# MHV RULES AND PURE YANG-MILLS AT ONE LOOP

Konstantinos Zoubos

Queen Mary, University of London

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**Based on:** A. Brandhuber, B. Spence, G. Travaglini & KZ, arXiv:0704.0245

# MOTIVATION: HIDDEN SIMPLICITY

- Setting: Yang-Mills perturbation theory
- Feynman diagrams are notoriously inefficient in calculating scattering amplitudes for many external particles
- It helps to ask the right questions
- SPINOR HELICITY FORMALISM
  - Fix external helicities
  - For $p^2=0$, decompose $\sigma^{\mu}_{\alpha\dot{\alpha}} p_{\mu} = p_{\alpha\dot{\alpha}} = \lambda_{\alpha}\tilde{\lambda}_{\dot{\alpha}}$

    [$\lambda_{\alpha}$ : Positive Chirality, $\tilde{\lambda}_{\dot{\alpha}}$ : Negative Chirality]
  - Spinor Products: $\langle ij\rangle = \epsilon_{\alpha\beta}\lambda_i^{\alpha}\lambda_j^{\beta}$

    $[ij] = \epsilon_{\dot{\alpha}\dot{\beta}}\tilde{\lambda}_i^{\dot{\alpha}}\lambda_j^{\dot{\beta}}$
- COLOUR ORDERING

  Decouple colour trace $\Rightarrow$ Partial Amplitudes

# MHV AMPLITUDES

- At tree level, amplitudes with 0 or 1 negative helicity gluons vanish
- Thus "Maximally Helicity Violating" amplitudes have two negative helicities
- Parke - Taylor formula

$$A_{MHV}(i,j) = \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

- Witten (2003) explored localisation properties of such amplitudes in the associated twistor space
- $\mathbb{CP}^3$: $(\lambda,\mu)$, where $\tilde{\lambda}\to i\frac{\partial}{\partial\mu}$
- Incidence relation: $\mu^{\dot\alpha} + x^{\dot\alpha\alpha}\lambda_\alpha = 0$

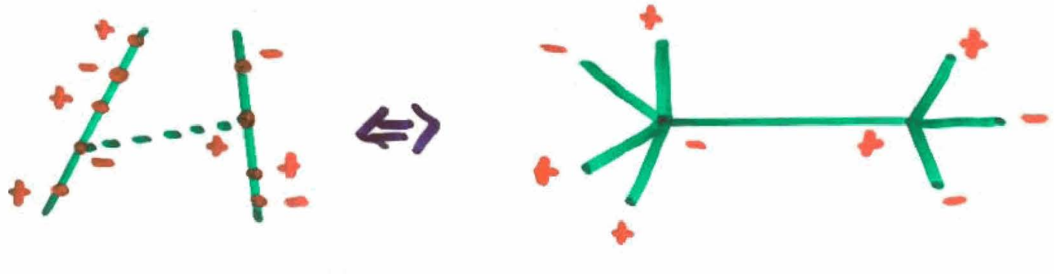
# MHV VERTICES

- Generic YM tree amplitudes supported on degree $d=q-1$ curves in twistor space

  $q$: # OF NEGATIVE HELICITY GLUONS

- MHV amplitudes ⇒ degree one curves. Incidence relation suggests to think of them as points in Minkowski space

- Cachazo-Svrček-Witten (2004): Construct tree-level non-MHV amplitudes by suitably joining MHV vertices. Propagators are just $1/p^2$



TWISTOR SPACE SPACETIME

- Simplifies calculations of tree amplitudes
- SUSY not necessary; Works for QCD too!

# OFF-SHELL CONTINUATION

- To think of MHV amplitudes as vertices, need to take some of the momenta off-shell

- How are the corresponding spinors defined?

- CSW use an arbitrary reference spinor $\tilde{\eta}_{\dot{\alpha}}$ :

$$\lambda_\alpha = p_{\alpha\dot{\alpha}}\,\tilde{\eta}^{\dot{\alpha}}$$

- Equivalently, for a null vector $\eta = \eta_\alpha \tilde{\eta}_{\dot{\alpha}}$ ,

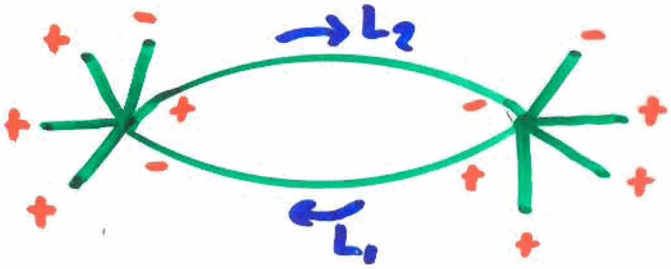
$$L = \ell + z\eta \qquad \left(z = \frac{L^2}{2L\cdot\eta}\right)$$

- Need to show covariance ($\eta$-independence)

# MHV RULES AT ONE LOOP?

- Twistor space picture ill-understood

- However spacetime picture works as expected!
  Brandhuber, Spence & Travaglini (2004) showed how to combine two tree MHV vertices to form the 1-loop MHV amplitude in N=4 SYM

$\sum \int dM$ 



- Reproduces result of Bern et al (1994)

- Has been extended to lower supersymmetry and "cut-constructible" parts of non-SUSY amplitudes

