Black Holes in Non-Relativistic Gravity

Univ. of Crete

(in progress)

with E. Kiritsis

Recent interest in non-relativistic QFT: Lorentz invariance explicitly broken

Horava 0901.3775, 0812.4287
: quantum gravity proposal of Lifshitz type

$$\vec{x} = \ell \vec{x}', \quad t = \ell^z t' \qquad UV\ (z>1)$$

by adding higher spatial (no time) derivatives

→ Power-counting renormalizability

E.g. $$S_{free} = \int dt\, d^D x \left[\frac{1}{2} \dot{\phi}^2 - \frac{1}{2} (\partial_i^z \phi)^2 \right]$$

$$\phi(\vec{x},t) = \ell^{\frac{z-D}{2}} \phi'(\vec{x}',t') \to G(\omega,\vec{k}) \sim \frac{1}{\omega^2 - \vec{k}^{2z}}$$

$$S = \frac{1}{\kappa^2} \int dt\, d^D x \left[\dot{g}_{ij}^2 - \cdots \right]$$

$$\sim \frac{1}{\kappa^2} \cdot \ell^{z+D} \cdot \left[\frac{1}{\ell^{2z}} - \cdots \right]$$

$$\sim \frac{1}{\kappa^2} \cdot \ell^{D-z} \quad - \cdots$$

$$\kappa \sim \ell^{\frac{D-z}{2}}$$

$z = D = 3$ makes κ dim-less

$$S \sim \frac{1}{\kappa^2} \int dt\, d^D x \left[\frac{1}{\ell^6} - \cdots \right]$$

$$\cdots \sim \nabla_k R_{ij} \nabla^k R^{ij},\ \nabla_k R_{ij} \nabla^i R^{jk},$$

$$R \Delta R,\ R^{ij} \Delta R_{ij}$$

$$R^3,\ R^i_j R^j_k R^k_\ell,\ R R_{ij} R^{ij}$$

Horava's gravity :
Preferred foliation of spacetime

space-dependent changes in time LOST

$$N(t, x^i)\ ,\ N^i(t, x^j),\ g_{ij}(t, x^k)$$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$
$$= -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$
$$= (-N^2 + N^i N_i) dt^2 + 2 N_i dx^i dt + g_{ij} dx^i dx^j$$

- Confusion with N : Is $N = N(x, t)$ or only $N = N(t)$

- $N = N(t)$ "projectable" :
  unnecessary extra condition
  (still consistent)
  Different analysis, non-local, mimic DM, full degree, BH, "

$$S_H = \int dt\, d^3x \sqrt{g}\, N \left[T - V(g_{ij}, g_{ij,k}, \cdots) \right]$$

- $T = \frac{2}{\kappa^2} \left(K_{ij} K^{ij} - \lambda K^2 \right)$

$$= \frac{2}{\kappa^2} K_{ij}\, G^{ijkl} K_{kl}$$

$$K_{ij} = \frac{1}{2N} \left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \right)$$

$$G^{ijkl} = \frac{1}{2} \left(g^{ik} g^{jl} + g^{il} g^{jk} \right) - \lambda g^{ij} g^{kl}$$

- $V = \cdots\cdots\cdots + \zeta R_{ij} R^{ij} + \eta R^2 + \xi R + \sigma$

"Detailed-Balance"

E.g. $$S = \int dt\, d^D x \left[\frac{1}{2} \dot{\phi}^2 - \frac{1}{2} \left(\frac{\delta W}{\delta \phi} \right)^2 \right]$$

$$W[\phi] = \int d^D x \left[\frac{1}{2} \partial_i \phi \partial^i \phi + \frac{m}{2} \phi^2 \right]$$

$$V = \frac{\kappa^2}{8} \frac{1}{\sqrt{g}} \frac{\delta W}{\delta g_{ij}} G_{ijkl} \frac{1}{\sqrt{g}} \frac{\delta W}{\delta g_{kl}}$$

some $W[g_{ij}]$

$$\rightsquigarrow W = \frac{1}{w^2} \int \mathrm{Tr} \left(\Gamma \wedge \Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma \right)$$
$$+ \mu \int d^3 x \sqrt{g} \, (R - 2\Lambda)$$

$$\frac{1}{\sqrt{g}} \frac{\delta W}{\delta g_{ij}} = \frac{1}{w^2} C^{ij} - \frac{\mu}{2} \left(G^{ij} + \Lambda g^{ij} \right)$$

$$S_{DB} = \int dt\, d^3 x \sqrt{g} N \Big[\frac{2}{\kappa^2} \left(K_{ij} K^{ij} - \lambda K^2 \right)$$
$$- \frac{\kappa^2}{2 w^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2 w^2} \epsilon^{ijk} R_{il} \nabla_j R^l_k$$
$$- \frac{\kappa^2 \mu^2}{\cdots} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2 (1 - 4\lambda)}{\cdots} R^2$$

