Holographic Neutron Stars

Kyriakos Papadodimas
University of Amsterdam

work with Jan de Boer and Erik Verlinde

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Introduction

Neutron Stars in AdS/CFT

Our main goal is to construct a "neutron star" in AdS and understand its dynamics in the boundary CFT.

Motivations:

- **Gravitational Collapse in AdS/CFT**

  • Neutron Stars are dense objects which do collapse if they get too big.

  • Good starting point to study black hole formation.

- **Holography**

  • Macroscopic objects in AdS, no horizon, may be easier to understand holographically than black holes.

  • What is the meaning of the gravitational field of the star in the boundary theory?

  • How is their radial profile of the star encoded in the boundary theory?

Some more motivations

- High Density Gauge theories
- Astrophysics ?

An ideal Neutron star

- Real world neutron stars are complicated - Equation of State (nuclear matter) is not well understood.
- We are interested in an idealized neutron star:

  • A system of identical fermions, which interact only gravitationally.

  • Fermions do not have to be neutrons, so more generally we can call them "degenerate stars".
- Fermions do not collapse because of degeneracy pressure.
- Degenerate fermions have $P \neq 0$ even at $T = 0$.

The Chandrasekhar/Oppenheimer-Volkoff Limit

- Consider N degenerate fermions of mass $m_{fermion}$.
- Degeneracy pressure cannot support gravitational attraction when:

$$N_{crit} \gtrsim \left(\frac{m_{Planck}}{m_{fermion}}\right)^3 \tag{1}$$

- Compute the energy of N fermions in a box of size R as the sum of gravitational potential energy + kinetic energy. Then energy is unbounded from below for $N > N_{crit}$.
- More precisely: Einstein equations + degenerate fermionic fluid have no static solutions for $N > N_{crit}$.

Neutron stars and Black Holes

- If the mass of a degenerate star exceeds the Chandrasekhar/OV limit, then the star becomes unstable and starts collapsing.
- Understanding the endpoint of the collapse is in general a complicated question (for example in the real world "white dwarf" $\longrightarrow$ "neutron star" $\longrightarrow$ "quark star (??)" $\longrightarrow$ "black hole").
- For our idealized fermions the natural endpoint would be a black hole.

Degenerate stars in AdS

AdS/CFT correspondence

- Any theory of quantum gravity in AdS$_{d+1}$ space is holographically dual to a d-dimensional CFT.
- Consider AdS$_{d+1}$ in global coordinates

$$ds^2 = -(1+r^2)dt^2 + (1+r^2)^{-1}dr^2 + r^2 d\Omega_{d-1}^2 \qquad (2)$$

- Dual CFT lives on $S^{d-1} \times$ time
- Hilbert spaces of two systems are equivalent.
- Degenerate star in AdS corresponds to some state of the CFT on the cylinder.

States and Operators

- State Operator Map:
  states on S^{d-1} $\Leftrightarrow$ local operators at the origin in R^d.

$$\Phi(0)|0\rangle \Rightarrow |\Phi\rangle \tag{3}$$

- The energy of the state is given by the conformal dimension of the operator

$$E = \frac{\Delta}{R} \tag{4}$$

- Other quantum numbers of $|\Phi\rangle$ are determined by those of $\Phi(0)$.
- A neutron star in the bulk will correspond to a certain "neutron star operator" in the CFT.

Single-fermion states

- Consider a fermionic "single trace operator" $\Psi(x)$ of conformal dimension Δ_0.
- For example in the $\mathcal{N} = 4$ SYM it could be a descendant of a chiral primary:

$$\Psi(x) = \{Q, Tr\phi^k\} \sim Tr(\lambda\phi^k) \tag{5}$$

(which is protected at strong coupling)

- A state corresponding to a single Ψ fermion in its ground state can be represented by the operator

$$\Psi(0) \tag{6}$$

and has energy $E = \Delta_0$.

Excited single-fermion states

- The conformal descendants in the same multiplet correspond to a single fermion in an excited state (moving in AdS) and can be written as

$$\partial_{\mu_1}...\partial_{\mu_n}\Psi(0) \tag{7}$$

with energy $E = \Delta_0 + n$.

- The operator Ψ is dual to a field ψ in the bulk, which obeys a Klein-Gordon (or Dirac) equation

$$(\Box + m^2)\psi = 0 \tag{8}$$

with mass $m \approx \Delta_0$ for large Δ_0.

- The single particle states in the bulk correspond to solutions of this equation and have the form

$$\psi_{n,l,m}(r,\Omega) = f_{n,l}(r)Y_{n-2l,m}(\Omega) \tag{9}$$

Multi-fermion states

- Now we consider a state with 2 fermions. We want to find the corresponding operator.
- In AdS/CFT multi-particle states in the bulk are dual to multi-trace operators.
- Since the operator Ψ obeys Fermi statistics we have

$$\Psi(x)\Psi(y) = -\Psi(y)\Psi(x) \tag{10}$$

which implies

$$:\Psi(0)^2 := 0 \tag{11}$$

- The lowest energy state with 2 fermions made out of Ψ corresponds to the operator

$$:\Psi(0)\partial_\mu\Psi(0): \tag{12}$$

and has conformal dimension $2\Delta_0 + 1$.