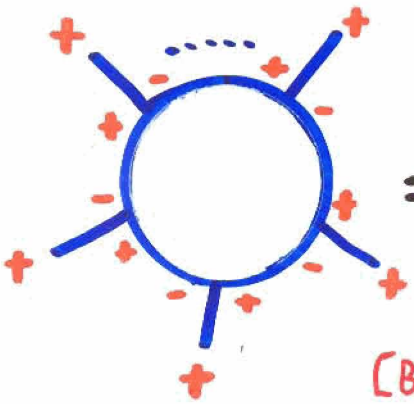
- For supersymmetric theories, BST (2005) showed:
  a) Covariance b) Discontinuities c) Soft, Collinear limits

# PROBLEMS AT ONE LOOP

- The BST procedure does not reproduce the rational parts of non-SUSY amplitudes

- Pure YM has amplitudes which cannot even be written down with MHV vertices!

- All-Plus :



$$= -\frac{i}{48\pi^2} \sum_{1\le i<j<k<l\le n} \frac{\langle ij\rangle[jk]\langle kl\rangle[li]}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

[Bern, Chalmers, Dixon, Kosower (1993)]

- Similar problem for the 1-minus amplitude

- Perhaps can extend off-shell and add as a new one-loop vertex in the theory?

  CSW′ (2004) : Naive extension doesn't work

- Could a lagrangian formulation help?

# THE MANSFIELD TRANSFORMATION

(2005)

- A similar transformation independently derived by Gorsky & Rosly (2005)

- MAIN IDEAS:

  - Consider Yang-Mills on the lightcone (restrict to physical polarisations)

  - Perform a canonical, nonlocal field redefinition

  - After the transformation, the lagrangian becomes an infinite expansion. Each term is an MHV vertex!

# YANG-MILLS ON THE LIGHTCONE

- LC coordinates: $x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$

$$z, \bar{z} = \frac{1}{\sqrt{2}}(x^1 \pm i x^2)$$

- LC gauge: $\eta \cdot A = A^- = 0$

$[\eta = (1,0,0,1)\ ,\ \eta^2 = 0]$

- YM lagrangian $\mathcal{L}_{YM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

$$\rightarrow \frac{1}{2}(\partial^- A^+)^2 - (\partial^- A^m)(\partial^m A^+) - i A^+ [\partial^- A^m, A^m]$$

$[m,n = 1,2]$

$$+ (\partial^- A^m)(\partial^+ A^m) \quad - \frac{1}{4} F_{mn} F^{mn}$$

- $A^+$ appears quadratically, no time $(x^-)$ derivatives: Can integrate out!

$$A^+ = (\partial^-)^{-2} (\partial^m \partial^- A^m + i [A^m, \partial^- A^m])$$

# LIGHTCONE YANG-MILLS LAGRANGIAN

- The final lagrangian depends only on the physical polarisations:

$$A_z \to + \quad , \quad \bar{A}_{\bar{z}} \to -$$

- Very simple structure:

$$\mathcal{L} = \mathcal{L}_{+-} + \mathcal{L}_{++-} + \mathcal{L}_{--+} + \mathcal{L}_{--++}$$

where:
$$\mathcal{L}_{+-} = -A_{\bar{z}} \Box A_z$$
$$\mathcal{L}_{++-} = 2i [A_z, \partial_+ \bar{A}_{\bar{z}}] (\partial_+)^{-1} (\partial_{\bar{z}} A_z)$$
$$\mathcal{L}_{--+} = 2i [\bar{A}_{\bar{z}}, \partial_+ A_z] (\partial_+)^{-1} (\partial_z \bar{A}_{\bar{z}})$$
$$\mathcal{L}_{--++} = -2 [\bar{A}_{\bar{z}}, \partial_+ A_z] (\partial_+)^{-2} [A_z, \partial_+ \bar{A}_{\bar{z}}]$$

- Note that this lagrangian contains a $(++-)$ vertex, which is not MHV

# MANSFIELD'S CHANGE OF VARIABLES

- We want to eliminate the $++-$ vertex!

- Mansfield accomplishes this by a field redefinition $A = A(B)$, $\bar{A} = \bar{A}(\bar{B}, B)$ after which the $\{$kinetic $+ \mathcal{L}_{++-}\}$ terms become a pure kinetic term:

$$\mathcal{L}_{+-}(A,\bar{A}) + \mathcal{L}_{++-}(A,\bar{A}) \longrightarrow \mathcal{L}_{+-}(B,\bar{B})$$

- Canonicality requirement: Would like the path integral measure $\int DA\, D\bar{A} = \int DB\, D\bar{B}$

- Conjugate momentum $p(A_z) \sim \partial_+ \bar{A}_{\bar{z}}$, so $\int DA\, D\bar{A} \sim (\mathrm{Det}\,\partial_+) \int DA\, Dp(A) = (\mathrm{Det}\,\partial_+) \int DB\, Dp(B)$ if $A \to B$ is canonical:

$$\partial_+ \bar{A}_{\bar{z}}(x^-, \vec{y}) = \int_\Sigma d^3x \, \frac{\delta B_z(x^-, \vec{x})}{\delta A_z(x^-, \vec{x})} \, \partial_+ \bar{B}_{\bar{z}}(x^-, \vec{x})$$

