Hawking radiation at strong coupling

Mukund Rangamani
Durham University

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Hawking radiation at strong coupling

- Veronika Hubeny, Don Marolf, MR: to appear

Outline

- Motivation
- CFTs on black hole backgrounds
- Phase structure: motivating black funnels
- Black Funnels in AdS
- Summary and discussion

Motivation

How do black holes radiate strongly coupled matter?

- Hawking radiation is conventionally discussed for perturbative fields.
- Quantify differences when matter is strongly coupled?
- Precise nature of the quantum stress tensor due to the radiation (matter)?
- How does the nature of the radiation change if the matter can be in distinct phases?

Motivation

Strong coupling dynamics in curved spacetime

- Characteristic behaviour of a strongly coupled field theory in a generic curved spacetime background.
- How does the matter react to the background curvature: differences in vacuum polarization effects, etc..
- Phase structure?

The basic framework:

- Attempt to use AdS/CFT to address some of these issues.

Motivation

Brane-world black holes

- Does one have a complete understanding of the potential set of brane-world black holes in induced gravity models?
- Are there new solutions apart from localized black holes and black strings?
- Why don't large localized black holes seem to exist?

Emparan, Fabbri, Kaloper

Fitzpatrick, Randall, Wiseman

Gregory, Ross, Zegers

- Induced gravity models will be treated as AdS/CFT with a UV cut-off.

CFTs on black hole backgrounds

Perturbative fields

- Consider a conformally coupled scalar field in a black hole background such as Schwarzschild.

$$ds^2_\partial = \gamma_{\mu\nu}\, dx^\mu\, dx^\nu = -{}^d\mathfrak{f}(r)\, dt^2 + \frac{dr^2}{{}^d\mathfrak{f}(r)} + r^2\, d\Omega^2_{d-2}\ .$$

$${}^d\mathfrak{f}_{\rm Schw}(r) = 1 - \frac{r_+^{d-3}}{r^{d-3}}\ .$$

- One can also consider the Schwarzschild-AdS black hole:

$${}^d\mathfrak{f}_{\rm SAdS}(r) = \frac{r^2}{\ell_d^2} + 1 - \frac{r_+^{d-3}}{r^{d-3}}\left(1 + \frac{r_+^2}{\ell_d^2}\right).$$

CFTs on black hole backgrounds

Perturbative fields

- On these black hole backgrounds we want to study the dynamics of a conformally coupled scalar field with action

$$\mathcal{S}_{\text{scalar}} = \int d^dx \sqrt{-\gamma} \left(\frac{1}{2} \gamma^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + \frac{d-2}{8\,(d-1)} R^{(\gamma)} \phi^2 \right)$$

- Of particular interest to us will be the nature of the quantum stress tensor, which encapsulates the physics of vacuum polarization, Hawking radiation etc..

CFTs on black hole backgrounds

Perturbative fields: Expectations

- Scalar radiation via the Hawking process at a temperature set by the black hole size.
- Stress tensor is thermal asymptotically ($r \to \infty$).

CFTs on black hole backgrounds

Perturbative fields: Expectations

- Scalar radiation via the Hawking process at a temperature set by the black hole size.
- Stress tensor is thermal asymptotically ($r \to \infty$).

- Thermal stress tensor is not regular on the event horizon.
- Correct quantum stress tensor should be regular at the horizon.
- Nothing singles out the event horizon: were the quantum stress tensor not regular there, we would be able to tell where the horizon was!

CFTs on black hole backgrounds

Perturbative fields: Results

- The general expectation is borne out by an explicit computation of quantum stress tensor.
- The computation was done by Page using the so called optical metric for static spacetimes:

Page '82

$$\overline{ds}^2_\partial = \overline{\gamma}_{\mu\nu}\, dx^\mu\, dx^\nu = -dt^2 + \frac{dr^2}{{}^d\mathfrak{f}(r)^2} + \frac{r^2}{{}^d\mathfrak{f}(r)}\, d\Omega^2_{d-2}\ .$$

- One finds a regular stress tensor using the optical metric using heat kernel techniques. The result is given as:

$$T^\mu_\nu = \frac{\pi^2\, T^4}{90}\left[A\left(\frac{r_+}{r}\right)\left(\delta^\mu_\nu - 4\,\delta^\mu_0\,\delta^0_\nu\right) + B\left(\frac{r_+}{r}\right)\left(3\,\delta^\mu_0\,\delta^0_\nu + \delta^\mu_1\,\delta^1_\nu\right)\right]$$

$$A(x) = \frac{1-(4-3\,x)^2\,x^6}{(1-x)^2}\ , \qquad B(x) = 24\,x^6$$

CFTs on black hole backgrounds

Perturbative fields: Results

- At large distances $r \gg r_+$ one has thermal radiation at

$$T_{\rm local} = \frac{1}{\sqrt{^d\mathfrak{f}(r)}}\, T_{\rm BH}$$

- On the black hole horizon the quantum stress tensor evaluates to:

$$T_\mu^{\ \nu} = \frac{2\,\pi^4}{15}\, T_{\rm BH}^4 \,{\rm diag}\,\{3,3,1,1\}$$

- Note that vacuum polarization renders local energy density negative at the horizon.

