Holographic methods and applications to black holes

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Introduction

This year will be the 10th anniversary of the AdS/CFT conjecture.

By now there are many known examples of gravity/gauge theory dualities.

Typically tests of the conjecture have been made at the conformal point, from the first work on matching non-renormalized quantities (protected operators, their correlation functions) through to the more recent work on integrability of the theory at the conformal point, in the planar limit.

But the most interesting physics comes from breaking conformal invariance ...

On the one hand one wants to use holographic engineering to open a window into gauge theories at strong coupling.

For example, there have been many attempts to construct geometries realizing duals of confining gauge theories with chiral symmetry breaking.

Moreover, black hole geometries are being used to probe the thermal physics of quark gluon plasmas at strong 't Hooft coupling, with an eye on RHIC.

In both cases one needs to extract quantitative precise results from the geometry - the answers are not already known from weak coupling computations!

Going in the other direction, from gauge theory data to geometry, there is a fundamental question at stake: how is the geometry reconstructed from gauge theory data?

At the more fundamental level, the significance of the AdS/CFT duality lies not so much on the specific examples but in the shift of paradigm for physical reality it implies. From this point of view specific examples are mostly useful laboratories – what is important is the general lessons one learns.

Physics is a quantitative science so any paradigm shift must come equipped with a new set of *precise* computational rules.

The basic principles underlying the gravity/gauge duality were laid down already in the foundational papers on AdS/CFT. Bringing these principles into their logical conclusion, however, has led to a long journey with many surprises and subtle issues to resolve.

In the first part of this talk I will review recent progress and discuss the current status of holographic methods, while afterwards I will move to discuss applications to black hole physics, where the shift in paradigm is already happening ...

Black holes and Holography

Defining questions in gravity for the last 30 years have been:

• Why does a black hole have entropy proportional to its horizon area?

• Is there information loss because of black holes?

• How does one resolve spacetime singularities, such as those inside black holes or in Big Bang cosmologies?

Recently, the gauge/gravity duality motivated a new idea in black holes physics, the fuzzball proposal which, if true, it would provide answers to these questions.

According to this proposal, associated with any black hole there are an exponential number of horizon-free non-singular solutions that look like the black hole asymptotically but generically differ from it up to the horizon scale.

These solutions represent the “microstates” of the black hole; the original black hole provides only the “average” description of the system.

This proposal would resolve black hole puzzles because:

• The entropy of the black hole would be of standard statistical origin. The physics of black holes would then be no different than that of a distant star, with temperature and entropy being of statistical origin.

• There are no horizons and therefore no information loss. Incomimg matter would escape back to infinity at late times.

• Spacetime is non-singular. The black hole singularity is an artifact of the coarse-grained description.

In the second part of this talk I will describe progress towards promoting this interesting idea to a physical quantitative model using holographic methods.

The talk will be based on a number of papers:

- How to determine the dual field theory data given an asymptotically $AdS \times X$ geometry and vice versa.

  Kaluza-Klein holography with M Taylor, 0603016;

  Coulomb branch vevs with M Taylor, 0604169;

  Holography for bubbling solutions with M Taylor, 0706.0216;

- Using AdS/CFT methods to test and make quantitative the fuzzball proposal.

  Fuzzball solutions for black holes and D1-brane–D5-brane microstates with M Taylor, 0609154;

  Holographic anatomy of fuzzballs with I Kanitscheider, M Taylor, 0611171;

  Fuzzballs with internal excitations with I Kanitscheider, M Taylor, 0704.0690

Recovering gauge theory data from a geometry

So, given a geometry conjectured to be dual to a certain gauge theory, what would one like to compute?

To be definite, suppose that we have an asymptotically $AdS_5 \times X_5$ geometry and we have already identified the CFT that is dual to $AdS_5 \times X_5$. The geometry is then expected to be dual to a QFT that in the UV flows to this CFT.

Then one would first like to deduce the vacuum structure corresponding to the geometry, namely the vevs and parameters of deformations.

For example, in the dual of a confining theory one would like to extract the vev of the gluino condensate $\left\langle \lambda\bar{\lambda} \right\rangle$.

However, to compute such vevs quantitatively is quite subtle...

For example, for a given geometry one can often see the existence of a gluino condensate qualitatively using the ”linearized” AdS/CFT dictionary developed in 1998-1999: normalizable modes in the near boundary asymptotics of bulk fields ”yield” the corresponding vevs.

This relationship is however imprecise - vevs are non-linear in sugra asymptotics.