# THE MANSFIELD EXPANSION

- These two requirements (elimination of $\mathcal{L}_{++-}$ and canonicality) are sufficient to determine the change of variables

- $A, \bar{A}$ are power series in $B, \bar{B}$:

$$A = B + a_1 B^2 + a_2 B^3 + \cdots$$

$$\bar{A} = \bar{B} + \bar{a}_1 \bar{B} B + \bar{a}_2 B \bar{B} + \bar{a}_3 \bar{B} B^2 + \bar{a}_4 B \bar{B} B + \bar{a}_5 B^2 \bar{B} + \cdots$$

- Crucially, $\bar{A}$ is linear in $\bar{B}$. This means that, when inserted in $\mathcal{L}_{--+} + \mathcal{L}_{--++}$, all vertices will be of MHV type:

$$\mathcal{L}(A,\bar{A}) = \underbrace{\mathcal{L}_{+-} + \mathcal{L}_{++-}} + \mathcal{L}_{--+} + \mathcal{L}_{--++}$$

$$\Downarrow$$

$$\mathcal{L}(B,\bar{B}) = \mathcal{L}_{+-} + \mathcal{L}'_{--+} + \mathcal{L}'_{--++} + \mathcal{L}'_{--+++} + \cdots$$

- We have an MHV lagrangian!

# MANSFIELD-ETTLE-MORRIS COEFFICIENTS

- Ettle & Morris (2006) found a closed form for the coefficients in the expansion of $A, \bar{A}$:

$$A(\vec{p}) = \sum_{n=1}^{\infty} \int \frac{d^3p'}{(2\pi)^3} \cdots \frac{d^3p^n}{(2\pi)^3} (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{p}' - \cdots - \vec{p}^n)\, \Upsilon(\vec{p}, 1 \cdots n)\, B(\vec{p}')B(\vec{p}^2)\cdots B(\vec{p}^n)$$

$$\bar{A}(\vec{p}) = -\sum_{n=1}^{\infty} \int \frac{d^3p'}{(2\pi)^3} \cdots \frac{d^3p^n}{(2\pi)^3} (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{p}' - \cdots - \vec{p}^n)\, \Upsilon(\vec{p}, 1 \cdots n) \frac{1}{(p_+)^2} \sum_{s=1}^{n} (p_+^s)^2 B(\vec{p}') \cdots \bar{B}(\vec{p}^s) \cdots B(\vec{p}^n)$$

- The coefficients $\Upsilon(\vec{p}; 1 \cdots n) := \Upsilon(\vec{p}; \vec{p}', \vec{p}^2, \ldots, \vec{p}^n)$ are surprisingly simple:

$$\Upsilon(\vec{p}; 1 \cdots n) = (\sqrt{2}ig)^{n-1} \frac{p_+}{\sqrt{p_+^1 p_+^n}} \frac{1}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n-1, n \rangle}$$

- EM check that the expansion produces the MHV vertices

# OFF-SHELL CONTINUATION (LIGHTCONE)

- MHV rules require an arbitrary null reference vector $\eta = \eta_\alpha \tilde{\eta}_{\dot{\alpha}}$
- On the lightcone, it is natural to identify $\eta$ with the vector defining the lightcone surface
- Then can show that

$$\langle i j \rangle = \frac{\sqrt{2}}{\sqrt{p_+^i p_+^j}} \left[ p_+^i p_z^j - p_+^j p_z^i \right]$$

$$[ i j ] = \frac{\sqrt{2}}{\sqrt{p_+^i p_+^j}} \left[ p_+^i p_{\bar{z}}^j - p_+^j p_{\bar{z}}^i \right]$$

- This defines the spinor brackets for off-shell momenta

# WHY DOES IT WORK?

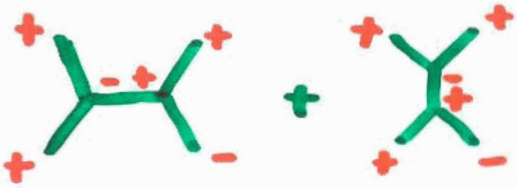
- $\mathcal{L}_{+-} + \mathcal{L}_{++-} = -\bar{A}_{\bar{z}}\Box A_z + 2i[A_z, \partial_+ \bar{A}_{\bar{z}}](\partial_+)^{-1}\partial_{\bar{z}} A_z$

  is a lightcone version of SELF-DUAL YM

- So Mansfield has mapped SDYM to a free theory!

- Makes sense at tree level: SDYM is a <u>classically</u> integrable system

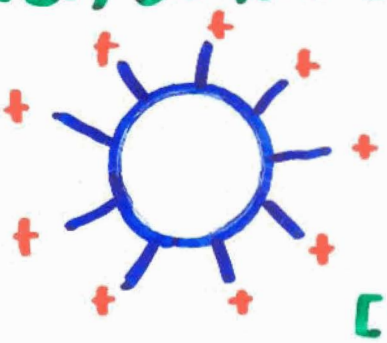
- Tree amplitudes vanish:



$= 0$

- $\mathcal{L}_{SDYM} \to \mathcal{L}_{FREE}$ consistent at tree level

- However, SDYM does have <u>one-loop</u> amplitudes!



