Is Weinberg’s Theorem violated?

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Weinberg's Theorem hep-th/0602442

- Gravity + Inflaton + free scalars + free spinors + free vector fields
- Quantum corrections to the primordial power spectum
- Effects down by GH^2
- Largest possible IR enhancement is powers of ln[a(t)]

Result of Ford, Wu and Ng gr-qc/0608002

- Gravity + Inflaton + Conformal Matter
- Extra contribution due to conf. goes like $[1/a_{beg}]^2$
- Because $a_{end} = 1$ (for them) we all GUESS effect $\sim [a(t)/a_{beg}]^2$
- TWO PUZZLES:
  1. How can Weinberg's bound be violated?
  2. How can conformal matter get a big IR enhancement?

Strength of Quantum Gravity

- Coupling constant $G = 1/M_P^2$
- Corrections go like $GE^2 = (E/M_P)^2$
- TRIVIALLY large if $E > M_P$ but
  - Don't know physics at those scales
  - Perturbation theory isn't valid
- Can SOMETIMES get IR enhancement for $E << M_P$

Strength of Quantum Loops

- Classical response to virtual particles
- More virtuals = larger response
- TWO FACTORS give density of virtuals
  1. Time virtuals persist (controlled by Energy-Time Uncertainty Principle)
  2. Rate at which they emerge (controlled by kinetic term in Lagrangian)

How does it work for FRW?

- $ds^2 = -dt^2 + a^2(t)\, dx^i\, dx^i$
- PERSISTENCE TIME
  - $H(t) > 0$ increases it
  - Maximum effect for $q(t) < 0$ (inflation)
- EMERGENCE RATE
  - Conformal invariance reduces it

E-Time Unc. Principle for Flat Space

- $\Delta t \, \Delta E > 1$ to resolve ΔE
  - Hence $\Delta t \, \Delta E < 1$ to NOT resolve
- Virtual pair has $\Delta E = 2(m^2 + k^2)^{1/2}$
  - Hence can last $\Delta t < 1/2(m^2 + k^2)^{-1/2}$
- Eg: Vacuum polarization
  - Most for $e^{\pm}$ because smallest m
  - Smallest k's live longest ➔ EM stronger at shorter distances (less polarization)

E-Time Unc. Principle for FRW

- $ds^2 = -dt^2 + a^2(t)\, dx^i\, dx^i$
  - $k = 2\pi/\lambda$ still conserved
  - But physics depends on $k/a(t)$
  - $E(t) = [m^2 + k^2/a^2(t)]^{1/2}$
- $\Delta t\, \Delta E \rightarrow \int_t^{t+\Delta t} dt'\, E(t') < 1$
- $m=0 \rightarrow \int_t^{t+\Delta t} dt'/a(t')$
  - Bounded wrt Δt for inflation!
  - Any m=0 virtuals live forever!

What about the EMERGENCE RATE?

- Conformal invariance, a killer symmetry:

$\mathcal{L} \xrightarrow{g'_{\mu\nu}=\Omega^2 g_{\mu\nu}} \mathcal{L}' \xrightarrow{\text{field redefinition}} \mathcal{L}$

  - EM in 4D

    $\mathcal{L} = -1/4\, F_{\mu\rho} F_{\nu\sigma} g^{\mu\nu} g^{\rho\sigma} g^{1/2}$

    $\mathcal{L}' = -1/4\, F_{\mu\rho} F_{\nu\sigma} g^{\mu\nu} g^{\rho\sigma} g^{1/2} \Omega^{D-4}$, $F'_{\mu\rho} = F_{\mu\rho}$

  - Massless conformal coupled scalar (mcc)

$$-\frac{1}{2}\partial_\mu\phi\partial_\nu\phi\sqrt{-g} - \frac{1}{4}\frac{D-2}{D-1}R\phi^2\sqrt{-g}$$

Killer Symmetry Suppresses Emergence Rate

- $-dt^2+a^2dx^2=a^2(-d\eta^2+dx^2) \rightarrow a\,d\eta=dt$
- Conformal invariance ➔ same locally (in conformal coordinates) as flat
  - Hence $dN/d\eta = \Gamma_{flat}$
  - Hence $dN/dt = \Gamma_{flat}/a(t)$
- Any m=0, conf. virtuals that emerge DO live forever, but few emerge!

Maximum IR Enhancements

- Requirements:
  - PERSISTENCE TIME → m=0 + inflation
  - EMERGENCE RATE → no conf. invariance
- Realized by:
  - Massless, minimally coupled scalars
  - Gravitons
- NOT by conformal matter
  - How did Wu, Ng and Ford get anything?

They initially used Raychaudhuri Eqn.

