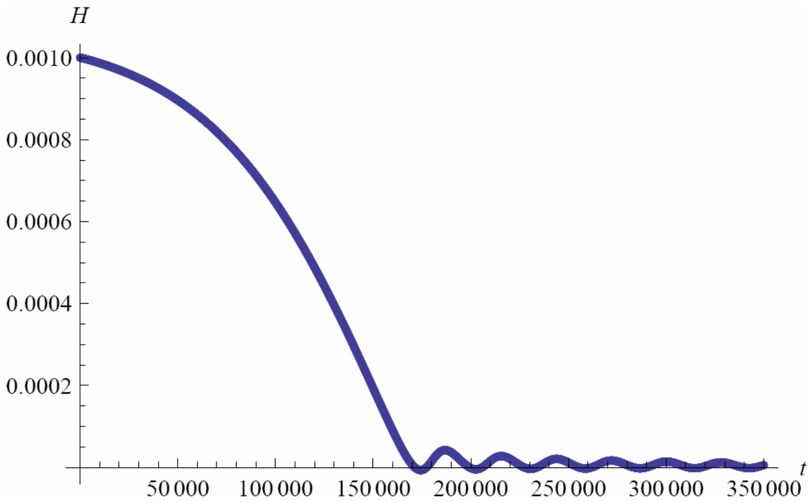
- $X = -\int^t du\, a^{-3} \int^u dv\, a^3 R$
- Criticality

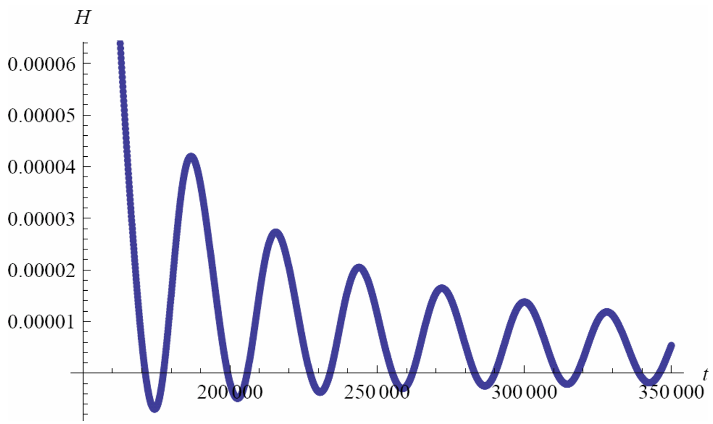
  $p = \Lambda^2 f(-G\Lambda X) = \Lambda/8\pi G$
- Evolution of $X(t)$
  - Falls steadily to X_c
  - Then oscillates with constant period and decreasing amplitude
  - For all $f(x)$ growing w/o bound

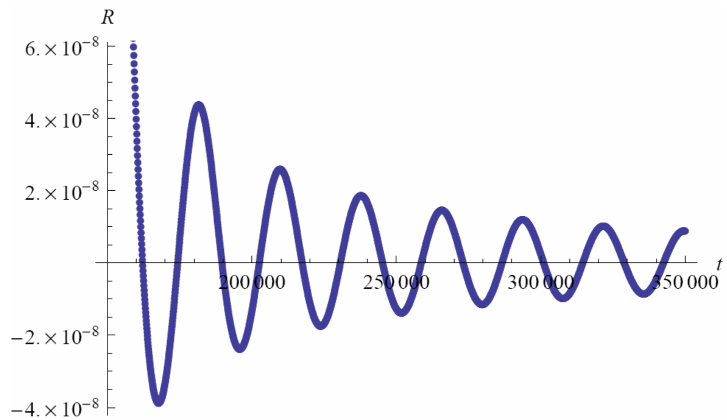




Inflation Ends, H(t) goes < 0, R(t) oscillates about 0







Analytic Treatment ($\epsilon \equiv G\Lambda$)

- $2\, dH/dt + 3\, H^2 = \Lambda[1 - 8\pi\epsilon f(-\epsilon X)]$
- $X(t) = X_c + \Delta X(t)$
  - $f \approx f_c - \epsilon \Delta X\, f'_c$
  - $2dH/dt + 3\, H^2 \approx 24\pi\epsilon^2\, f'_c\, \Delta X$
- Use $R = 6\, dH/dt + 12\, H^2$
  - L.H.S. $= R/3 - H^2$
  - $\Delta X = 1/\Box\, R - X_c$
- Act $\Box = -[d/dt + 3H]d/dt$ to localize
  - $[(d/dt)^2 + 2H(d/dt) + \omega^2]R \approx 0$
  - $R(t) \approx \sin(\omega\, t)/a(t)$
  - $\omega^2 = 24\pi\epsilon^2\Lambda f'_c$ (agrees with plots!)