$$\sim \sum_{1\le i<j<k<\ell\le n} \frac{\langle ij\rangle[jk]\langle k\ell\rangle[\ell i]}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}$$

[Same as all-plus in full YM]

# MANSFIELD AT ONE-LOOP ?

- By eliminating the ++- vertex of SDYM, we seem to have lost the all-plus amplitudes!
- Mansfield's approach does not seem consistent at 1-loop, even for SDYM
- For full YM, the usual problems with MHV (one-minus, rational terms) are still there
- Not a problem for supersymmetric theories, all of these amplitudes vanish
- In the following I will concentrate on the all-plus amplitude, which can be treated in the context of pure (non-SUSY) self-dual Yang-Mills

# TOWARDS THE MISSING AMPLITUDES

- Subtleties even at tree level: EQUIVALENCE THEOREM violation? $\langle A\bar{A}\cdots A\rangle_A \neq \langle B\bar{B}\cdots B\rangle_B$ but ET normally guarantees amplitudes are the same
- Analysed by Ettle, Fu, Fudger, Mansfield & Morris (2007): Recover three-point tree vertex
- Need to specify a suitable regularisation scheme before performing the transformation (In LCYM, all-plus is an $\epsilon/\epsilon$ effect [BST 2006])
- EFFMM choose dimensional regularisation
- We looked for a purely four-dimensional regularisation: Preserve the nice properties of Mansfield's original approach

# THE LIGHTCONE WORLDSHEET

- We were inspired by an approach to gauge/string duality initiated by Bardakci & Thorn 2001.

- Improves on early dual model ideas of the worldsheet as made of large planar diagrams [Nielsen-Olesen '70, Sakita-Virasoro '70, 't Hooft '74]

- Aim is to map an arbitrary planar diagram to a precise 4d worldsheet configuration


- Complicated worldsheet action

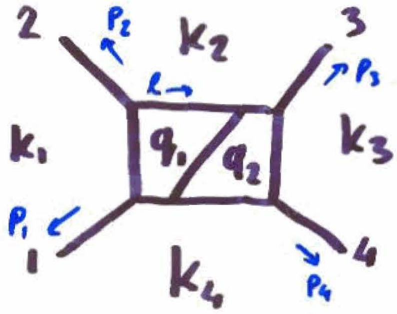
- Early stages: Understand how to perform lightcone gauge theory computations in a "worldsheet-friendly" way

# MAIN FEATURES

- In Chakrabarti, Qiu & Thorn (2005) these ideas are applied to explicit calculations in lightcone Yang-Mills

- The computations are essentially as in usual lightcone field theory, with two main differences:

  - CQT use the dual (region) momentum representation:




$$p_1 = k_1 - k_4$$
$$p_2 = k_2 - k_1$$
$$\vdots$$
$$l = q_1 - k_2$$

The line momenta are differences of the region momenta (consistent for planar diagrams)

# THE "WORLDSHEET FRIENDLY" REGULATOR

- To regulate an $\ell$-loop integral, CQT introduce in the integrand a factor of

$$e^{-\delta \sum_{i=1}^{\ell} \vec{q}_i^{\,2}}$$

- Here $\vec{q}^{\,2} = 2 q_z q_{\bar{z}}$, i.e. affects only the two transverse directions

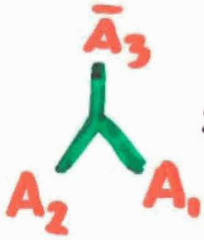
- Breaks even more of the Lorentz symmetry than the usual lightcone

- Breaks the shift symmetry of the region momenta in the $z, \bar{z}$ directions

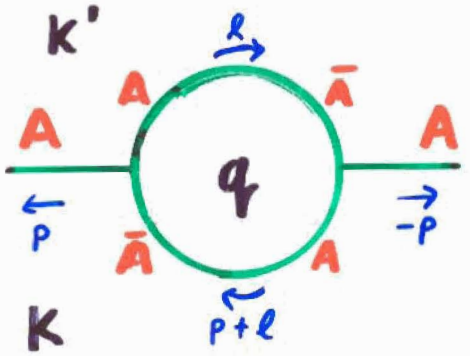
- Looks very "QFT-unfriendly". However CQT show that it reproduces known (helicity-violating and helicity-conserving) results at 1-loop

# THE GLUON SELFENERGY

- Following CQT, compute the helicity flipping gluon selfenergy in LCYM

- Need just ++− vertex : 

$= -2g\frac{p_+^3}{p_+^1 p_+^2}[p_+^1 p_{\bar{z}}^2 - p_+^2 p_{\bar{z}}^1]$

- $\Pi^{++} =$ 

$p = k' - k \,,\; \ell = q - k'$

- Calculation with the worldsheet-friendly regulator $\exp[-\delta(2q_z q_{\bar{z}})]$ results in a finite answer as $\delta \to 0$ :

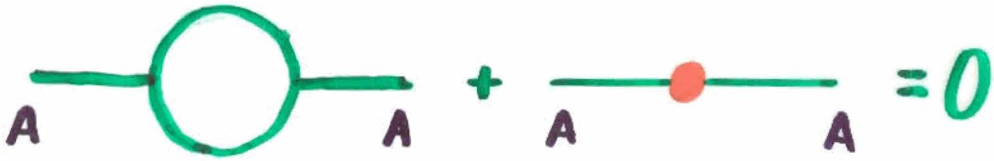
$$\Pi^{++} = \frac{g^2 N}{12\pi^2}\left[(k_{\bar{z}})^2 + (k'_{\bar{z}})^2 + k_{\bar{z}} k'_{\bar{z}}\right]$$

- Violates Lorentz invariance! Also explicit dependence on region momenta

- Can we repair this?