"Degenerate operators" at infinite N

- More generally the operator corresponding to the ground state of many fermions in the bulk is

$$\Phi = \Psi \prod_i \partial_i \Psi \prod_{i,j} \partial_i \partial_j \Psi \ldots \prod_{i_1,\ldots,i_{n_F}} \partial_{i_1} \ldots \partial_{i_{n_F}} \Psi \qquad (13)$$

- n_F is # of "filled shells" = "Fermi Level"
- The total number of particles is

$$\mathcal{N} = \sum_{n=0}^{n_F} \binom{n+d-1}{d-1} = \binom{n_F+d}{d} \qquad (14)$$

for AdS_{d+1}, and the total conformal dimension (energy)

$$\Delta = \sum (n + \Delta_0) \binom{n+d-1}{d-1} = \Delta_0 \mathcal{N} + \frac{d n_F}{d-1} \mathcal{N} \qquad (15)$$

First comments on interactions

- If we keep # fermions and Δ fixed and send $N \to \infty$ then our previous analysis is exact.

- This is based on 't Hooft large N counting and the factorization of correlators

$$\langle \Psi(x)\Psi(y)\mathcal{O}(z)\rangle \sim \frac{1}{N} \tag{16}$$

- The $\Delta =$ fixed, $N \to \infty$ limit is under control but not so interesting because our system is essentially a free Fock space of fermions.

- However: if we scale the number of fermions to infinity at the same time as $N \to \infty$, it is possible that the 't Hooft $\frac{1}{N}$ suppression is compensated by the large number of diagrams between pairs of particles.

The scaling limit

- What limit do we want to take?
- In the bulk the gravitational backreaction of an object of mass M is of the order

$$\frac{G_{newton} M}{\ell^2} \tag{17}$$

where ℓ is the radius of AdS$_5$.

- We also have $G_{newton} \sim \frac{1}{N^2}$.
- To have nontrivial backreaction we need $M \equiv \Delta \sim N^2$.
- We consider "degenerate fermionic operators" of conformal dimension $\Delta = \mu N^2$.
- When $\mu \ll 1$ the system is approximately free, for $\mu \sim 1$ very complicated.

Bulk without Backreaction

Quantum states in AdS

- Operators of the form $\partial_\mu....\partial_\nu\Psi$ are single fermion states in the bulk with wavefunctions

$$\psi_{n,l,m}(r,\Omega) = f_{n,l}(r)Y_{n-2l,m}(\Omega) \tag{18}$$

- The composite degenerate operator

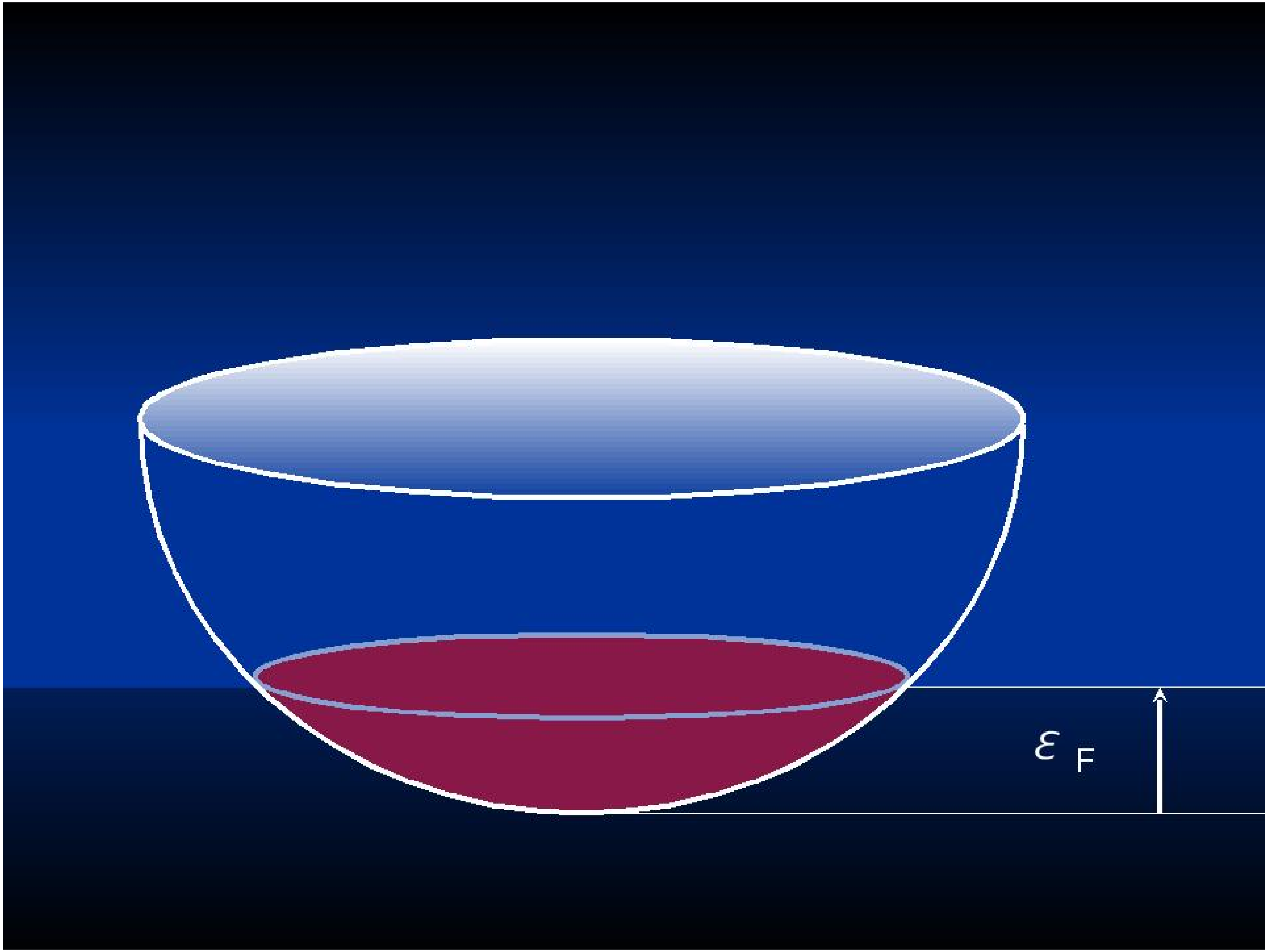
$$\Phi = \Psi\prod_i \partial_i\Psi\prod_{i,j}\partial_i\partial_j\Psi ... \prod_{i_1,...,i_{n_F}}\partial_{i_1}...\partial_{i_{n_F}}\Psi \tag{19}$$