Tensor Perturbations

- No change from usual eqn

  $$\ddot{x} + 3\,H\,\dot{x} + k^2/a^2\,x = 0$$

- Of course a(t) is unusual . . .
  - Oscillations in H(t)
  - And H(t) drops below zero!
- But this happens at the end of inflation
  - Little effect on far super-horizon modes

Origin of Scalar Perturbations

1. In Fundamental QG Inflation
   - $\mathcal{L} = 1/16\pi G\ (R - 2\Lambda)(-g)^{1/2}$
   - Two h_{ij}'s can make a scalar!
     E.g. Graviton KE: $\dot{h}_{ij}\ \dot{h}_{ij} + \nabla h_{ij}\ \nabla h_{ij}$
   - Usually negligible but if IR logs make homogeneous ~ O(1) maybe perts ~ $O(G\Lambda)$
2. In Phenomenological Model
   - $T_{\mu\nu}[g] = p\ g_{\mu\nu} + (\rho + p)\ u_\mu u_\nu$
   - $p = \Lambda^2 f(-G\Lambda/\Box\ R)$ fixed by retarded BC
   - But ρ and u_i at t=0 not fixed by $D^\mu T_{\mu\nu} = 0$

Analysis (in conformal coords)

- 0^{th} order: $2a''/a^3 - a'^2/a^4 = \Lambda[1 - 8\pi\epsilon f(-\epsilon X_0)]$
- $h_{\mu\nu}dx^\mu dx^\nu = -2\phi d\eta^2 - 2B_{,i}dx^i d\eta - 2[\psi\delta_{ij} + E_{,ij}]dx^i dx^j$
  - $\Phi = \phi - a'/a\,(B-E') - (B'-E'')$
  - $\Psi = \psi + a'/a\,(B-E')$
- G_{ij} Eqn $\rightarrow$ $\Psi = \Phi$ and

  $2/a^2\,\Phi''+6a'/a^3\,\Phi'+[4a''/a^3-2a'^2/a^4]\Phi = -8\pi\epsilon^2\Lambda\, f'(-\epsilon X_0)$

  $\times\, 1/\Box_0\,[\nabla^2/a^2\,\Phi - 6/a^2\,\Phi'' - 24\,a'/a^3\,\Phi' - 4/a^2\,X_0'\,\Phi']$

$d^2\Phi/dt^2 + 4H d\Phi/dt + (2dH/dt + 3H^2)\Phi = -8\pi\varepsilon^2\Lambda f'(-\varepsilon X(t))$ NL

- Early ➔ $f'(-\varepsilon X(t)) << 1$
  - + de Sitter ➔ $\Phi_1 = 1/a$ and $\Phi_2 = 1/a^3$
  - Same for all k's
- Late ➔ $f'(-\varepsilon X(t)) \approx f_c'$
  - Oscillates with constant frequency ω

    $$d^2\Phi/dt^2 \approx -\omega^2\, 1/\Box\, [d^2\Phi/dt^2]$$
  - Amplitude seems constant (numerically)
- Energy transfer to matter crucial

After Inflation

- Model driven by $X = 1/\Box R$
  - Oscillations & $H < 0$ ➔ efficient reheating
  - $H = 1/2t$ ➔ $R = 6\, dH/dt + 12 H^2 = 0$
- QG ends inflation, reheats & then turns off for most of cosmological history
  - $X(t) = -\int^t du\, a^{-3} \int^u dv\, a^3 R \rightarrow X_c$

Two Problems at Late Times

Eventually matter dominates

- H(t) goes from 1/(2t) to 2/(3t)
- $R = 6dH/dt + 12H^2$ from 0 to $3/(4t^2)$
- $X = 1/\Box R$ from X_c to $X_c - 4/3 \ln(t/t_{eq})$

1. The Sign Problem:

   This gives further screening!

