Brane Cosmology and the Generalised Dark Radiation

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• Motivation:

non-compact

What is the cosmological evolution of a brane Universe in a higher-dimensional bulk space-time?

V.A. Rubakov, M.E. Shaposhnikov, Phys.Lett.B125:136-138,1983

• Working framework:

The Randall-Sundrum model (five space-time dimensions)

L. Randall, R. Sundrum, Phys.Rev.Lett.83:4690-4693,1999
P. Binetruy, C. Deffayet, U. Ellwanger, D. Langlois, Phys.Lett.B477:285-291,2000

• Possible effects:

– Energy exchange between the brane and the bulk

$$\dot{\rho} + 3\frac{\dot{R}}{R}(\rho + p) = J \propto T^0_{\ 4}$$

– Mirage effects: Influence of the bulk matter on the brane evolution

A. Kehagias, E. Kiritsis, JHEP 9911:022,1999

The Randall-Sundrum model (II)

L. Randall, R. Sundrum, Phys.Rev.Lett.83:4690-4693,1999

Action:

$$S = \int d^5x\sqrt{-g}\left(\Lambda + M^3 R\right) + \int d^4x\sqrt{-\hat{g}}\,(-V)$$

Metric:

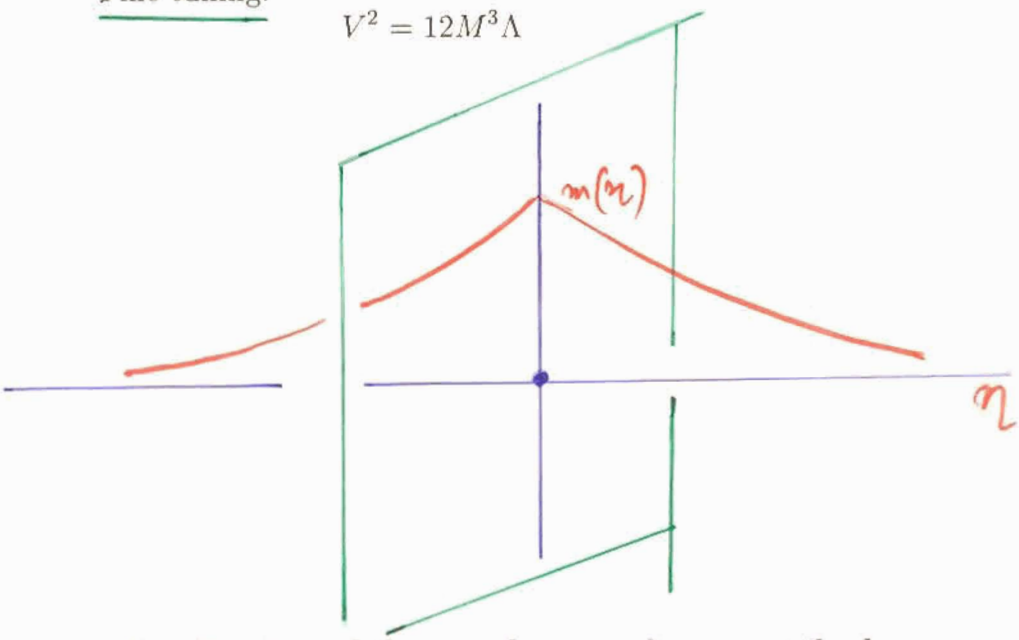
$$ds^2 = -m^2(\eta)d\tau^2 + m^2(\eta)d\Omega_k^2 + d\eta^2$$

Warp factor:

$$m^2(\eta) = \exp\left(-2k|\eta|\right) \qquad k = \Lambda/V$$

Fine tuning:

$$V^2 = 12M^3\Lambda$$



- **Localization of the massless graviton near the brane**
- **Newtonian potential on the brane:**

$$V(r) = \frac{1}{4\pi M_{\rm Pl}^2}\frac{1}{r} + \mathcal{O}(1/r^3) \qquad M_{\rm Pl}^2 = 12M^6/V$$

Brane cosmology

P. Binetruy, C. Deffayet, U. Ellwanger, D. Langlois, Phys.Lett.B477:285-291,2000

Action:

$$S = \int d^5x\sqrt{-g}\left(\Lambda + M^3 R\right) + \int d^4x\sqrt{-\hat{g}}\left(-V + \mathcal{L}_b\right)$$