- $d\theta/dt=-R_{\mu\nu}u^{\mu}u^{\nu}- 1/3\theta^2-\sigma_{\mu\nu}\sigma^{\mu\nu}+\omega_{\mu\nu}\omega^{\mu\nu}+(u^{\nu}{}_{;\mu}u^{\mu})_{;\nu}$
- $\theta(t)=\#\int \# \, dt \, [R_{\mu\nu}u^{\mu}u^{\nu}]_q$
  - $R_{\mu\nu}=8\pi G(T_{\mu\nu}-1/2g_{\mu\nu}T)$
  - $(T_{\mu\nu})_{RW}=a^{-2}(T_{\mu\nu})_{flat}$
  - $d\rho/dt+\theta(p+\rho)=0 \longrightarrow$

    $\delta\rho/\rho_0= -(1+w) \int d\eta \, \theta/a(\eta)$
- Cannot ignore $(u^{\nu}{}_{;\mu}u^{\mu})_{;\nu}$ because it cancels parts of $-R_{\mu\nu}u^{\mu}u^{\nu}$

Cannot Ignore the Acceleration Term

- Co-moving gauge : $u^\mu=\delta^\mu{}_0 \longrightarrow g_{00}=-1$

  $ds^2=-dt^2+2ah_{0i}dx^idt+a^2(\delta_{ij}+h_{ij})dx^idx^j$

  - $\theta=u^\mu{}_{;\mu}=d/dt\ \ln(g^{1/2})=3H+1/2\ d/dt\ h_{ii}$
  - $\delta\theta=1/2\ d/dt\ h_{ii}$ due to conformal matter

$$u^\mu = -\frac{g^{\mu\nu}\partial_\nu\varphi}{\sqrt{-g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi}} \ .$$

$$u^0 = 1 + O(\Delta^2)\ ,\ u^i = \frac{h_{0i}}{a} + \frac{\partial_i\delta\varphi}{a^2\dot{\varphi}_0} + O(\Delta^2)\ .$$

$$h_{0i}(t,\vec{x}) = -\partial_i\left[\frac{\delta\varphi(t,\vec{x})}{a(t)\dot{\varphi}_0(t)}\right] + O(\Delta^2)\ .$$

$$R_{00} = -3\dot{H} - 3H^2 - \frac{1}{2}\left[\ddot{h}_{ii} + 2H\dot{h}_{ii}\right] + \frac{1}{a}\left[\dot{h}_{0i,i} + Hh_{0i,i}\right] + O(\Delta^2)\ .$$

$$(u^\nu{}_{;\mu}u^\mu)_{;\nu} \longrightarrow \frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}g^{\nu k}\dot{g}_{k0}\right) = \frac{1}{a}\left[\dot{h}_{0i,i} + Hh_{0i,i}\right] + O(\Delta^2)\ .$$

Really h_{ii} Mixes with $\delta\varphi$

- First order perturbation :

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} + V_0''\delta\varphi + \frac{1}{2}\dot{\varphi}_0\dot{h}_{ii} = 0$$

$$-\kappa^2\dot{\varphi}_0\delta\dot{\varphi} + \frac{1}{2}\kappa^2 V_0'\delta\varphi - \frac{\nabla^2}{a^2}\frac{\partial}{\partial t}\left[\frac{\delta\varphi}{\dot{\varphi}_0}\right] - \frac{1}{2a^2}\frac{\partial}{\partial t}\left[a^2\dot{h}_{ii}\right] = \frac{1}{2}\kappa^2 T_{00}^{\mathrm{conf}}$$

$$(\delta\varphi)_1 = \dot{\varphi}_0 \qquad , \qquad (\dot{h}_{ii})_1 = 6\dot{H}$$

$$\delta\varphi(x) \equiv \dot{\varphi}_0(t)\Phi(x)$$

$$\dot{h}_{ii} = 6\dot{H}\Phi - \left(4\frac{\ddot{\varphi}_0}{\dot{\varphi}_0} + 6H\right)\dot{\Phi} - 2\ddot{\Phi}$$

$$\left\{\partial_t^2 + \left(5H + 2\frac{\ddot{\varphi}_0}{\dot{\varphi}_0}\right)\partial_t + 4\dot{H} + 6H^2 + 4H\frac{\ddot{\varphi}_0}{\dot{\varphi}_0} - 2\frac{\ddot{\varphi}_0^2}{\dot{\varphi}_0^2} + 2\frac{\dddot{\varphi}_0}{\dot{\varphi}_0} - \frac{\nabla^2}{a^2}\right\}\dot{\Phi} = \frac{1}{2}\kappa^2 T_{00}^{\mathrm{conf}}$$