Whilst in some cases the linearized approximation gives an answer which is qualitatively correct, there are many known cases where it gives qualitatively incorrect results, e.g. typically one finds $E \neq 0$ for a susy vacuum.

The holographic formulas for the vevs (and other n-point functions) can be in derived in all generality for arbitrary backgrounds that are asymptotically $AdS_p \times X^p$. Here I will only summarize the answer and then move to discuss applications.

The derivation starts from the basic principles of gravity/gauge theory duality that relates bulk fields to boundary gauge invariant operators and the bulk partition function to the generating functional of boundary correlators and leads to exact formulas that give the vevs of gauge invariant operators in terms of the coefficients in the asymptotic expansion of bulk fields.

On the way one had to understand

1. how to deal with the infinities that appear in the naive implementation of the program,

2. how to properly take into account the compact part of the geometry.

The first issue is dealt with by the formalism of holographic renormalization, which is the precise gravitational analogue of QFT renormalization, and the second with ”Kaluza-Klein” holography.

To understand the holographic formulas one needs to know some facts about asymptotically (locally) AdS spacetimes.

These spacetimes solve the Einstein equations with a negative cosmological constant and have the following asymptotic (Fefferman-Graham) form

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2} g_{ij}(x,r) dx^i dx^j$$

where

$$g_{ij}(x,r) = g_{(0)ij} + r^2 g_{(2)ij} + ... + r^d \left(\log r^2 h_{(d)ij} + g_{(d)ij}\right) + ...$$

This is an expansion in r (the conformal boundary of the spacetime is located at $r = 0$). A covariant way to organize this expansion is in terms of eigenfunctions of scale transformations [Papadimitriou, KS, 2004].

The underlying structure of the renormalized holographic correlators is best exhibited in a radial Hamiltonian formalism, where the radial coordinate plays the role of time. (This is also the most efficient way to do actual computations.) [Papadimitriou, KS, (2004)]

A fundamental property of asymptotically locally AdS spacetimes is that scale transformations are part of the asymptotic symmetries and therefore every covariant quantity can be decomposed into a sum of terms each having a definite scaling.

In particular, the radial canonical momentum π can be decomposed into eigenfunctions of scale transformations π_k of weight k and furthermore each of these eigenfunctions is related to the asymptotic coefficients by (in general) non-linear relations.

The precise relationship between vevs and supergravity asymptotics can be summarized as follows. Let $\mathcal{O}^I_k$ label the set of operators of dimension k. Then

$$\left\langle \mathcal{O}^I_k \right\rangle = \pi^I_k + \sum c^I_{I_1 \cdots I_j} \pi^{I_1}_{k_1} \cdots \pi^{I_j}_{k_j}.$$

Here π^I_k is the part of radial canonical momentum π^I that has weight k. Recall that it can be expressed non-linearly in terms of field asymptotics.

The terms non-linear in canonical momenta arise whenever $k = \sum k_i$; the constants $c^I_{I_1 \cdots I_j}$ are related to extremal correlators.

These formulae are derived by first implementing a non-linear gauge invariant KK map to $(d+1)$ dimensions and then applying (standard) $(d+1)$-dimensional holographic renormalization formulae.

1. Expand the perturbations of $10d$ fields about the $AdS_{d+1} \times N$ background in harmonics of the compact manifold.

$$\Phi = \varphi_B + \phi$$

and

$$\phi(x,y) = \sum \psi^I(x) Y^I(y)$$

where Φ denotes collectively all fields, φ_B is the $AdS_{d+1} \times N$ solution, ϕ are the fluctuations and $Y^I(y)$ denotes collectively all spherical harmonics (scalar, vector, tensor) and their derivatives.

2. Insert the expansion in $10d$ equations of motion and find the equations that gauge invariant combinations satisfy.

Not all fluctuations are independent: there are diffeomorphisms that map them to each other and to the background solution. To deal with this issue we developed a systematic procedure to construct gauge invariant combinations:

$$\hat{\psi}^I = a^I_J \psi^J + b^I_{JK} \psi^j \psi^J + \cdots$$

where a^I_J, b^I_{JK} are numerical coefficients. The gauge invariant combinations are the physical KK modes and they satisfy equations of the form

$$\begin{aligned}(\Box - m_I^2)\hat{\psi}^I &= D^I_{JK} \hat{\psi}^J \hat{\psi}^K + E^I_{JK} D_\mu \hat{\psi}^J D^\mu \hat{\psi}^K \\ &\quad + F^I_{JK} D_{(\mu} D_{\nu)} \hat{\psi}^J D^{(\mu} D^{\nu)} \hat{\psi}^K + \cdots\end{aligned}$$

where $D^I_{JK}, E^I_{JK}, F^I_{JK}$ are numerical coefficients.