# THE CQT COUNTERTERM

- CQT add an explicit counterterm to the action, to precisely cancel the helicity-flipping selfenergy




- This is the only counterterm required for SDYM. In full YM need also $\Pi^{--}$, $\Gamma^{++-}$, $\Gamma^{+--}$

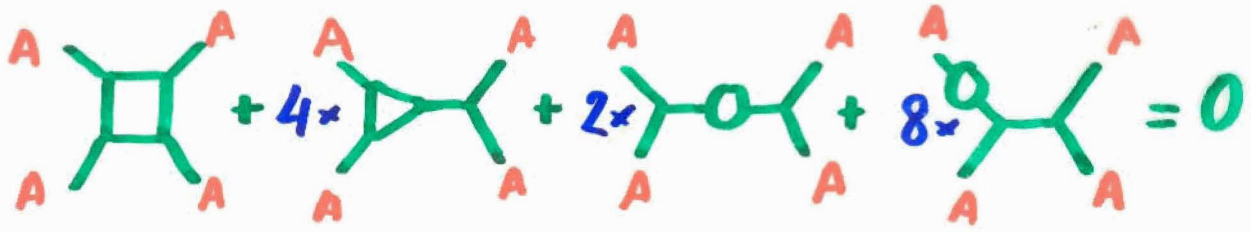
- The complete SDYM action is now:

$$\mathcal{L}_{SDYM} = \mathcal{L}_{+-} + \mathcal{L}_{++-} + \mathcal{L}_{CT}$$

- In dimensional regularisation, none of this would happen. $\Pi^{++}=0$ and there is no need for counterterms

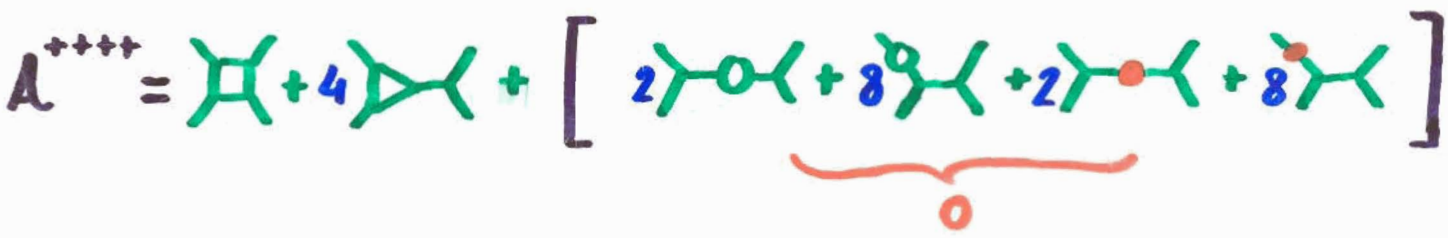
# A REMARKABLE IDENTITY

- CQT observe the following result (attributed to Zvi Bern)




- The contributions to the all-plus 4-point amplitude (all external legs on-shell) sum up to zero!

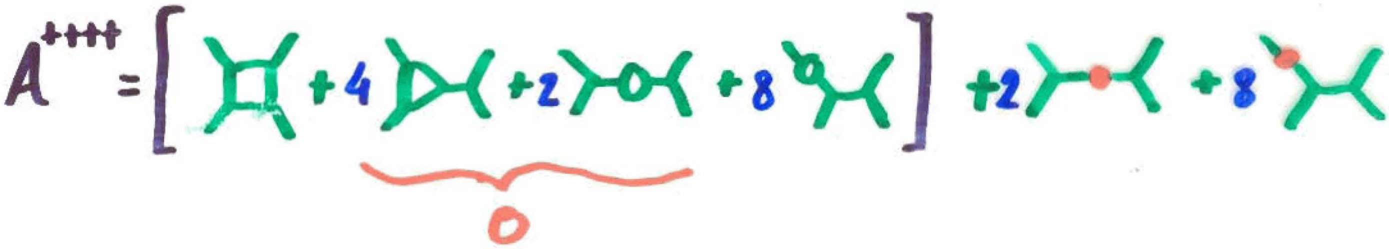
- To recover the correct, nonzero result for this amplitude, we need to include the counterterms:




- This maps to the calculation using dimensional regularisation (e.g. BST '06)

# ONE-LOOP ALL-PLUS FROM A COUNTERTERM

- We can shift the bracket in the last equation:




- We can thus calculate the complete 1-loop 4-point all-plus amplitude just from tree diagrams with counterterm insertions!

- In this regularisation, the all-plus 1-loop amplitude arises from tree-level $\mathcal{L}_{SDYM}$ plus the counterterm lagrangian $\mathcal{L}_{CT}$

- Can we combine this with the Mansfield transformation?