Metric:

$$ds^2 = -m^2(\tau,\eta)d\tau^2 + a^2(\tau,\eta)d\Omega_k^2 + d\eta^2$$

Friedmann equation:

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{1}{6M_{\rm Pl}^2}\left[\tilde{\rho}\left(1+\frac{\tilde{\rho}}{2V}\right) + \tilde{\rho}_d\right] - \frac{k}{R^2} + \lambda$$

$$R(\tau) = a(\tau, \eta = 0) \qquad \lambda = (V^2/12M^3 - \Lambda)/12M^3 = 0$$

Dark radiation:

$$\tilde{\rho}_d = \frac{12M^3}{\pi^2 V}\frac{\mathcal{M}}{R^4}$$

Conservation equations:

$$\dot{\tilde{\rho}} + 3\frac{\dot{R}}{R}\left(\tilde{\rho} + \tilde{p}\right) = 0 \qquad \tilde{p} = \tilde{w}\tilde{\rho}$$

$$\dot{\tilde{\rho}}_d + 4\frac{\dot{R}}{R}\tilde{\rho}_d = 0$$

Brane cosmology with matter in the bulk

C. van de Bruck, M. Dorca, C.J.A.P. Martins, M. Parry, Phys.Lett.B495:183-192,2000
E. Kiritsis, G. Kofinas, N. Tetradis, T.N. Tomaras, V. Zarikas JHEP 0302:035,2003
N. Tetradis, Phys.Lett.B569:1-6,2003

Action:

$$S = \int d^5x\sqrt{-g}\left(\Lambda + M^3 R + \mathcal{L}_B\right) + \int d^4x\sqrt{-\hat{g}}\left(-V + \mathcal{L}_b\right)$$

Brane evolution equations:

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{1}{6M_{\rm Pl}^2}\left[\tilde{\rho}\left(1+\frac{\tilde{\rho}}{2V}\right)+\tilde{\rho}_d\right] - \frac{k}{R^2} + \lambda$$

$$\dot{\tilde{\rho}} + 3\frac{\dot{R}}{R}\left(\tilde{\rho}+\tilde{p}\right) = -2T^0_{\ 4}$$

$$\dot{\tilde{\rho}}_d + 4\frac{\dot{R}}{R}\tilde{\rho}_d = 2\left(1+\frac{\tilde{\rho}}{V}\right)T^0_{\ 4} - \frac{24M^3H}{V}T^4_{\ 4}$$

- For $\frac{T^4_{\ 4}}{T^0_{\ 4}}\sqrt{\frac{\tilde{\rho}}{V}} \ll 1$ the last two equations become:

$$\left[\dot{\tilde{\rho}} + 3\frac{\dot{R}}{R}\left(\tilde{\rho}+\tilde{p}\right)\right] + \left[\dot{\tilde{\rho}}_d + 4\frac{\dot{R}}{R}\tilde{\rho}_d\right] = 0$$

- **For $\tilde{\rho}_d < 0$ accelerated expansion is possible**

Assumption 2a

$$T^0{}_4 = T^0{}_4(\tilde{\rho}) \qquad T = 2T^0{}_4$$

Attractive fixed point

$$3H_*(1+\tilde{w})\tilde{\rho}_* = -T(\tilde{\rho}_*)$$

$$H_*^2 = 2\gamma\tilde{\rho}_* + \chi_* \qquad \tilde{w} < 1/3$$

$$2H_*\chi_* = \gamma T(\tilde{\rho}_*)$$

$$T(\tilde{\rho}_*) = -\frac{3\sqrt{\gamma}}{\sqrt{2}}(1+\tilde{w})(1-3\tilde{w})^{1/2}\tilde{\rho}_*^{3/2} < 0$$

$$H_*^2 = \frac{1-3\tilde{w}}{4}\gamma\tilde{\rho}_*$$

$$\chi_* = -\frac{3(1+\tilde{w})}{4}\gamma\tilde{\rho}_* \qquad < 0$$

Exponential expansion $R \sim e^{H_* t}$

Assumption 2b

(N. Tetradis 2002)

$$T^0{}_4 \sim -R^{-q} \quad < 0$$

Solution (Attractor)

$$\tilde{\rho} = \frac{C_1}{R^s}$$

$$s = \frac{2}{3}q$$

$$\chi = \frac{C_2}{R^s}$$

$$\frac{C_1}{C_2} \sim -\frac{4-s}{3(1+\tilde{w})-s}$$

$$\frac{1}{H^2}\frac{\ddot{R}}{R} = 1 - \frac{q}{3}$$

Accelerated expansion requires $q < 3 \rightsquigarrow \chi < 0$

Exact solutions

Use a **different** system of coordinates:

P. Kraus, JHEP 9912:011,1999

$$
\begin{aligned}
ds^2 &= -m^2(\tau,\eta)d\tau^2 + a^2(\tau,\eta)d\Omega_k^2 + d\eta^2 && \text{Gauss normal} \\
&= -n^2(t,r)dt^2 + r^2 d\Omega_k^2 + b^2(t,r)dr^2 && \text{Schwarzschild}
\end{aligned}
$$

Location of the brane: $r = a(\tau, \eta = 0) = R(\tau)$

Strategy:

- Find a solution of Einstein equations in the bulk

- Embed the brane

Example: AdS-star

P.S. Apostolopoulos, N. Tetradis, Class.Quant.Grav.21:4781-4792,2004

$$
\frac{1}{b^2(r)} = k + \frac{1}{12M^3}\Lambda r^2 - \frac{1}{6\pi^2 M^3}\frac{\mathcal{M}(r)}{r^2}
$$

$$
\frac{d\mathcal{M}}{dr} = 2\pi^2 r^3 \rho(r)
$$

Brane evolution:

$$
H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{1}{6M_{\rm Pl}^2}\left[\tilde{\rho}\left(1 + \frac{\tilde{\rho}}{2V}\right) + \frac{1}{6\pi^2 M^3}\frac{\mathcal{M}(R)}{R^4}\right] - \frac{k}{R^2} + \lambda
$$

If $\rho = p = 0$, then $\mathcal{M}(r)$ =constant (AdS-Schwarzschild bulk)

The exact solution for an arbitrary bulk content

P.S. Apostolopoulos, N. Tetradis, Phys.Rev.D71:043506,2005
P.S. Apostolopoulos, N. Tetradis, Phys.Lett.B633:409-414,2006

- Every slice is characterized by a length scale ℓ

  Gauss normal system: $\ell = a(\tau, \eta)$

  Schwarzschild system: $\ell = r$

- Effective Friedmann equation:

$$H^2 = \left(\frac{\dot{\ell}}{\ell}\right)^2 = \frac{1}{6M_{\rm Pl}^2}\tilde{\rho}\left(1+\frac{\tilde{\rho}}{2V}\right) + \frac{1}{6\pi^2 M^3}\frac{\mathcal{M}(\ell,\tau)}{\ell^4} - \frac{k}{\ell^2} + \lambda$$

- Generalised comoving mass of the bulk fluid

$$\mathcal{M} = \int_{\ell_0=0}^{\ell} 2\pi^2 \rho \ell^3 d\ell + \mathcal{M}_0$$

$\rho \equiv T_{AB}u^A u^B$: bulk energy density as measured by the bulk observer.

- **Generalized dark radiation**

$$\rho_d = \frac{12M^3}{\pi^2 V}\frac{\mathcal{M}(\ell,\tau)}{\ell^4}$$

- Raychaudhuri equation:

$$\dot{H} = -H^2 - \frac{1}{12M_{\rm Pl}^2}\left[(\tilde{\rho}+3\tilde{p}) + \frac{2\tilde{\rho}^2+3\tilde{\rho}\tilde{p}}{V}\right] - \frac{1}{6\pi^2 M^3}\frac{\mathcal{M}(\ell,\tau)}{\ell^4} - \frac{1}{M^3}\bar{p} + \lambda$$

$\bar{p}$: bulk pressure as measured by the brane observer.

Acceleration ?