corresponds to a Slater determinant of many fermions in the bulk.

- We will work in a limit where the number of fermions goes to infinity.

Trapped Fermionic Gas in AdS

Bulk without Backreaction



Fluid Approximation

- If number of fermions is very large it is better to describe the state in a fluid approximation

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu} \tag{20}$$

- ρ and P are given by the Equation of State of a degenerate fermionic gas.
- It is OK to use the flat space EOS even in AdS, if the number of fermions is very large.

Fermionic Equation of State

- Local Fermi Energy μ_F given by

$$\mu_F = \sqrt{m_f^2 + k_F^2} \tag{21}$$

- Energy density

$$\rho = \int_0^{k_F} k^{d-1}\sqrt{k^2+m^2}\,\frac{dk}{(2\pi)^d} \tag{22}$$

- Pressure

$$P = \frac{1}{d}\int_0^{k_F} \frac{k^{d+1}}{\sqrt{k^2+m^2}}\,\frac{dk}{(2\pi)^d} \tag{23}$$

- Particle number density $n = \frac{k_F^d}{d(2\pi)^d}$

The star, no self-gravitation

- Start with fixed, non-dynamical background metric

$$ds^2 = -A(r)^2 dt^2 + B(r)^2 dr^2 + r^2 d\Omega_3^2 \qquad (24)$$

- Consider a spherically symmetric profile $\mu_F(r)$ for the fluid.
- Equation for hydrostatic equilibrium

$$\nabla^\mu T_{\mu\nu} = 0 \qquad (25)$$

- Solution is given by "Tolman factor"

$$\mu_F(r) = \frac{\epsilon_F}{A(r)}, \qquad \epsilon_F = const \qquad (26)$$

- Edge of the star when $\mu_F(R) = m_F$. At that point the pressure and density go to zero.

Particle Number and Energy

- As a check of the fluid approximation we can compute the total mass and particle number

$$
\begin{aligned}
M &= \int_0^R dr A(r) B(r) r^3 \rho(r) \\
\mathcal{N} &= \int_0^R dr B(r) r^3 n(r)
\end{aligned}
\tag{27}
$$

where $\rho(r), n(r)$ are to be computed using the profile

$$
\mu_F(r) = \frac{\epsilon_F}{A(r)}, \qquad \epsilon_F = const \tag{28}
$$

- Computing the integrals one finds agreement with the exact counting in the gauge theory.