CFTs on black hole backgrounds

Beyond perturbation theory: Expectations

- Even when the matter fields are strongly coupled one should obtain a stress tensor that is regular on the horizon and is thermal asymptotically.
- The interpolation between the two regimes will be sensitive the details of the system.
- Also take into account the phase structure of the QFT.

- To understand these issues we are going to embed the problem within the AdS/CFT correspondence, which is the cleanest framework to discuss these issues.

The AdS/CFT story

QFT + Gravity $\rightsquigarrow$ Classical gravity

- Consider a situation in the AdS/CFT framework where the boundary field theory is on a non-dynamical black hole background $\mathcal{B}_d$ with metric $\gamma_{\mu\nu}$.
- Dynamics of these field theories is governed at strong coupling by ``asymptotically locally AdS'' geometries which asymptote to $\mathcal{B}_d$.
- This involves finding bulk spacetimes $\mathcal{M}_{d+1}$ which have as their conformal boundary the preferred manifold $\mathcal{B}_d$.

- One has to therefore find solutions to Einstein's equations with a negative cosmological constant which has the correct boundary conditions.

Known solutions for boundary black holes

- Consider the case where the boundary is the asymptotically flat Schwarzschild black hole.
- One solution which satisfies the field equations is the AdS black string

$$ds^2 = z^2 \, ds^2_{\partial,\mathrm{Schw}} + \frac{dz^2}{z^2} \ .$$

- This solution is singular on the Poincare horizon (due to the black hole singularity which extends along z.
- Ignoring this issue we can ask what the induced stress tensor is on the boundary.
- The result is simply:

$$T_\mu^{\ \nu} = 0$$

The AdS/CFT story

Schwarzschild boundary conundrums:

- The vanishing stress tensor for the AdS black string is at odds with expectations of thermal Hawking radiation.
- Anticipate strong vacuum polarization effects due to the strongly interacting matter.
- Rather curious that the conventional Hawking thermal spectrum is not reproduced.
- A similar result can be derived for the boundary Schwarzschild-AdS black hole.
- Note that this result in particular implies that if we make the boundary gravity dynamical then it would see no back-reaction from the strongly coupled matter sector!

Phase structure of the CFT

Resolution: Confinement vs. Deconfinement

- The vanishing stress tensor for the AdS black string seems to suggest that the dual field theory is actually in the confined phase, i.e.,

$$0 \sim \mathcal{O}(1)$$

Phase structure of the CFT

Resolution: Confinement vs. Deconfinement

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- Typically in the AdS/CFT context, the boundary field theories on Minkowski space doesn't undergo a confinement-deconfinement transition.
- At any non-zero temperature, the preferred phase is the deconfined phase with $\mathcal{O}(N^2)$ free energy.

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- At any non-zero temperature, the preferred phase is the deconfined phase with $\mathcal{O}(N^2)$ free energy.

- It is somewhat reassuring that the black string geometry actually is classically unstable, indicating that it is not a local minimum of the free energy.

Phase structure of the CFT

Whither the Deconfined phase?

- The stable phase at any finite temperature ought to be the deconfined phase.
- This phase has a thermal stress tensor asymptotically and should give rise to a strongly coupled version of the Page stress tensor.
- In AdS/CFT this should just amount to a multiplicative renormalization of the free field result (the famous 3/4).
- Question then remains: what is the bulk solution which encapsulates the physics of a deconfined field theory plasma in equilibrium with the thermal Hawking radiation of the boundary black hole?

Motivating the Black Funnel

- The deconfined phase is dual to a Black Funnel geometry.
  - Without the boundary black hole the spacetime dual of the deconfined phase is the Schwarzschild AdS black hole.
  - Induces the correct thermal stress tensor on the boundary.
  - Expect the bulk spacetime to behave similarly when we are asymptotically far away from the black hole on the boundary.
  - Denote boundary radial coordinate as r and the bulk direction as z.
  - Then as $r \to \infty$ expect that the bulk solution for all z behaves like the Schwarzschild AdS black hole.
  - In addition the bulk horizon should smoothly connect to the horizon on the boundary.