3. These equations reduce to $(d+1)$-dimensional field equations upon use of a non-linear field transformation, the non-linear KK map, which can be integrated into an action.

The non-linear KK map

$$\Psi^I = \hat{\psi}^I + J^I_{JK}\hat{\psi}^J\hat{\psi}^K + L^I_{JK}D_\mu\hat{\psi}^J D^\mu\hat{\psi}^K + \cdots$$

leads to equations

$$(\Box - m_I^2)\Psi^I = \lambda^I_{JK}\Psi^J\Psi^K + \cdots$$

that can be integrated to an action, and holographic renormalization can then be carried out.

4. Certain finite boundary terms in the 10d action must also be taken into account; these lead to the non-linear terms in the vevs related to extremal correlators.

Holographic map

The expressions for the 1-point functions can be written explicitly in terms of the asymptotics of the $10d$ solutions.

Thus, to obtain the vevs for a given solution, one expands it to high enough order in the radial coordinate and decomposes deviations from the $AdS_{d+1} \times N$ background into harmonics.

These deviations can then be inserted into the holographic 1-pt functions to obtain the vevs.

The map is constructed perturbatively in the number of fields, with only a finite number of fields participating in the computation of the vev of a given operator. The number however increases with the operator dimension.

Remarks

• This method allows one to extract, using algebraic manipulations only, all QFT data from any supergravity solution which is asymptotically $(AdS \times N)$. These can then be used to identify the holographic dual.

• Any precise computation in gravity/gauge duality requires these techniques; this holds also for non-local operators and corresponding bulk branes/strings. [KS, Taylor (2002)][Karch, O'Bannon, KS (2005)].

Applications

Our method allows strong tests of gravity/gauge theory duality beyond the conformal point.

Example: $\mathcal{N} = 4$ SYM on the Coulomb branch.

$\mathcal{N} = 4$ SYM has a Coulomb branch corresponding to giving a vacuum expectation value (vev) to the scalars subject to the condition

$$[X^{i_1}, X^{i_2}] = 0.$$

A useful parametrization of the CB branch is in terms of vevs of the chiral primary operators,

$$\mathcal{O}^{I_1} = C^{I_1}_{i_1 \cdots i_k} \mathrm{Tr}(X^{i_1} \cdots X^{i_k}),$$

where C^{I_1} is a totally symmetric traceless rank k tensor of $SO(6)$.

In the large N limit, a generic point on the CB is described by a smooth (unit normalized) distribution of eigenvalues, $\rho(x)$, and the vevs can be computed exactly

$$\langle \mathcal{O}^{I_1} \rangle = \mathcal{N}(k) \int d^6 x \rho(x) (C^I_{i_1 \cdots i_k} x^{i_1} \cdots x^{i_k}),$$

where $\mathcal{N}(k)$ is a specific numerical coefficient.

These vevs are known to be protected from quantum corrections, because of the 16 supercharges. The challenge for holography is to extract these vevs from the corresponding supergravity solution.

The decoupling limit of multi-center D3-brane solutions is the corresponding holographic dual,

$$ds^2 = H(x_\perp)^{-1/2} dx_{||}^2 + H(x_\perp)^{1/2} dx_\perp^2$$

The harmonic function and its asymptotic expansion corresponding to a smooth distribution $\rho(x)$ of D3 branes is

$$H = L^4 \int d^6 y \frac{\rho(y)}{|x-y|^4} = \sum_{k,I} h_{kI} \frac{Y_k^I}{r^{k+4}},$$

where

$$h_{KI} = 2^k (k+1) L^4 \int d^6 x \rho(x) \left(C^I_{i_1 \cdots i_k} x^{i_1} \cdots x^{i_k} \right)$$

Following the steps outlined above one arrives at

$$\left\langle \mathcal{O}_k^I \right\rangle = \frac{N^2}{\pi^2} \frac{(k-2)\sqrt{(k-1)}}{2^{k/2}\sqrt{k}(k+1)L^4} h_{kI}$$

Upon use the value of h_{kI} we obtain exact agreement with the weak coupling result!

A similar analysis of the LLM bubbling solutions provides a precise map between geometries and $1/2$ BPS states of $\mathcal{N}=4$ SYM on $R\times S^3$ and a very impressive agreement between gravitational and QFT computations of vevs. [KS, Taylor, 0706.0216]

More generally, these techniques can be used to identify the field theory duals of previously known solutions ...