Our action

$$S = \int dt\, d^3x \sqrt{g}\, N \Big[\alpha (K_{ij}K^{ij} - \lambda K^2) + \beta C_{ij}C^{ij} + \gamma \epsilon^{ijk} R_{il} \nabla_j R^l_k + \zeta R_{ij}R^{ij} + \eta R^2 + \xi R + \sigma \Big]$$

$$\text{IR}: S \to \frac{1}{16\pi G} \int d^4x \sqrt{g_4} (R_4 - 2\tilde{\Lambda})$$

But at what distance does the first correction appears?

- arbitrary couplings, $\xi \to 1$
- $\lambda = 1 \to R_{(4)} + R^2_{(3)} + \dots$

Cosmology

$$ds^2 = -N^2(t)dt^2 + a^2(t)\gamma_{ij}dx^i dx^j$$

0904.0829 Calgani
0904.1334 Kiritsis, GK

Background Cosmology

$$3\alpha(3\lambda-1)H^2 = \rho - \sigma - \frac{6k\xi}{a^2} - \frac{12k^2(J+3\ldots}{a^4}$$

- $\sigma_{DB} = \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)} > 0$ (for $\lambda > \frac{1}{3}$)
  $\rightsquigarrow$ eff. cosm. constant < 0
- extra term scaling as dark radiation may provide bounce
- If extra cubics

$$S_{R^3} = \int dt\, d^3x \sqrt{g}\, N\left[\delta_1 R_{ij} R^i_k R^{jk} + \delta_2 R_{ij}R^{ij}R + \delta_3 R^3\right]$$

$$\rightsquigarrow (3\lambda-1)H^2 = \cdots - (3\delta_1 + 3^2\delta_2 + 3^3\delta_3)\frac{(2k)^3}{a^6}$$

$\rightsquigarrow$ even stronger effect of

- In the cosmological era where six-derivative terms dominate the dynamics, the speed of light is almost infinite → horizon problem
- Effects of spatial curvature amplified in early universe $\frac{k^3}{a^6}$, if same order as matter early, suppress curvature contributions later → flatness problem
- Scalars at early times have zero dimension → fluctuations scale-independent.

Approximate study gives scale-invariant perturbations generated.

0904.2190 Mukohyama

0905.3821 Gao, Wang further
Brandenberger, Riotto
(breaking D-B maintains scale-

Black Holes

$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_k^2$$

- 0904.1595 Lu, Mei, Pope

Detailed-Balance

$$f(r) = 1 - \frac{2\Lambda}{3} r^2 + c\, r^{\frac{2\lambda \pm \sqrt{6\lambda - 2}}{\lambda - 1}}$$

$$N(r) = r^{-\frac{1+3\lambda \pm \sqrt{6\lambda - 2}}{\lambda - 1}} \sqrt{f}$$

For a special slight deformation from D-B Schwarzschild is recovered

Also thermodynamics has been studied

• 0905.0477 Kehagias, Sfetsos
Detailed-balance for quadratic couplings, cosmological constant=0

$$N^2 = f = 1 + \frac{16\mu^2}{\kappa^2} r^2 - \frac{16\mu^2}{\kappa^2}\sqrt{r(c+r^3)}$$

$$\simeq 1 - \frac{c}{r} + O\left(\frac{1}{r^4}\right)$$

Mathematically, versions of detailed-balance make the system linear

Our solution

$$ds^2 = -\hat{N}(r)^2 f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_k^2$$