Green's Function Solution

- The formal solution: $d\Phi/dt \equiv \Psi$

$$\widetilde{G}(t,t',k) = \frac{\theta(t-t')}{W(t',k)}\left\{\Psi_2(t,k)\Psi_1(t',k) - \Psi_1(t,k)\Psi_2(t',k)\right\}$$

$$\dot{\Phi}(x) = \int d^D x'\, G(x;x')\frac{\kappa^2}{2}T_{00}^{\rm conf}(x')$$

- It cannot be solved exactly but can be related to solution of Mukhanov equation

$$\left[\partial_t^2 + H\partial_t + \frac{k^2}{a^2} - \left(\frac{\ddot{\theta}+H\dot{\theta}}{\theta}\right)\right]Q_i = 0 \quad , \quad \theta(t) \equiv H/(a\sqrt{-\dot{H}})$$

$$\Psi_i(t,k) = \frac{Q_i(t,k)}{a^2(t)\sqrt{-\dot{H}(t)}}$$

  - Before the 1st horizon crossing :

$$Q_\pm(t,k) = e^{\pm ik\int^t dt'/a(t')}$$

  - After the 1st horizon crossing :

$$Q_s(t) = \theta(t) \quad , \quad Q_\ell(t) = \theta(t)\int^t \frac{dt'}{a(t')\theta^2(t')}$$

Asymptotic Results for $d\Phi/dt$

- $d\Phi/dt$ is BIG at early times
- $d\Phi/dt$ is SMALL at late times
- $\Phi(t) = \Phi_0 + \int_0^t dt' \, d\Phi/dt'$
- THIS EXPLAINS THEIR BIG $\Phi(\text{end})$!
  - Not from late times when $k/a(t) << M_P$
  - From EARLY times when $k/a(t) >> M_P$

Explaining the Two Puzzles

- No violation of Weinberg's Thm
  - Their $[1/a(beg)]^2$ is NOT $[a(t)/a(beg)]^2$
  - It's really $[k/a(beg)]^2$
- No IR enhancement from conf. matter
  - Effect from early times when $k/a >> M_P$
  - All modes give big effects in this regime!

Trans-Planckian Issue

- No paradoxes, but is it RIGHT?
- Problems with k/a(beg) >> M_P
  - We don't know fundamental physics there
  - Perturbation theory isn't valid
- We agree to disagree
  - Wu, Ng & Ford: start at beginning of inflation & use pert. theory ➔ big corrections from early times
  - Miao & Woodard: assume modes emerge from trans-Planck regime in vacuum ➔ only small corrections from late times

Bua's Bound

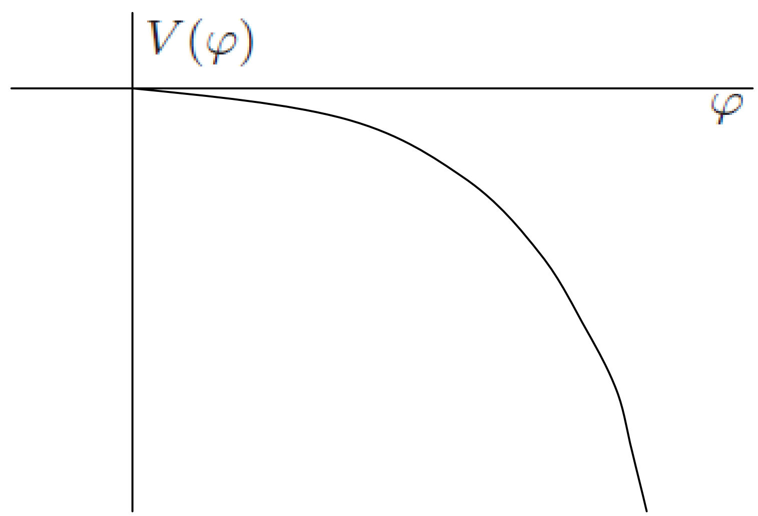
- Bua Chaicherdsukul (hep-th/0611352) :

  No large effects to power spectrum from fermions or gauge particles, no matter how they couple

Counter example to Bua's bound --- Yukawa theory

$$\mathcal{L} = -\frac{1}{2}\partial_\alpha\varphi\partial_\beta\varphi g^{\alpha\beta}\sqrt{-g} - \frac{1}{2}\delta\xi\varphi^2 R\sqrt{-g} - \frac{1}{4!}\lambda\varphi^4\sqrt{-g}$$
$$+i\overline{\psi}e^{\beta}{}_{b}\gamma^b\mathcal{D}_\beta\psi\sqrt{-g} - f\varphi\overline{\psi}\psi\sqrt{-g}$$

- Integrate out fermions at leading Log. Order and compute V_{eff}
  - Scalar induces Fermion mass
  - Fermion vacuum E < 0
  - Unbounded below!



- When can we get big QFT effects?

Conclusions

- No Violation to Weinberg's theorem
- Big quantum corrections due to conformal matter come from highly Trans-Planckian regime
  - Only MMC scalars and gravitons get big IR enhancements
- Agree to disagree about significance
- QFT corrections CAN be big
  - No general rule yet