Holography

- Generalized dark radiation:

$$\rho_d = \frac{12M^3}{\pi^2 V}\frac{\mathcal{M}(\ell,\tau)}{\ell^4}$$

- Pressure of the generalized dark radiation:

$$p_d = \frac{\rho_d}{3} + \frac{8M^3}{V}\tilde{p}$$

- Effective Friedmann equation:

$$H^2 = \left(\frac{\dot{\ell}}{\ell}\right)^2 = \frac{1}{6M_{\rm Pl}^2}\left[\tilde{\rho}\left(1+\frac{\tilde{\rho}}{2V}\right)+\rho_d\right] - \frac{k}{\ell^2} + \lambda$$

- Conservation equation:

$$\left[\dot{\tilde{\rho}} + 3H(\tilde{\rho}+\tilde{p})\right]\left(1+\frac{\tilde{\rho}}{V}\right) = -\left[\dot{\rho}_d + 3H(\rho_d + p_d)\right]$$

Examples

P. Apostolopoulos, N. Tetradis, Class.Quant.Grav.21:4781-4792,2004
N. Tetradis, Class.Quant.Grav.21:5221-5232,2004
P. Apostolopoulos, N. Brouzakis, E. Saridakis, N. Tetradis, Phys.Rev.D72:044013,2005

If we define $p_d = w_d\rho_d$, there are configurations with:

- $w_d > 1/3$: non-relativistic bulk matter in an AdS-Tolman-Bondi geometry
- $w_d = 1/3$: AdS-Schwarzschild bulk geometry

  AdS-Vaidya bulk with energy exchange between the brane and a radiation field in the bulk.

- $w_d = 0$: generalised AdS-Vaidya bulk;
- $w_d = -1/3$: global monopole in an AdS bulk;
- $w_d = -1$: constant field with a non-zero potential in the bulk (effective cosmological constant).

Particular examples

AdS-Vaidya bulk

Hebecker, March-Russell 2001
Langlois, Sorbo, Rodriguez-Martinez 2002
Leeper, Maartens, Sopuerta 2003
N. Tetradis 2004

Bulk frame

$$ds^2 = -n^2(u,r)\,du^2 + 2\epsilon\, du\, dr + r^2 d\Omega_k^2$$

$$n^2(u,r) = \frac{1}{12M^3}\Lambda r^2 + k - \frac{1}{6\pi^2 M^3}\frac{\mathcal{M}(u,r)}{r^2}$$

Brane frame

$$H^2 = \frac{1}{144M^6}\tilde{\rho}^2 + \frac{1}{6M_{pl}^2}\tilde{\rho} + \frac{1}{6\pi^2 M^3}\frac{\mathcal{M}}{R^4} - \frac{k}{R^2} + \lambda$$

$$\dot{\tilde{\rho}} + 3H(\tilde{\rho}+\tilde{p}) = -\frac{12M^3}{\pi^2 V}\frac{\partial\mathcal{M}/\partial\tau}{R^4}\frac{1}{1-\epsilon\frac{12M^3H}{V}+\frac{\tilde{\rho}}{V}}$$

$$\mathcal{M}(\tau, R(\tau)) = \mathcal{M}(u(\tau,\eta=0), R(\tau,\eta=0))$$

$\mathcal{M}(u,r) = \mathcal{M}(u)$: mirage radiation

$\mathcal{M}(u,r) = \mathcal{M}(u)\,r$: mirage dust (dark matter)

Bulk energy-momentum tensor in the bulk frame

$$T^0{}_0 = T^4{}_4 = \Lambda - \frac{1}{2\pi^2}\frac{\mathcal{M}_{,r}}{r^3}$$

$$T^1{}_1 = T^2{}_2 = T^3{}_3 = \Lambda - \frac{1}{6\pi^2}\frac{\mathcal{M}_{,rr}}{r^2}$$

$$T^0{}_4 = \frac{1}{2\pi^2}\frac{\mathcal{M}_{,u}}{r^3}$$

AdS-Tolman-Bondi bulk

(Apostolopoulos, Brouzakis, Saridakis, Tetradis
hep-th/0502115)

Bulk frame

$$ds^2 = -dt^2 + b^2(t,r)\,dr^2 + s^2(t,r)\,d\Omega_2^{\,2}$$

$$b^2(t,r) = \frac{s_{,r}^{\,2}(r,t)}{k + f(r)}$$

$$T^A{}_B = \mathrm{diag}\left(\Lambda - \rho(t,r), \Lambda, \Lambda, \Lambda, \Lambda\right)$$