Motivating the Black Funnel

- The deconfined phase is dual to a Black Funnel geometry.

$z = 0$

$r \to \infty$

$z \downarrow$

A Conjecture: field theory on Schwarzschild bg

- The stable deconfined phase is dual to a Black Funnel geometry.
- Corresponds to the phase of the CFT in equilibrium with the thermal Hawking radiation.
- The unstable deconfined phase is dual to the AdS black string.

Black Funnels & the phase structure of the CFT

A Conjecture: field theory on Schwarzschild bg

- The stable deconfined phase is dual to a Black Funnel geometry.
- Corresponds to the phase of the CFT in equilibrium with the thermal Hawking radiation.
- The unstable deconfined phase is dual to the AdS black string.

Extensions:

- Can be extended to situations where the field theory has a confining phase and to boundary metrics which have different asymptotics.

Black Funnels: Examples

Black funnels in AdS_3

- The simplest explicit example of black funnels can be obtained when the bulk is three dimensional.
- The non-dynamical boundary metric can be taken to the be the two dimensional black hole with metric

$$ds^2 = -\tanh^2 r\, dt^2 + dr^2$$

- Einstein's equations (with -ve cc) can be easily solved.
- Work in Fefferman-Graham coordinates for simplicity and take the metric ansatz to be

$$ds^2 = \frac{1}{z^2}\left(-f(r,z)\, dt^2 + g(r,z)\, dr^2 + dz^2\right)$$

Black Funnels: Examples

Black funnels in AdS_3

- The metric functions turn out to be:

$$\begin{aligned} f(r,z) &= \frac{1}{16}\left(4\,\tanh r + z^2\,\frac{1-2\,r_+\,\cosh^4 r}{\sinh r\,\cosh^3 r}\right)^2 \\ g(r,z) &= \left(1 + z^2\,\frac{2\,r_+\cosh^4 r - 1 - 4\,\sinh^2 r}{4\,\sinh^2 r\,\cosh^2 r}\right)^2 \end{aligned}$$

- Naively we have a one-parameter family of solutions, but the parameter r_+ is fixed by demanding regularity:

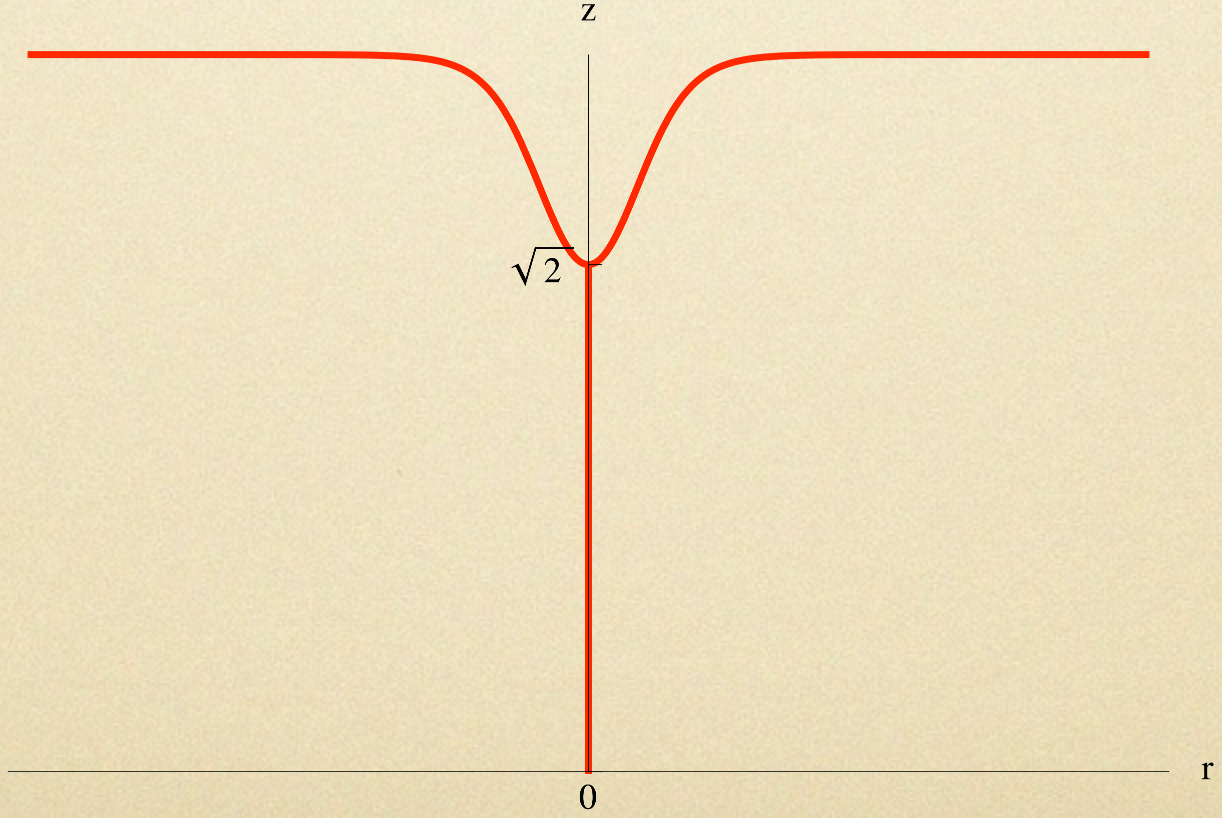
$$T_{\text{bdy}} = \frac{1}{2\,\pi} \implies r_+ = \frac{1}{2}$$