- $3J + 8\eta = 0$

$$f(r) = k + \frac{\xi}{4(J+3\eta)} r^2 \pm \sqrt{r\left[c + \frac{2}{3J}\left(\sigma + \frac{6\xi^2}{J}\right) r^3\right]}$$

$$\hat{N}(r) = 1$$

Schwarzschild recovered

- deformed D-B, Lu, Mei, Pope
- $\sigma = 0$ Kehagias, Sfetsos

- $\xi + 3\eta = 0$

$$f(r) = k + \frac{\sigma}{6\xi} r^2 + \frac{\xi}{\eta} r^2 y(r)$$

$$\left|\sqrt{1-3y} \mp 1\right| e^{\pm\sqrt{1-3y}} = \left(\frac{r_0}{r}\right)^3$$

$$\hat{N}(r) = \frac{e^{\oplus(\sqrt{1-3y}-1)}}{\sqrt{1-3y}}$$

- Schwarzschild recovered
- singularity at $r=0$
- Entropy regular as $r_+ \to 0$

- $(J+3\eta)(3J+8\eta) \neq 0$

$$f(r) = k + \frac{\xi}{4(J+3\eta)} r^2 + g(r)$$

- $J(J+3\eta) > 0$

$$\ln\left[\left(\frac{r}{r_0}\right)^3 \left|\sqrt{1 - \frac{A}{B}\frac{g^2}{r^4}} \mp \frac{C}{2\sqrt{B}} \frac{|g|}{r^2}\right|\right] =$$

$$= \frac{2\sqrt{A}}{C} \arcsin\left(\pm\sqrt{\frac{A}{B}} \frac{|g|}{r^2}\right)$$

$$\hat{N}(r) = \sqrt{\frac{C}{6}} \left(1 - \frac{A}{B}\frac{g^2}{r^4}\right)^{-\frac{1}{2}} \times$$

$$\times e^{\mp\frac{2\sqrt{A}}{C}\left[\arcsin\left(\sqrt{\frac{A}{B}}\frac{|g|}{r^2}\right) - \arcsin\sqrt{\frac{2A}{3C}}\right]}$$

$A, B, C = \ldots J, \eta, \xi, \sigma \ldots$

- r bounded from below (regular)
- Schwarzschild recovered $\prec r^2$
- $\sigma = 0 \to$ for one correct branch

- $T(r_+)$, $M(r_+)$, $S(r_+)$
- For J, η small, maybe simpler

- $J(J+3\eta) < 0$

$$\left|\sqrt{B - A\frac{g^2}{r^4}} \mp \frac{C}{2}\frac{|g|}{r^2}\right| \left(\frac{r}{\tilde{r}_0}\right)^3 =$$

$$= \left|\sqrt{B - A\frac{g^2}{r^4}} \pm \sqrt{-A}\,\frac{|g|}{r^2}\right|^{-\frac{2\sqrt{-A}}{C}}$$

$$\hat{N}(r) = N_0\left(\frac{r_0}{r}\right)^3 \frac{\left(B - A\frac{g^2}{r^4}\right)^{-\frac{1}{2}}}{\left|\sqrt{B - A\frac{g^2}{r^4}} \mp \frac{C}{2}\frac{|g|}{r^2}\right|}$$

- solution singular $r \to 0$
- $C(J,\eta) > 0 \rightsquigarrow$ Schwarzschild $\prec r^2$

  $C \quad < 0 \rightsquigarrow$ semi-Schwarzsc.

- $\sigma = 0 \rightarrow \cdots$
- T, M, S

[illegible]

More comments

- BH's found are also solutions of the theory
$$\int d^4x \left[R_{(4)} + \sigma + \xi \, R_{(4)ij} R^{ij}_{(4)} + \eta \, R^2_{(4)} \right]$$
- Adding ... $R^3_{(3)}$ ... the singular bh's found hopefully will become regular
- Thermodynamics for non-relativistic bh's complicated.

Diffent ptcls can have different dispersion relations at high energies, see horizon at different distances, see different temperatures. More understanding.

Conclusions

- Cosmology may allow regular bouncing solutions
- Solution of horizon problem without inflation
- Scale-invariant pert/tions can be generated without graceful exit
- BH without shift, full study of non-linear system with arbitrary couplings, general solution found
- (Almost) always asymptotically $\frac{1}{r} + \mathcal{O}\left(\frac{1}{r^4}\right)$
- One region in parameters, regular sln., other singular
- Thermodynamics partially