Einstein's equations

$$s_{,t}^{\,2}(t,r) = \frac{1}{6\pi^2 M^3}\frac{\mathcal{M}(r)}{s^2} - \frac{1}{12M^3}\Lambda s^2 + f(r)$$

$$\mathcal{M}_{,r}(r) = 2\pi^2 s^3 \rho\, s_{,r}$$

Brane frame

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{1}{144M^6}\tilde{\rho}^2 + \frac{1}{6M_{Pl}^2}\tilde{\rho} + \frac{1}{6\pi^2 M^3}\frac{\mathcal{M}(\tau)}{R^4} - \frac{k}{R^2}$$

$$\dot{\tilde{\rho}} + 3H\tilde{\rho} = -2T^0{}_4$$

$$\mathcal{M}(\tau) = \mathcal{M}\left(r(\tau, \eta=0)\right)$$

$$T^0{}_4 = -\rho\, t_{,\tau} t_{,\eta}$$

AdS - monopole bulk

$$\mathcal{L}_B(\varphi) = -\frac{1}{2}\varphi^{\alpha}{}_{;C}\varphi_{\alpha;D}g^{CD} - \frac{\lambda}{4}\left(\varphi^{\alpha}\varphi_{\alpha} - \varphi_0^2\right)^2$$

$$\varphi^{\alpha} = \varphi_0 f(r)\frac{x^{\alpha}}{r}$$

For $r \to \infty$

$$T^0{}_0 = T^4{}_4 = \Lambda - \frac{3\varphi_0^2}{2r^2}$$

$$T^1{}_1 = T^2{}_2 = T^3{}_3 = \Lambda - \frac{\varphi_0^2}{2r^2}$$

Brane frame $(R \to \infty)$

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{1}{6M_{pl}^2}\tilde{\rho} + \left(\frac{\varphi_0^2}{4M^3} - 1\right)\frac{1}{R^2}$$

$$\dot{\tilde{\rho}} + 3H\left(\tilde{\rho} + \tilde{p}\right) = 0$$

Brane cosmology with induced gravity

G.R. Dvali, G. Gabadadze, M. Porrati, Phys.Lett.B485:208-214,2000
P.S. Apostolopoulos, N. Tetradis, hep-th/0604014

Action:

$$S = \int d^5x\sqrt{-g}\left(\Lambda + M^3 R + \mathcal{L}_B\right) + \int d^4x\sqrt{-\hat{g}}\left(-V + M^3 r_c \hat{R}_4 + \mathcal{L}_b\right)$$

Effective Friedmann equations for $\tilde{\rho}, \tilde{\rho}_d \ll V$:

$$H^2 = \frac{2(1+\epsilon)(1+k_c r_c)}{r_c^2} + \frac{1+\epsilon+k_c r_c}{k_c r_c(1+k_c r_c)}\frac{\tilde{\rho}}{6\,(M^3/k_c)} - \frac{\epsilon}{1+k_c r_c}\frac{\tilde{\rho}_d}{6\,(M^3/k_c)} - \frac{k}{\ell^2}$$

Two possibilities: $\epsilon = \pm 1$

- $\epsilon = -1$:

$$H^2 = \frac{1}{6M_{\rm Pl}^2}\left(\tilde{\rho} + \tilde{\rho}_d\right) - \frac{k}{\ell^2} \qquad\qquad M_{\rm Pl}^2 = M^3(r_c + 1/k_c)$$

- $\epsilon = +1$:

$k_c r_c \ll 1$:

$$H^2 = \frac{4}{r_c^2} + \frac{1}{6M_{\rm Pl}^2}\left(\tilde{\rho} - \frac{k_c r_c}{2}\tilde{\rho}_d\right) - \frac{k}{\ell^2} \qquad\qquad M_{\rm Pl}^2 = M^3 r_c/2$$

$k_c r_c \gg 1$:

$$H^2 = \frac{4k_c}{r_c} + \frac{1}{6M_{\rm Pl}^2}\left(\tilde{\rho} - \tilde{\rho}_d\right) - \frac{k}{\ell^2} \qquad\qquad M_{\rm Pl}^2 = M^3 r_c$$

Unstable accelerating branch (?)

Summary

- A viable **brane cosmology** in an infinite bulk is possible if there is gravity localization.
- The Randall-Sundrum model provides the only known working framework.
  Includes backreaction
- The matter content of the bulk is reflected in the **generalised radiation** term of the effective Friedmann equation.
- The effective equation of state of the generalised radiation can vary greatly.
- Accelerated expansion on the brane requires negative pressure or a negative mass in the bulk.
- The induced gravity scenario (DGP model) has a self-accelerating branch with interesting properties. Unfortunately, this branch suffers from instabilities.
- Gauss-Bonnet term in the bulk