Including Backreaction

The Tolman-Oppenheimer-Volkoff equations

- We assume spherically symmetric and static solutions

$$ds^2 = -A(r)^2 dt^2 + B(r)^2 dr^2 + r^2 d\Omega_3^2 \tag{29}$$

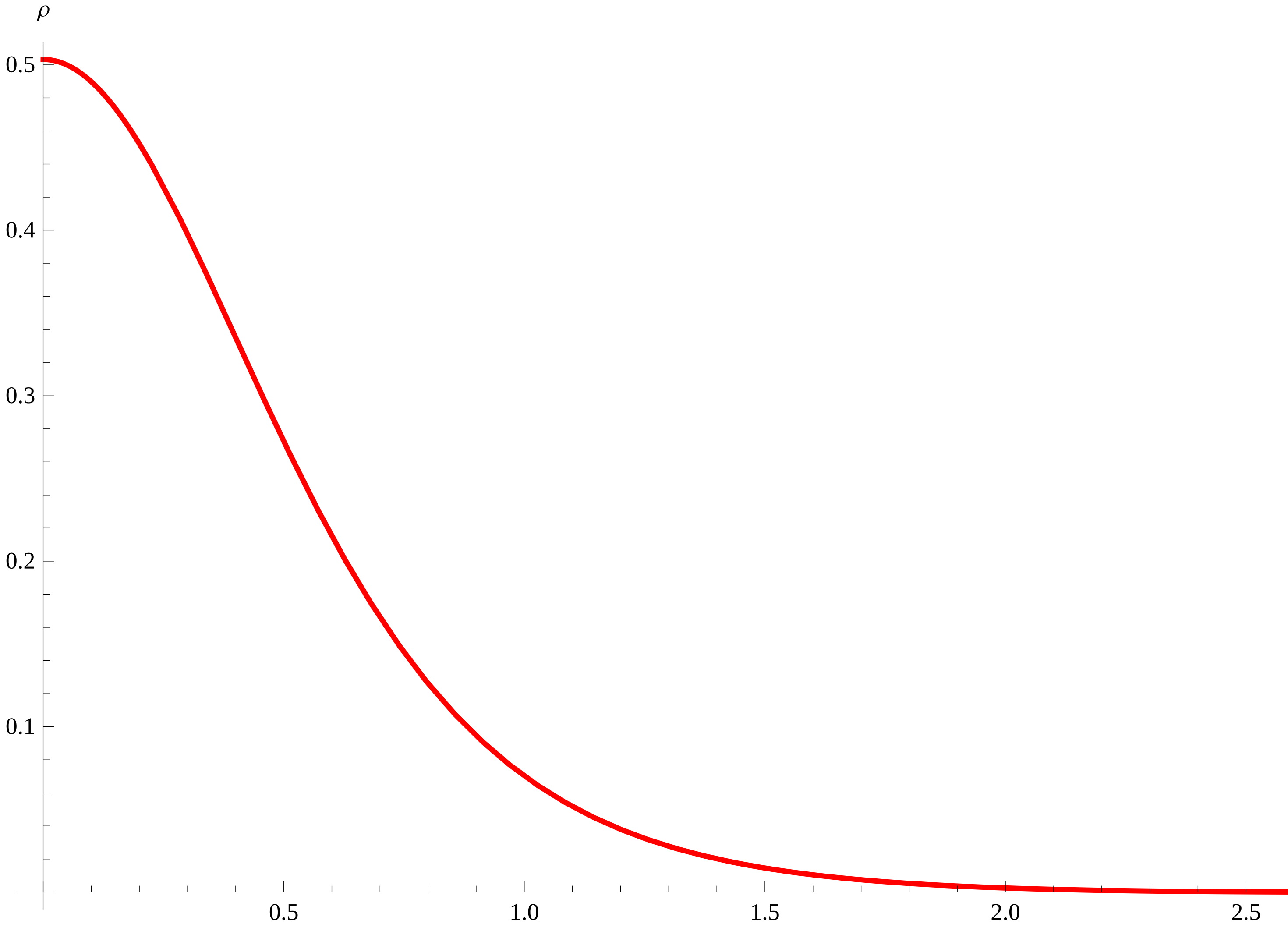
- And we have to solve Einstein's equations

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} &= 8\pi G T_{\mu\nu} \\ T_{\mu\nu} &= (\rho + P) u_\mu u_\nu + P g_{\mu\nu} \end{aligned} \tag{30}$$

- We start with a density ρ_0 at the center of the star and then integrate the equations outwards.

Numerical Solution: Density profile

Including
Backreaction

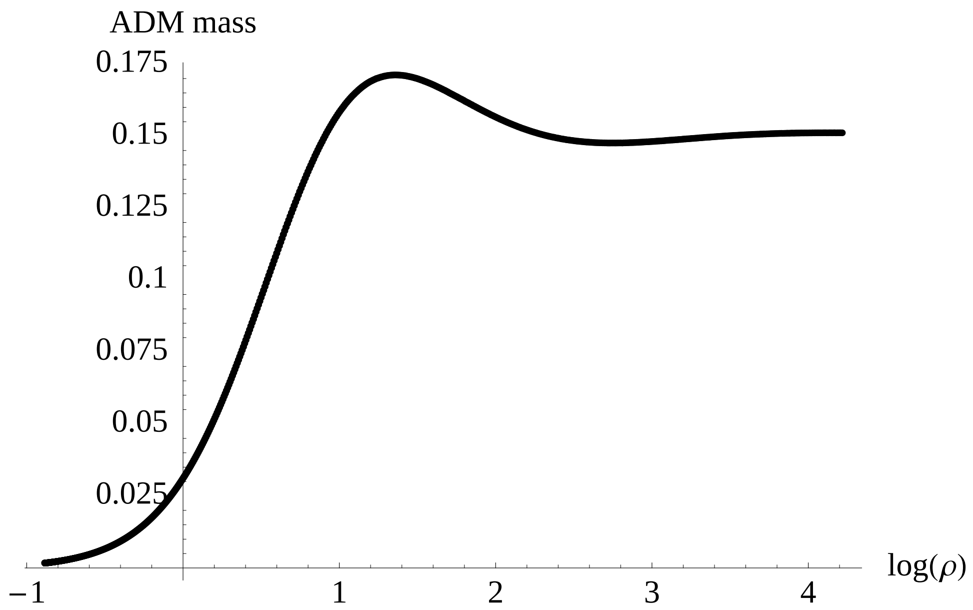


Numerical Solutions

- One finds spherical star-like objects.
- For fixed total number of fermions, increasing G_{newton} makes the star more compact.
- Increasing G_{newton} lowers the ADM mass $\rightarrow$ binding energy
- What happens if we put too many fermions? Are the stars stable?