And can be combined with supersymmetric classification tools to develop holographic engineering, the systematic construction of holographic duals.

The fuzzballs program

1. Review of the fuzzball proposal

2. General 2-charge fuzzball solutions

3. Holographic anatomy of fuzzballs

4. Implications for the fuzzball program

The fuzzball proposal

According to this proposal, associated with any black hole there are an exponential number of horizon-free solutions that look like the black hole asymptotically but generically differ from it up to the horizon scale. [Lunin, Mathur (2001)]

These solutions represent the “microstates” of the black hole; the original black hole provides only the “average” description of the system.

The AdS/CFT correspondence supports this picture, at least for supersymmetic black holes. [KS, Taylor] (2006)

More precisely, the gravity/gauge duality relates a given asymptotically AdS geometry to either a deformation of the CFT or the CFT in a non-trivial vacuum characterized by the expectation values of gauge invariant operators. Conversely, one expects that for any stable state of the CFT there exists an asymptotically AdS solution, whose asymptotics encode the vevs of gauge invariant operators in that state.

If the field theory is in a pure state, there is no entropy and one expects the corresponding geometry to be horizon-less. This is the fuzzball proposal.

There is however no guarantee that the geometry should be well-described by supergravity alone, i.e. weakly curved everywhere.

The most basic questions are:

• Can one find enough such geometries for each black hole?

• What properties should such geometries have to be associated with black hole microstates?

• Can one show quantitatively how black hole properties emerge upon coarse-graining?

Answering these questions in full generality is currently out of reach.

However, one may focus on the simplest possible non-trivial set up, the D1-D5 system.

For this system an exponential number of horizon-free non-singular solutions was found by Lunin, Mathur (2001). This provided enough solutions to account for a fraction of the black hole entropy.

General 2-charge fuzzball solutions

We recently obtained in [Kanitscheider, KS, Taylor, 0704.0690] the general 2-charge D1-D5 solutions for IIB supergravity on T^4 and K3 by dualizing the F1-P geometries of [Callan etal], [Dabholkar etal] (1995)

D1-D5 solutions of IIB/T^4 are obtained from F1-P solutions of IIB/T^4 by use of standard S and T dualities.

D1-D5 solutions of IIB/K3 are obtained from F1-P solution of the heterotic string on T^4. These are first mapped to P-NS5 solution of IIA/K3 by string-string duality and then we proceed by S and T dualities.

The solutions are characterized by a curve in N-dimensional space.

- $N = 24$ for the K3 solutions; most general such solution.

- $N = 8$ for the T^4 solutions; most general solution carrying only bosonic excitations in T^4. [Solution with fermionic condensates can be obtained by dualizing the solution of [Taylor (2005)].]

The curves represent the profile of the string in the original F1-P system as well as the profile of the charge waves in the case of the heterotic F1-P system.

The solutions of Lunin-Mathur (2001) are a subset of these solutions, characterized by a curve in R^4 representing the blown-up of the naive geometry to a supertube in the transverse space.

• All fields of IIB sugra are non-trivial in the solution.

• The solution is determined in terms of 26 (K3)/10 (T^4) harmonic functions, $(K, f_5, A_i, \mathcal{A}, \mathcal{A}^{\alpha -})$, which in turn are determined by the curve $F^i(v), \mathcal{F}(v), \mathcal{F}^{\alpha -}(v)$. For example,

$$f_5 = \frac{Q_5}{L} \int_0^L \frac{dv}{(x_i - F_i(v))^2}; \qquad \mathcal{A} = -\frac{Q_5}{L} \int_0^L \frac{\partial_v \mathcal{F} dv}{(x_i - F_i(v))^2}.$$

• The solution has the same mass and conserved charges as the naive (singular) D1-D5 system.

• It has non-zero angular momentum provided the curves F_i are non-zero, and carries multipole moments.

• F^i are associated with transverse excitations, $\mathcal{F}, \mathcal{F}^{\alpha -}$ with excitations in the internal manifold.

• It is non-singular and horizon-free as long as $F^i(v)$ does not self-intersect and have only isolated zeros.

• When $F^i = 0$ the solutions collapses to the naive solution.

• In the ”decoupling limit” the solutions become asymptotically $AdS_3 \times S^3 \times T^4/K3$ so one may use KK holography to identify what these solutions are dual to.