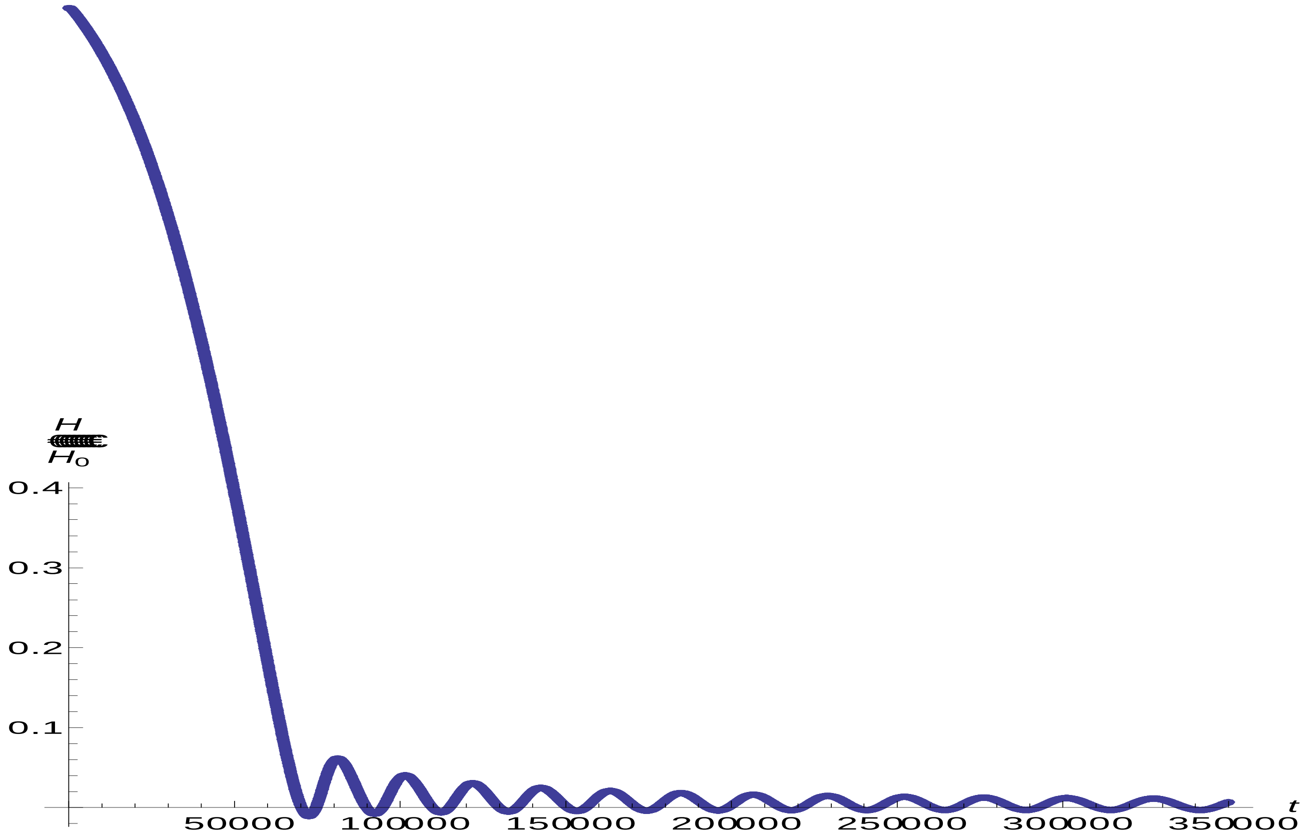
2. The Magnitude Problem:

   $p \approx -\Lambda/G\,(G\Lambda)^2 f_c' \Delta X \approx -10^{86} p_0 \times f_c' \Delta X$

Magnitude Problem: Too many Λ's

- $p = \Lambda^2 f(-G\Lambda\, 1/\Box\, R)$
  - Dangerous changing initial Λ^2
  - But can do $-G\Lambda\, 1/\Box[R] \rightarrow -G/\Box[\text{“}\Lambda\text{”}R]$
- Properties of “Λ”
  - Approximately Λ during inflation
  - Approx. R by onset of matter domination
  - No change to initial value problem
  - Invariant functional of metric
- Many choices but “Λ” = R(t/10) works
  - Can specify invariantly

Same as before with "Λ" = ¼ R(t/10)



Sign Problem: $R(t) > 0$

- $p = \Lambda^2 f(-G/\Box[\text{“}\Lambda\text{”} R])$
- Need to add term to “Λ” R inside [ ]
  - Nearly zero during inflation & radiation
  - Comparable to R^2 after matter
  - Opposite sign
- Many choices but $\Box R$ works
  - $R = 4/(3t^2) \rightarrow \Box R = -8/(3t^4)$

Conclusions

- Advantages of QG Inflation
  1. Based on fundamental IR theory → GR
  2. Λ not unreasonably small!
  3. Λ starts inflation naturally
  4. QG back-reaction stops

     Simple idea: Grav. Int. E. grows faster than V
  5. 1 free parameter: Λ
- But tough to use → Phenom. Model

$T_{\mu\nu}[g] = p\, g_{\mu\nu} + (\rho+p)\, u_\mu u_\nu$

- Guess $p[g] = \Lambda^2 f(-G\Lambda X)$
  - $X_1 = 1/\Box R$
  - Infer ρ and u_i from conservation
- Homogeneous evolution: (generic f)
  - X falls to make p cancel $-\Lambda/8\pi G$
  - Then oscillate with const. period & decreasing amp.
- Reheats to radiation dom. (R=0)
  - Matter dom. ➔ $R \neq 0$
  - $\Lambda X_2 = 1/\Box\, [\text{“}\Lambda\text{”}\, R + \Box R]$ can give late acceleration
- Perturbations
  - Little change to observable tensors
  - Scalars differ but still not clear