The Chandrasekhar bound in AdS

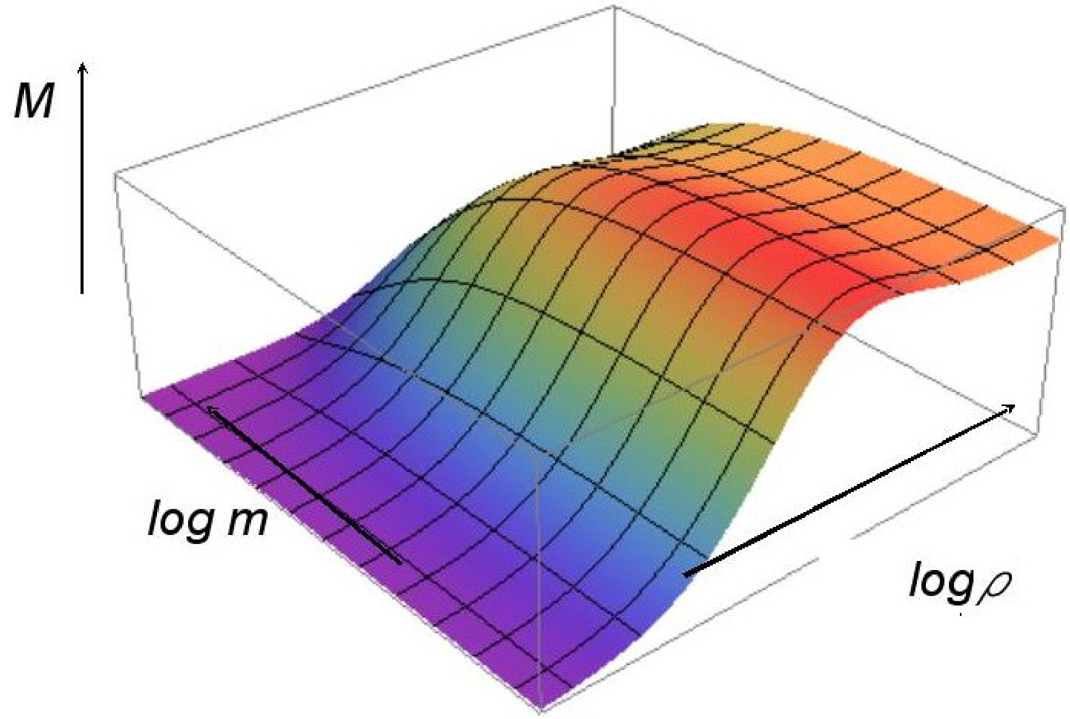
- Below we plot the ADM mass as a function of the density at the center.



- Solutions for which $\frac{dM}{d\rho_0} < 0$ are unstable towards radial density perturbations.
- There is a maximum critical mass M_c of a degenerate fermionic star in AdS.

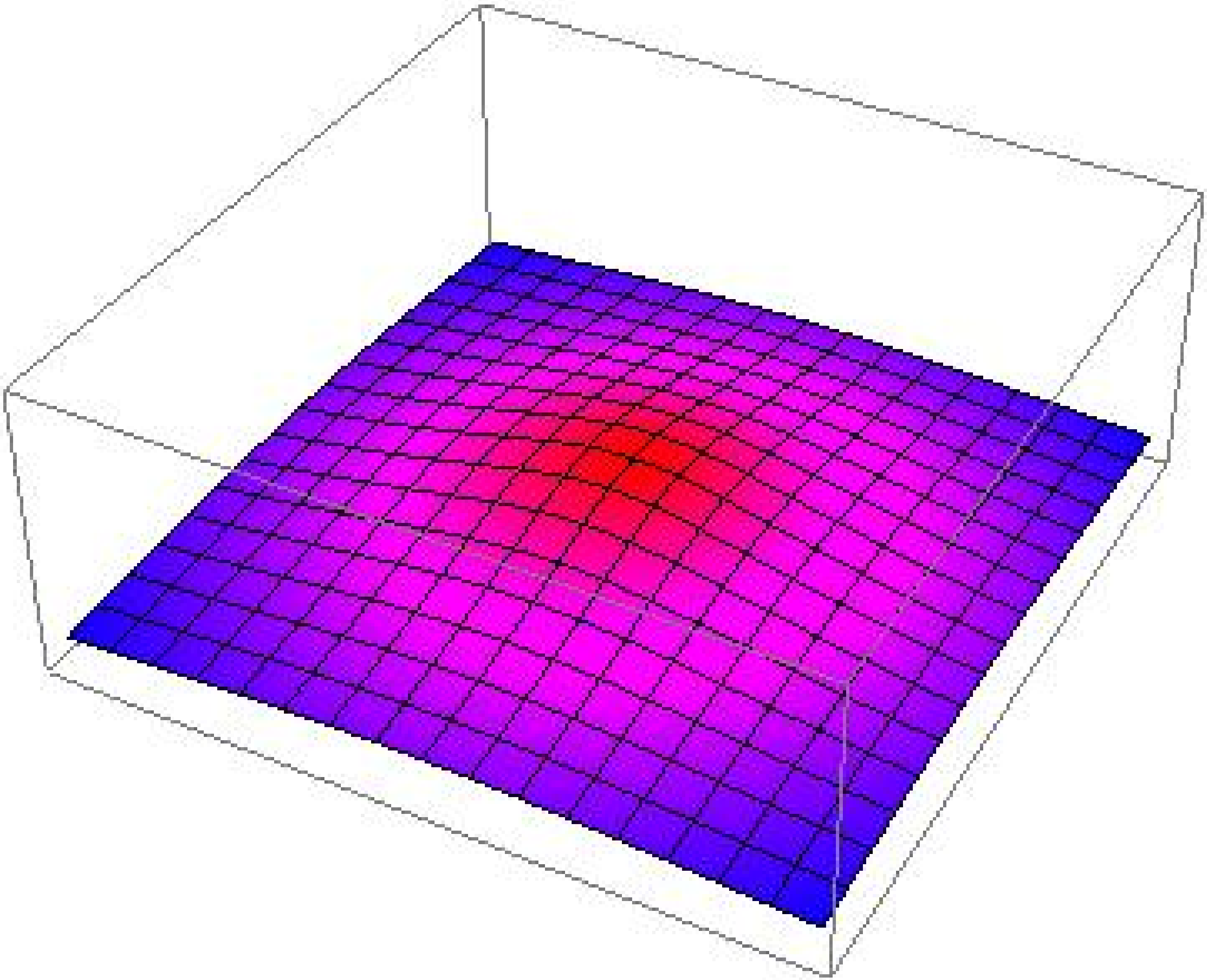
Dependence on fermion mass

Including
Backreaction



- The critical mass depends on the mass of the fermions
- For light fermions the critical star is bigger than AdS radius and relativistic, while for heavy fermions it is smaller and non-relativistic.