Often people use only the symmetries, along with the R charges (angular momenta), in identifying the dual but there is an infinite amount of other data , namely vevs of chiral primary operators encoded in the geometry; these capture the ”multipoles”.

$$
\begin{aligned}
ds^2 &= \frac{f_1^{1/2}}{\tilde{f}_1 f_5^{1/2}}[-(dt - A_i dx^i)^2 + (dy - B_i dx^i)^2] \\
&\quad + f_1^{1/2} f_5^{1/2} dx_i dx^i + f_1^{1/2} f_5^{-1/2} ds^2_{M^4}, \\
e^{2\Phi} &= \frac{f_1^2}{f_5 \tilde{f}_1},\ B^{(2)}_{ty} = \frac{\mathcal{A}}{f_5 \tilde{f}_1},\ B^{(2)}_{\bar{\mu} i} = \frac{\mathcal{A}\mathcal{B}^{\bar{\mu}}_i}{f_5 \tilde{f}_1}, \\
B^{(2)}_{ij} &= \lambda_{ij} + \frac{2\mathcal{A} A_{[i} B_{j]}}{f_5 \tilde{f}_1},\ B^{(2)}_{\rho\sigma} = f_5^{-1} k^\gamma \omega^\gamma_{\rho\sigma}, \quad C^{(0)} = -f_1^{-1}\mathcal{A}, \\
C^{(2)}_{ty} &= 1 - \tilde{f}_1^{-1},\ C^{(2)}_{\bar{\mu} i} = -\tilde{f}_1^{-1}\mathcal{B}^{\bar{\mu}}_i,\ C^{(2)}_{ij} = c_{ij} - 2\tilde{f}_1^{-1} A_{[i} B_{j]}, \\
C^{(4)}_{tyij} &= \lambda_{ij} + \frac{\mathcal{A}}{f_5 \tilde{f}_1}(c_{ij} + 2A_{[i} B_{j]}), \quad C^{(4)}_{\bar{\mu} ijk} = \frac{3\mathcal{A}}{f_5 \tilde{f}_1}\mathcal{B}^{\bar{\mu}}_{[i} c_{jk]}, \\
C^{(4)}_{ty\rho\sigma} &= f_5^{-1} k^\gamma \omega^\gamma_{\rho\sigma},\ C^{(4)}_{ij\rho\sigma} = (\lambda^\gamma_{ij} + f_5^{-1} k^\gamma c_{ij})\omega^\gamma_{\rho\sigma}, \\
C^{(4)}_{\rho\sigma\tau\pi} &= f_5^{-1}\mathcal{A}\epsilon_{\rho\sigma\tau\pi},
\end{aligned}
$$

Now the harmonic functions characterizing the solution can be expanded near the AdS boundary in terms of spherical harmonics as

$$f_5 = Q_5 \sum_{k,I} \frac{f^I_{5k} Y^I_k}{r^{2+k}}, \qquad f^I_{5k} \sim \int_0^L dv C^I_{i_1 \cdots i_k} F^{i_1} \cdots F^{i_k}$$

where k labels the degree of the harmonic and I its degeneracy; $C^I_{i_1 \cdots i_k}$ is the corresponding symmetric traceless tensor.

Using the methods of KK holography, vevs of chiral primary operators can be expressed in terms of the field asymptotics, and hence in terms of the coefficients in the expansions of the harmonic functions.

Results

The vev of the stress energy tensor is (non-trivially) zero,

$$\langle T_{ij} \rangle = 0$$

in agreement with the fact that the fuzzballs are conjectured to be dual to Ramond ground states in the CFT.

Furthermore, there are non-zero vevs for the R symmetry currents and for scalar chiral primaries. Schematically,

$$\left\langle \mathcal{O}_k^I \right\rangle \sim f_{5k}^I + \cdots$$

Any proposal for the dual must reproduce these vevs.

Field theory dual

The dual CFT is a deformation of a sigma model with target space the symmetric orbifold of the compactification manifold.

• The holographic results invalidate naive proposals that associate each of the fuzzball solutions (specified by a curve F^i) to a single R-ground state.

• We proposed a precise map that associates a given supergravity solution to a specific superposition of R-ground states. The exact form of the superposition depends on the curves determining the solution.

• This proposal passes all kinematical tests and all accessible dynamical tests.

• Given such a precise proposal, one can now envision deriving black hole properties by coarse graining the fuzzball geometries.