Black Funnels: Examples

Black funnels in AdS$_3$

- There is a bulk event horizon located at

$$z_H(r) = \frac{2\ \cosh r}{\sqrt{\cosh^2 r + 1}}\ , \quad \&\ \ r = 0$$



Black Funnels: Examples

Black funnels in AdS_3

- Given the solution we can use holographic techniques to find the induced stress tensor on the boundary:

$$T_{tt} = \alpha \tanh^2 r \left(\frac{\cosh^2 r - 3}{2 \cosh^2 r} \right) , \qquad T_{rr} = \alpha \frac{\cosh^2 r + 1}{2 \cosh^2 r}$$

- α is a normalization constant measuring the effective number of degrees of freedom.

$$\alpha \sim \frac{\ell}{\ell_p}$$

- This stress tensor is thermal asymptotically far from the black hole and regular on the event horizon as expected.

Black Funnels: Examples

Black funnels in AdS_4 with BTZ boundary

- The simplest non-trivial example where we can find explicit solution and in fact examine the full phase structure is when the boundary is a BTZ black hole.

$$ds^2 = -(r^2 - r_+^2)\, dt^2 + \frac{dr^2}{r^2 - r_+^2} + r^2\, d\varphi^2$$

- Obtain all the requisite solutions via a set of solution generating tricks.

Black Funnels: Examples

Black funnels in AdS_4 with BTZ boundary

◉ Recall that for the global AdS_4 spacetime with the standard Einstein Static Universe $\mathbf{R} \times \mathbf{S}^2$ boundary one has the following solutions dual the various phases:

✤ Confined phase: Thermal AdS spacetime
✤ Deconfined phase: Large Schwarzschild AdS black hole
✤ Unstable phase: Small Schwarzschild AdS black hole

Black Funnels: Examples

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◉ By an analytic continuation + conformal transformation one can map the ESU to BTZ.

◉ Applying the same trick to the bulk solutions, we recover the desired black funnel geometries.

Black Funnels: Examples

Black funnels in AdS_4 with BTZ boundary

In particular, starting with the known class of solutions

$$ds^2 = -f(\rho)\, d\tilde{t}^2 + \frac{d\rho^2}{f(\rho)} + \rho^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