Including
Backreaction



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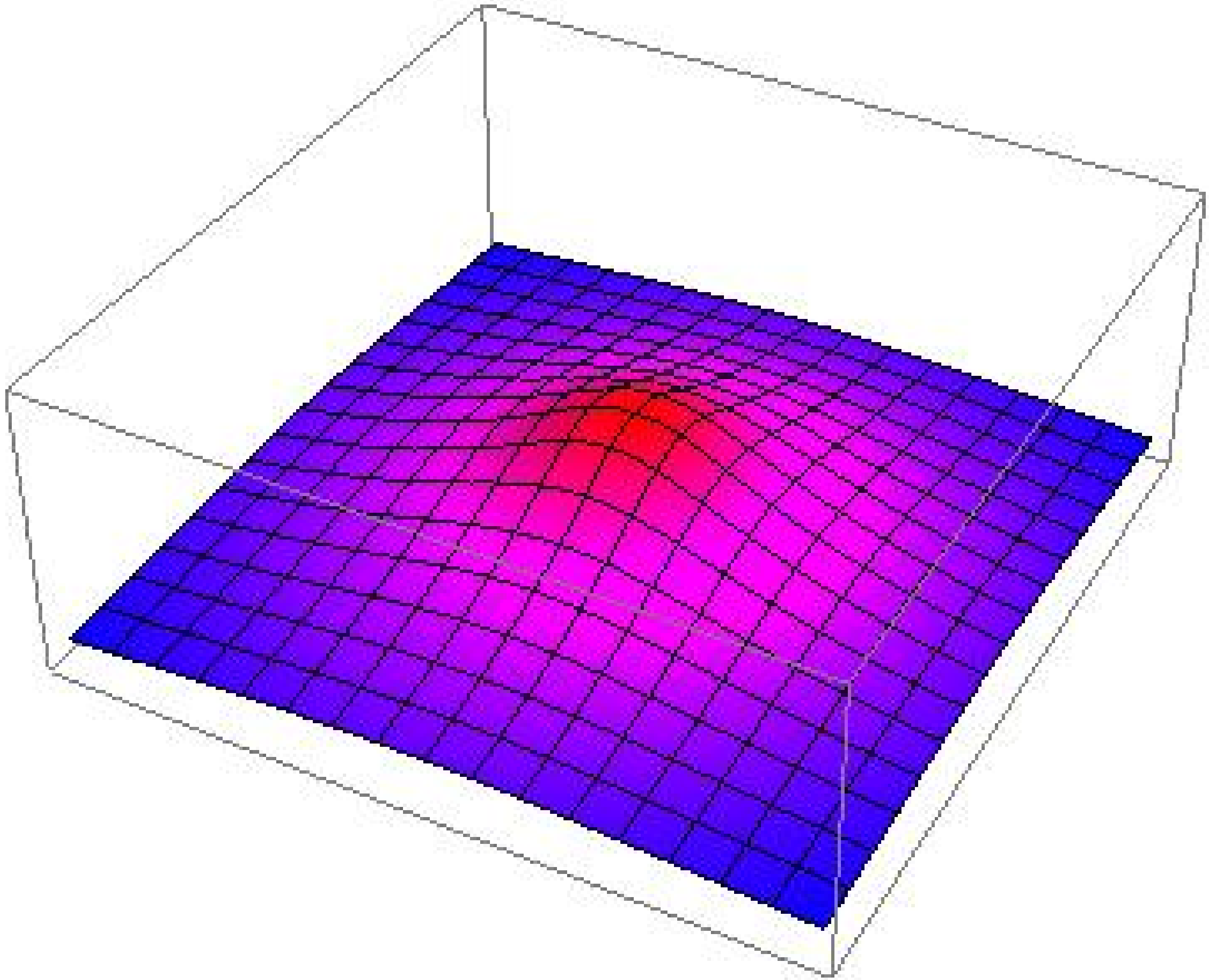