# COMBINING CQT AND MANSFIELD

- I will apply the Mansfield transformation $A \to B$, $\bar{A} \to \bar{B}, B$ on the SDYM action regularised à la CQT.

$$\mathcal{L}^{(r)}_{SDYM} = \underbrace{\mathcal{L}_{+-} + \mathcal{L}_{++-}}_{\mathcal{L}_{+-}(B,\bar{B})} + \mathcal{L}_{CT}(A)$$

$$\mathcal{L}_{+-}(B,\bar{B}) + \mathcal{L}_{CT}(B)$$

- In full YM this guarantees we will get the MHV vertices correctly.

- $\mathcal{L}_{CT}$ has a very unusual structure. It can be written as:

$$\mathcal{L}_{CT} = -\frac{g^2 N}{12\pi^2}\int_{\Sigma} dk^i dk^j\, A^i{}_j(k^i, k^j)\left[(k^i_z)^2 + (k^j_z)^2 + k^i_z k^j_z\right] A^j{}_i(k^j, k^i)$$

- Depends on the sum of region momenta, not just differences! (nonlocal)

# MANSFIELD TRANSFORM OF $\mathcal{L}_{CT}$

- $\mathcal{L}_{CT}$ depends only on $A_z$, not on $\bar{A}_{\bar{z}}$. So we will need only the expansion of A in terms of B

- We need to account for all possible cyclic orderings. Schematically:

$$AA \to \left(\int B_1 B_2\right)\left(\int B_3 B_4\right) + \left(\int B_2 B_3\right)\left(\int B_4 B_1\right)$$
$$+ \left(\int B_1 B_2 B_3\right) B_4 + \left(\int B_2 B_3 B_4\right) B_1 + \left(\int B_3 B_4 B_1\right) B_2 + \left(\int B_4 B_1 B_2\right) B_3$$

- Recalling the EM expansion:

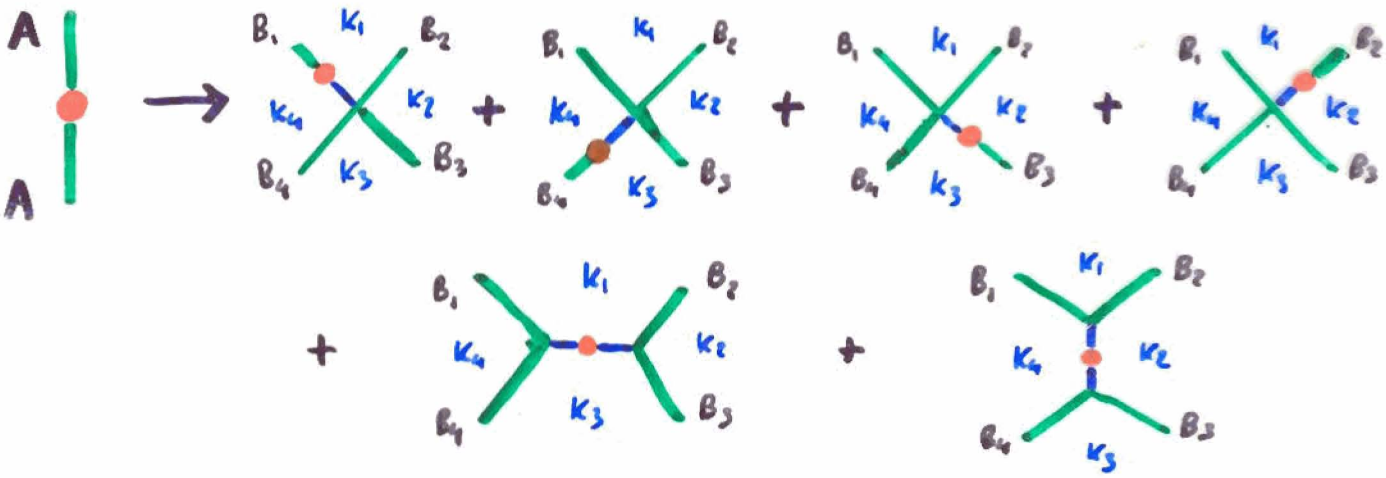
$$A(\vec{p}) = B(\vec{p}) - 2ip_+ \int \frac{d^3p^1 d^3p^2}{(2\pi)^3} \delta^{(3)}(\vec{p}-\vec{p}^1-\vec{p}^2) \frac{1}{\sqrt{p_+^1 p_+^2}} \frac{1}{\langle 12 \rangle} B(\vec{p}^1) B(\vec{p}^2)$$

$$+ 4p_+ \int \frac{d^3p^1 d^3p^2 d^3p^3}{(2\pi)^6} \delta^{(3)}(\vec{p}-\vec{p}^1-\vec{p}^2-\vec{p}^3) \frac{1}{\sqrt{p_+^1 p_+^3}} \frac{1}{\langle 12 \rangle \langle 23 \rangle} B(\vec{p}^1) B(\vec{p}^2) B(\vec{p}^3)$$

$$+ \mathcal{O}(B^4)$$

# SUMMING ALL CONTRIBUTIONS

- We can represent the combinations pictorially as:




- The total contribution can be written as:

$$V^{++++} = -4\int dp^1 \cdots dp^4\, \delta(p^1+p^2+p^3+p^4)\ \mathbf{A}\ \mathrm{tr}[B(p_1)B(p_2)B(p_3)B(p_4)]$$

where: $(k_i = k_{\bar{z},i})$

$$A = \frac{1}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}\left[\frac{p_+^1}{\sqrt{p_+^2 p_+^4}}(k_1^2+k_4^2+k_1k_4)\langle 12\rangle\langle 41\rangle + \frac{p_+^4}{\sqrt{p_+^1 p_+^3}}(k_4^2+k_3^2+k_4k_3)\langle 34\rangle\langle 41\rangle\right.$$

$$+\frac{p_+^3}{\sqrt{p_+^2 p_+^4}}(k_2^2+k_3^2+k_2k_3)\langle 23\rangle\langle 34\rangle + \frac{p_+^2}{\sqrt{p_+^1 p_+^3}}(k_1^2+k_2^2+k_1k_2)\langle 12\rangle\langle 23\rangle$$

$$\left.+\frac{(p_+^2+p_+^3)^2}{\sqrt{p_+^1 p_+^2 p_+^3 p_+^4}}(k_1^2+k_3^2+k_1k_3)\langle 12\rangle\langle 34\rangle + \frac{(p_+^2+p_+^1)}{\sqrt{p_+^1 p_+^2 p_+^3 p_+^4}}(k_2^2+k_4^2+k_2k_4)\langle 23\rangle\langle 41\rangle\right]$$

# THE FOUR-POINT ALL-PLUS VERTEX

- Manipulations using spinor identities, momentum conservation and region momentum identities (e.g. $(k_1^2+k_2^2+k_1k_2)=\underbrace{(k_1-k_4)}_{p^1_{\bar{z}}}(k_1+k_2+k_4)+(k_2^2+k_4^2+k_2k_4)$) result in:

$$V^{++++} = -\int dp^1 \cdots dp^4 \, \delta(p^1+p^2+p^3+p^4) \times$$

$$\times \left[ \frac{[12][34]}{\langle 12\rangle\langle 34\rangle} + f(p_i^2) \right] \mathrm{tr}(B_1 B_2 B_3 B_4)$$

- So we have recovered the missing all-plus one-loop amplitude from an all-plus vertex in the lagrangian

- This was for four-point. How about the n-point case?

# OFF-SHELL STRUCTURE

- We would like to decouple region momenta from actual momenta. Find:
  - a) Quadratic region momentum dependence vanishes
  - b) Linear region momentum dependence is very simple:

$$V_K^{(4)} = -\frac{3}{16}\,\frac{(12)+(23)+(34)+(41)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}\left[\sum_{i=1}^{4} k_{\bar{z}}^{i}\right]\left[\sum_{i=1}^{4}\frac{(p_i)^2}{p_+^i}\right]$$

- Left with only line momenta:

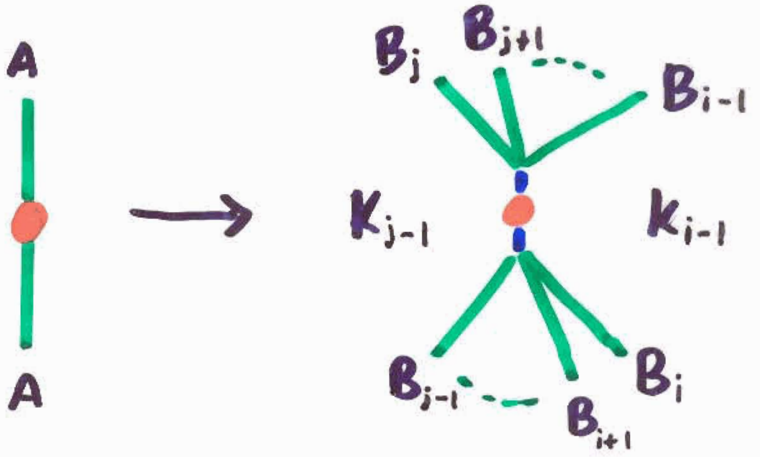
$$\begin{aligned} V_p^{(4)} = \frac{1}{8}\Big[ & p_+^4\sqrt{p_+^2 p_+^4}\,[(\bar{p}_1+\bar{p}_2)(\bar{p}_1-\bar{p}_2)+(\bar{p}_3+\bar{p}_2)(\bar{p}_3-\bar{p}_2)]\langle 34\rangle\langle 41\rangle \\ & +p_+^1\sqrt{p_+^1 p_+^3}\,[(\bar{p}_2+\bar{p}_3)(\bar{p}_2-\bar{p}_3)+(\bar{p}_4+\bar{p}_3)(\bar{p}_4-\bar{p}_3)]\langle 41\rangle\langle 12\rangle \\ & +p_+^2\sqrt{p_+^2 p_+^4}\,[(\bar{p}_3+\bar{p}_4)(\bar{p}_3-\bar{p}_4)+(\bar{p}_1+\bar{p}_4)(\bar{p}_1-\bar{p}_4)]\langle 12\rangle\langle 23\rangle \\ & +p_+^3\sqrt{p_+^3 p_+^1}\,[(\bar{p}_4+\bar{p}_1)(\bar{p}_4-\bar{p}_1)+(\bar{p}_2+\bar{p}_1)(\bar{p}_2-\bar{p}_1)]\langle 23\rangle\langle 34\rangle \\ & -(p_+^2+p_+^3)(p_+^1+p_+^4)[(\bar{p}_3-\bar{p}_2)(\bar{p}_1-\bar{p}_4)-(\bar{p}_1+\bar{p}_2)^2]\langle 12\rangle\langle 34\rangle \\ & -(p_+^3+p_+^4)(p_+^2+p_+^1)[(\bar{p}_4-\bar{p}_3)(\bar{p}_2-\bar{p}_1)-(\bar{p}_2+\bar{p}_3)^2]\langle 23\rangle\langle 41\rangle\Big]\cdot\frac{1}{\sqrt{p_+^1\cdots p_+^4}\,\langle 12\rangle\cdots\langle 41\rangle} \end{aligned}$$