the following set of transformations

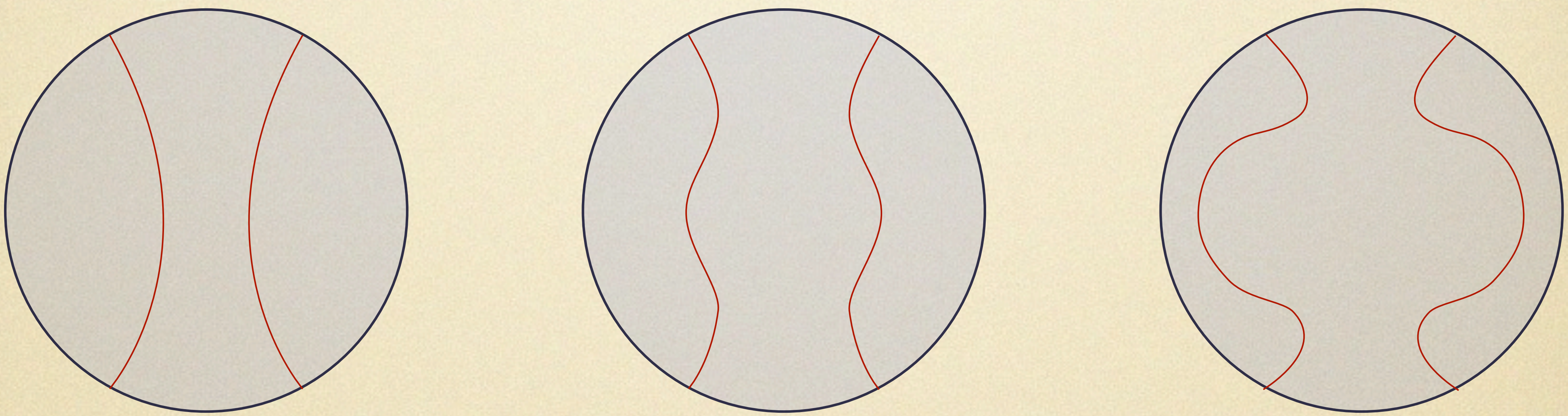
$$\tilde{t} = i\,\chi\ , \qquad \varphi = i\,t\ , \qquad \tilde{r} = \frac{1}{w}$$

takes us to the full set of solutions dual to BTZ on the boundary:

$$ds^2 = \frac{\rho^2}{w^2}\left[-(w^2-1)\,dt^2 + \frac{dw^2}{w^2-1} + w^2\,\frac{f(\rho)}{\rho^2}\,d\chi^2\right] + \frac{d\rho^2}{f(\rho)}$$

Black Funnels: Examples

Black funnels in AdS_4 with BTZ boundary



AdS black string
Confined phase

Small Black Funnel
Unstable phase

Large Black Funnel
Deconfined phase

Black Funnels: Examples

Black funnels in AdS_4 with BTZ boundary

While the ``funnelnality'' of the solution is not manifest, one can compute the boundary stress tensor having fixed the non-normalizable part to coincide with BTZ metric.

$$f(\rho) = 1 + \rho^2 - \frac{M}{\rho}\,, \qquad M = \rho_+\,(1+\rho_+^2)$$

$$T_\mu^{\ \nu} = \frac{M}{3\,r^3}\,\mathrm{diag}\{1,1,-2\}$$

- The stress tensor is regular on the horizon, but seems to behave somewhat strangely as $r \to \infty$.
- But this in fact turns out to be the precise form computed for perturbative fields.

Black Funnels: Examples

Black funnels in AdS_4 with BTZ boundary

For the BTZ geometry

$$ds^2 = -(r^2 - r_+^2)\, dt^2 + \frac{dr^2}{r^2 - r_+^2} + r^2\, d\varphi^2$$

the calculation of the quantum stress tensor for scalar fields leads to the result:

$$\begin{aligned} T_\mu^{\ \nu} &= \frac{A(r_+)}{3\, r^3}\, \mathrm{diag}\{1, 1, -2\}\ , \\ A(r_+) &= \frac{\sqrt{2}}{32\pi} \sum_{n=1}^{\infty} \frac{\cosh(2\pi\, n\, r_+) + 3}{(\cosh(2\pi\, n\, r_+) - 1)^{\frac{3}{2}}} \end{aligned}$$

whose form coincides with the holographic computation.

Steif '93

Black Funnels: Examples

Black funnels in AdS_4 from the AdS C-metric

- One can also find black funnel like solutions within the AdS C-metric family.
- These turn out to correspond to boundary black holes which are asymptotically $\mathbf{R} \times \mathbf{H}^2$.

- Once again the expectations regarding the stress tensor are borne out by explicit computation.

Summary & Discussion

Here, there be Black funnels

- Holographic techniques allow us for the first time to investigate the Hawking radiation of strongly coupled quanta.
- Analysis of the dual gravitational solutions leads to a new class of solutions which we've dubbed black funnels.
- Explicit solutions exist when the boundary is 2 dimensional, or BTZ or an exotic hyperbolic black hole.
- Not only can such solutions be constructed in a limited class of examples, but also they confirm our intuition regarding the quantum stress tensor due to curved spacetime effects.

Summary & Discussion

Realistic Black funnels?

- ◉ Black funnels with Schwarzschild boundary should exist (numerical relativists please step up)!
- ◉ Would be interesting to explore the explicit behaviour of these solutions and figure out the quantum stress tensor induced by strongly coupled $\mathcal{N} = 4$ SYM in this background.

There are important implications for:

- ✤ Plasma balls in confining theories
- ✤ Brane-world and induced gravity models.