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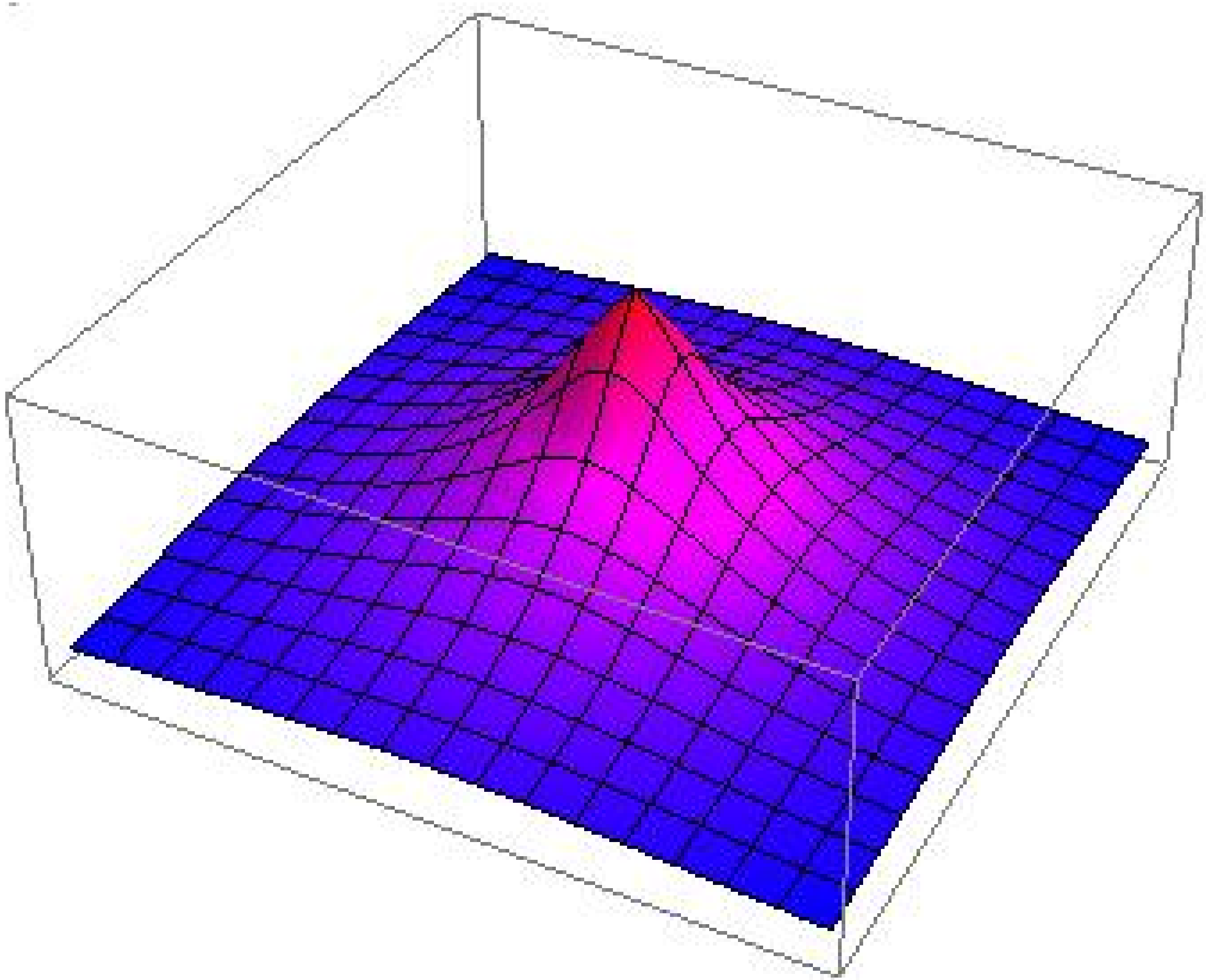

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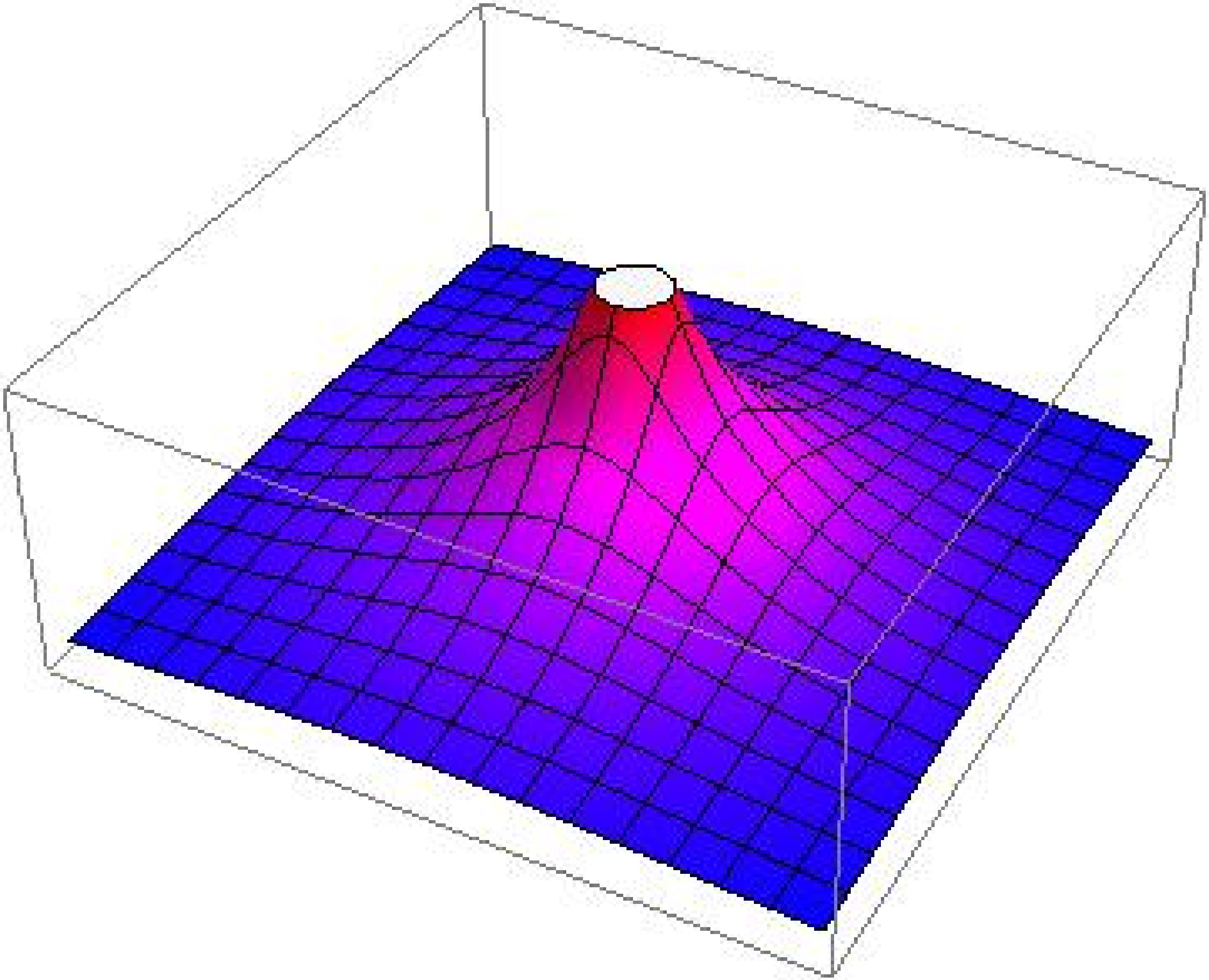