ON-SHELL REDUCES TO

$$\sim \frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

# THE GENERAL ALL-PLUS VERTEX

- Repeating for n-point, we find:




$[i,j=1,\dots,n]$ cyclic

- So can write as a double sum:

$$V^{(n)} = \frac{1}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle} \times \sum_{1\le i<j\le n} \frac{\langle j-1,j\rangle\langle i-1,i\rangle}{\sqrt{p_+^i p_+^{i-1} p_+^j p_+^{j-1}}} \left(k_+^{j-1}-k_+^{i-1}\right)^2 \left(k_{\bar{z},i-1}^2 + k_{\bar{z},j-1}^2 + k_{\bar{z},i-1}k_{\bar{z},j-1}\right)$$

- Is this equal to known result?

$$V \overset{?}{=} \frac{1}{\langle 12\rangle\cdots\langle n1\rangle} \sum_{1\le i<j<k<\ell\le n} \langle ij\rangle[jk]\langle k\ell\rangle[\ell i]$$

- Very similar to a conjecture of Bern, Dixon, Dunbar & Kosower (1996) on relation to MHV in N=4 SYM

# COLLINEAR AND SOFT LIMITS

- We have verified that the n-point expression has the correct collinear and soft behaviour

- Write as $A^{(n)} = \sum_{1\le i<j\le n} K_{i-1,j-1}\, \Upsilon(p; i,\cdots j-1)\, \Upsilon(-p; j,\ldots, i-1)$

$$\left[ K_{i,j} = k^2_{\xi,i} + k^2_{\bar\xi,j} + k_{\bar\xi,i}\,k_{\xi,j} \right]$$

- Collinear: $p_k \to zP$, $p_{k+1} \to (1-z)P$

  No contribution if $k, k+1$ on different $\Upsilon$'s

  For internal $k, k+1$:

$$\Upsilon(p; 12\cdots s) \to \frac{1}{z(1-z)} \frac{P_+^k}{(k,k+1)} \Upsilon(p; 1\ldots k, k+2, \ldots, s)$$

  Can show this is true for full $A^{(n)}$

- Soft: $p_k \to 0$. Want $A^{(n)}_{1\cdots n} \to S(k)\, A^{(n-1)}_{1\cdots k-1, k+1\cdots n}$

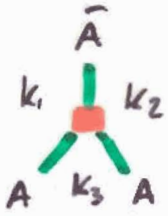
  with $S(k) = \dfrac{P_+^k\,(k-1,k+1)}{(k-1,k)(k,k+1)}$

  Easy to show from definition of $\Upsilon$'s

# ONGOING WORK

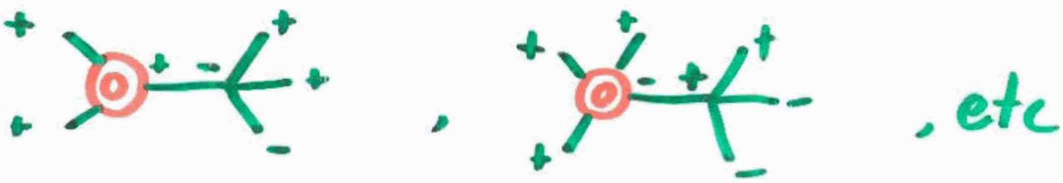
- Can this approach be applied to full Yang-Mills ?

- Have more counterterms in the CQT lagrangian : $\Pi^{++}, \Pi^{--}, \Gamma^{++-}, \Gamma^{--+}$

  ( e.g. $\Gamma^{++-} = \frac{g^3 N^{3/2}}{12\pi^2} (k_{\bar{z},1} + k_{\bar{z},2} + k_{\bar{z},3}) \Leftrightarrow$ 

  )

- Need to combine with MHV vertices




- Perhaps this will improve on the discussion in CSW (2), which had only an all-plus one-loop vertex ?

- No clear picture yet...

# CONCLUSIONS

- We have proposed an approach to obtaining the "missing" amplitudes in pure YM and possibly other non-supersymmetric theories

- A step towards completing the MHV formalism at the quantum level?

- The main ingredients are:

  - Lightcone
  - Mansfield's transformation (in 4d)
  - A strange, but purely 4d regulator
  - Lorentz-violating counterterms

- Several extra vertices, in addition to MHV, seem to be required

# OUTLOOK

- Is there a more "QFT-friendly" regulator that reproduces this?
- Can we obtain these vertices using DR?
- Relation to the approach of EFFMM?
- Does this shed any light on higher-loop amplitudes?
  (at least in the planar limit)
- Are MHV-rules fully consistent at the quantum level?
- Are they equivalent to Yang-Mills theory?