The proposal

Given a curve F^I we extract the Fourier coefficients,

$$F^I(v) = \sum_{n>0} \frac{1}{\sqrt{n}} (\alpha_n^I e^{-inv} + (\alpha_n^I)^* e^{inv}),$$

and consider the coherent state:

$$\left| F^I \right) = \prod_{n,I} \left| \alpha_n^I \right) ,$$

Contained in this coherent state are Fock states, such that

$$\prod (\hat{a}_{n_I}^I)^{m_I} \left| 0 \right\rangle , \qquad N = \sum n_I m_I .$$

Now retain only the terms in the coherent state involving these Fock states, and map the FP oscillators to CFT R operators via the dictionary

$$\frac{1}{\sqrt{2}}(\hat{a}_n^1 \pm i\hat{a}_n^2) \leftrightarrow \mathcal{O}_n^{R(\pm 1+1),(\pm 1+1)};$$
$$\frac{1}{\sqrt{2}}(\hat{a}_n^3 \pm i\hat{a}_n^4) \leftrightarrow \mathcal{O}_n^{R(\pm 1+1),(\mp 1+1)};$$
$$\hat{a}_n^\rho \leftrightarrow \mathcal{O}_{(\rho-4)n}^{R(1,1)}.$$

where $\mathcal{O}_n^{R(p,q)}$ are operators associated with (p,q) cohomology of the internal manifold and n is the twist.

Computing the vevs of gauge invariant operators in this state one find exact agreement with the gravity computation, within the approximations used.

In general one must go beyond supergravity to accurately describe the physics of the system.

• Most of the geometries contain small regions of high curvature, so they are not well-described by supergravity.

• When one includes geometries with internal excitations, the situation is even worse: geometries with only internal excitations $F^i(v) = 0$ collapse to the naive singular geometry. These should account for a finite fraction of the black hole entropy but are not visible in sugra approximation at all!

• Even the geometries that only involve macroscopic scales generically can only be resolved from each other by effects beyond supergravity; their vevs differ from each other by a very small amount ($\sim 1/N$ effects).

Implications for the fuzzball proposal

One could argue (or hope) that these problems are confined to the two charge system, which after all is not a macroscopic black hole.

However, on rather general grounds one can argue that these issues will persist in other cases.

The typical scale of the fuzzball geometry is linearly related to the angular momenta/R charges: the greater the angular momentum, the more the supertube blows up in the transverse R^4.

So when there are only internal excitations along X_4 there is no angular momentum in R^4 and the D1-D5 branes don't expand into a supertube.

Now in the D1-D5(-P) system the density of microstates d_{N,j^1,j^2} as a function of the R-charges is known and one may compare the total number of states, $d_N = \sum_{j^1,j^2} d_{N,j^1,j^2}$, with the states of zero R-charge, $d_{N,0,0}$. The salient feature is that (for large charges and the 2-charge system)

$$d_{N,0,0} \cong d_N / N.$$

Thus the zero-charge states are suppressed only polynomially with respect to the total (exponential) number of states. The typical state has zero R charge. This structure is similar in both the 2 and 3 charge cases.

Thus many 3-charge microstates have very small R charges and the corresponding geometric duals most likely have small scales. As in the 2 charge case, a finite fraction of the entropy would be associated with geometries carrying only excitations on X_4, not blown up at all in R^4.

What have we learned?

The success in constructing a map between horizon-free solutions and microstates consistently with the AdS/CFT correspondence provides the most stringent test of the fuzzball proposal to date.

Generically however one needs to go beyond the leading supergravity approximation to extract the physics.

Given that many of the fuzzball geometries are not likely well-described in supergravity, and explicitly constructing all of them for even the 3-charge system is difficult, perhaps we need a different approach to the problem.

We argued that it is likely there does not exist enough geometries, well-described and distinguishable in supergravity, to span the entire set of black hole microstates.

However a sufficiently representative basis may still exists. That is, suppose one chooses a single representative of the indistinguishable geometries, and assigns a measure to this geometry. Such basis of weighted geometries may be sufficient to obtain the black hole properties.

Thus to make progress within supergravity, one should understand quantitatively how typical is the state associated with any given fuzzball solution, which requires understanding the precise map between geometries and states.

Conclusions

We have seen how holography can increasingly be promoted to a very precise framework that can be used to compute gauge theory properties from geometry and vice versa.

In particular, we have discussed how to decode the hologram for asymptotically $AdS \times X$ geometries.

We have also used these techniques to scrutinize and further develop the fuzzball proposal for black holes.

Our results certainly support the overall picture. However the detailed correspondence between geometries and microstates is more complex than previously suggested.