The instability

- At the point where $\frac{dM}{d\rho_0} = 0$ there is a linearized perturbation of the solution with $\omega = 0$ which becomes tachyonic for larger ρ_0.
- This tachyonic mode corresponds to radial density waves, which move fermions towards the center and leads to the collapse of the star.
- There are no *static* solutions with $M > M_{crit}$ (except for Black Holes)

Endpoint of the collapse

- In AdS_5 the critical mass of a degenerate fermionic star is slightly above the "smallest big AdS black hole".
- Notice that the neutron star has $T = 0$ while the black hole $T > 0$.
- We have entropy production.

CFT intepretation

What is the collapse in the CFT

- Initial State: many fermions. These are gauge singlets or "glueballs" of the CFT.
- Final state: black hole, dual to a quark-gluon plasma phase.
- If we try to place too many "glueballs" on the 3-sphere, even at zero temperature, they will deconfine.
- Gravitational Collapse = High density deconfinement "phase transition" in the CFT.

Gravitational Backreaction in the CFT 1

CFT intepretation

- At infinite N the correlators of operators in the CFT factorize. This implies that the conformal dimension of a multi-trace operator is the sum of conformal dimensions of constituents

$$\Delta(:\phi_1\phi_2:) = \Delta(\phi_1) + \Delta(\phi_2) \tag{31}$$

- If we keep $\frac{1}{N}$ corrections then there will be an anomalous dimension or "binding energy"

$$\Delta(:\phi_1\phi_2:) = \Delta(\phi_1) + \Delta(\phi_2) - \frac{\delta}{N^2} \tag{32}$$

- To compute the binding energy $\frac{\delta}{N^2}$ we need the 4-point function $\langle\phi_1\phi_1\phi_2\phi_2\rangle$ to order $\frac{1}{N^2}$.

Gravitational Backreaction in the CFT 2

CFT intepretation

- In the bulk the gravitational backreaction can be understood as graviton exchange between the fermions.
- In AdS/CFT the bulk graviton $g_{\mu\nu}$ is mapped to the stress-energy tensor $T_{\mu\nu}$ of the CFT.
- So we expect that the gravitational interaction must be related to $T_{\mu\nu}$ exchange between ϕ_1, ϕ_2 in the CFT.
- The contribution of $T_{\mu\nu}$ to the 4-point function $\langle \phi_1 \phi_1 \phi_2 \phi_2 \rangle$ is fixed by conformal invariance.
- The Ward identities fix $\langle \phi_i \phi_i T \rangle \sim \Delta_i$ while $\langle TT \rangle \sim N^2$.

- Then one finds

$$\langle \phi_1 \phi_1 \phi_2 \phi_2 \rangle_T \sim \frac{\Delta_1 \Delta_2}{N^2} \tag{33}$$

  Which gives a binding energy of the order

$$\frac{\Delta_1 \Delta_2}{N^2} \sim \frac{G_N m_1 m_2}{\ell^2} \qquad ! \tag{34}$$

- This can be made more precise, one finds that the gravitational interaction between two particles in the bulk can be exactly reproduced by $T_{\mu\nu}$ exchange on the boundary (Conformal Partial Wave vs Witten diagram).

Summing up the corrections

- Since we can reproduce the gravitational interaction between 2 particles from the CFT it should be possilbe to sum up over all pairs.
- This should reproduce the gravitational binding energy of the star in the “Newtonian” approximation.
- To get the full General Relativity answer we also need the self-interactions of gravitons.
- The collapse should be visible even in the Newtonian limit.

Further Questions

CFT intepretation

- How big is the effect of other particles and interactions? Is there an AdS/CFT setup in which they can be neglected?
- How do correlators look in the presence of the star? (see also V. Hubeny, H. Liu, M. Rangamani)
- What is the tachyonic mode on the boundary CFT (radial instability) ?
- Can we reconstruct the bulk spacetime+star from the CFT with only $T_{\mu\nu}$ and Ψ?
- What do Einstein's equations mean in the CFT?
- Can we study the dynamics of the collapse in the CFT?