Sharif University of Technology and IPM
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Mainly based on:

- *F. Ardalan, H. Arfaei and N.S., IJMPA 21, 4161 (2006)*
- *A. Jafari Salim and N.S., PRD 73, 065023 (2006)*
- *N.S. and A. Jafari Salim, PRD 74, 085032 (2006)*

- Part I: Introduction to QED in a strong magnetic field (Strong QED)
  - ↪ *V.A. Miransky et al., PRD (1993-2003)*
    - ▷ Theory of Magnetic Catalysis
- Part II: NC-QED vs. Strong QED in the Lowest Landau Level (LLL)
  - ↪ *V.A. Miransky et al., PRD (2004-05)*
  - ↪ *A. Jafari Salim and N.S., PRD (2006)*
    - ▷ Effective action of strong QED in LLL vs. action of NC-QED
- Part III: NC anomalies vs. LLL anomaly
  - ↪ *F. Ardalan, H. Arfaei and N.S., IJMPA (2006)*
  - ↪ *N.S. and A. Jafari Salim, PRD (2006)*

Part I: Strong QED

or QED in a Strong Magnetic Field

- **Introduction**

In a strong magnetic field,

QED,

in addition to the familiar weak coupling phase,

has a nonperturbative strong coupling phase

characterized by

Dynamical Chiral Symmetry Breaking (DχSB)

Consequences of DχSB in Strong QED:

- Bound state formation in strong QED
  - Historically: Explanation for multiple correlated and narrow peak structure in $e^- e^+$ spectra observed in heavy ion collisions at GSI

    $\exists$ Strong and rapidly varying EM field in the neighborhood of colliding heavy ions

Consequences of DχSB in Strong QED:

- Electroweak standard model in a strong magnetic field
  - Strong magnetic field induces a transition from broken to sym. phase in EWSM
  - One obtains W and Z condensate solution (finite mass gap)
  - T_c for symmetry restoration is lower than the one without magnetic field
    ($T_c \simeq 1$ TeV)

Consequences of DχSB in Strong QED/QCD:

- Astrophysics of compact stars
  - Neutron stars (Quark Matter under extreme conditions T, μ, B)

    $\exists\ B \simeq 10^{15}$ Gauß leading to exotic phases of quark matter (*e.g.* color superconductive phases etc.)

Theory of Magnetic Catalysis of DχSB

$\hookrightarrow$ *Miransky et al. 1993-2003*

Theory of Magnetic Catalysis of DχSB

- As in non-relativistic QM, $B \neq 0$ ► Landau levels
- In a strong magnetic field a Lowest Landau Level (LLL) approximation is justified

►▷► Effective Quantum Field Theory in the LLL

Properties of the effective Field Theory in the LLL

Starting with a chiral invariant theory including massless particles

I. Dynamical mass generation ◄ χSB ► Bound state formation $\langle\bar{\psi}\psi\rangle \neq 0$

Solving the corresponding Schwinger-Dyson Eq. (Gap Eq.)

- In the quenched/ladder approximation (*i.e.* neglecting the effects of dynamical fermions)

$$m_{dyn.} = C\sqrt{eB}\exp\left(-\frac{\pi}{2}\left(\frac{\pi}{2\alpha}\right)^{1/2}\right) \qquad \Rightarrow \qquad m^2_{dyn.} \ll |eB|$$

- Numerical calculation of $\langle\bar{\psi}\psi\rangle$ in 3-dim. strong QED
  $\hookrightarrow$ *Farakos, Mavromatos et al. (1999)*

Properties of the effective Field Theory in the LLL

II. Dimensional reduction $D \to D-2$

Dynamics of 4-dim. QED in a strong B field $\simeq$ Dynamics of 2-dim. Schwinger model

- In ordinary Schwinger model $\hookrightarrow$ Photon mass $m_\gamma^2 = \frac{g^2}{\pi}$
- In Strong QED in the LLL approximation $\hookrightarrow$ Photon mass $M_\gamma^2 = \frac{|eB|}{2\pi}\frac{e^2}{\pi}$

Characteristics of Fermions and Photons in LLL

- QED Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \gamma^{\mu} \left(i\partial_{\mu} + eA_{\mu} \right) \psi \qquad \text{with} \qquad A_{\mu} = a_{\mu} + A_{\mu}^{ext.}$$

- In the symmetric gauge

$$A_{\mu}^{ext.} = \frac{B}{2} \left(0, x_2, -x_1, 0 \right) \qquad \Rightarrow \qquad \mathbf{B} = B \hat{\mathbf{e}}_3$$

Using Schwinger proper time formalism

▶▷▶ Full fermion and photon propagators

Characteristics of Fermions and Photons in LLL

Fermion propagator in a constant magnetic field

$$\mathcal{S}_F(x,y) = \exp\left(\frac{ieB}{2}\epsilon^{ab}x_a y_b\right) S_F(x-y), \qquad a,b=1,2$$

$$\tilde{S}_F(\mathbf{k}_\parallel, \mathbf{k}_\perp) = i e^{-\frac{\mathbf{k}_\perp^2}{|eB|}} \sum_{\mathbf{n}=0}^{\infty} (-1)^{\mathbf{n}} \frac{D_{\mathbf{n}}(eB,k)}{\mathbf{k}_\parallel^2 - m^2 - 2|eB|\mathbf{n}}, \qquad \mathbf{k}_\parallel = (k_0, k_3) \;\; \& \;\; \mathbf{k}_\perp = (k_1, k_2)$$

- ↷ $\mathbf{n}$ labels the Landau levels
- ↷ $D_n(eB,k)$ are some Laguerre polynomials

- ▶ In the IR region , with $|\mathbf{k}_\parallel|, |\mathbf{k}_\perp| \ll \sqrt{|eB|}$, the physics is dominated by the dynamics in the LLL with $\mathbf{n}=\mathbf{0}$

  Higher Landau levels with $\mathbf{n} > \mathbf{0}$ decouple in this approximation

Full Fermion propagator in LLL approximation

$$\mathcal{S}_F(x,y) = S_{\parallel}(\mathbf{x}_{\parallel} - \mathbf{y}_{\parallel}) P(\mathbf{x}_{\perp}, \mathbf{y}_{\perp})$$

- <u>the longitudinal part</u>

$$S_{\parallel}(\mathbf{x}_{\parallel} - \mathbf{y}_{\parallel}) = \int \frac{d^2 k_{\parallel}}{(2\pi)^2}\, e^{i\mathbf{k}_{\parallel}\cdot(\mathbf{x}-\mathbf{y})^{\parallel}}\ \frac{i\mathcal{O}}{\gamma^{\parallel}\cdot\mathbf{k}_{\parallel} - m}, \qquad \mathcal{O} \equiv \frac{1}{2}\left(1 - i\gamma^1\gamma^2 \mathsf{sign}(eB)\right)$$

- <u>the transverse part</u>

$$P(\mathbf{x}_{\perp}, \mathbf{y}_{\perp}) = \frac{|eB|}{2\pi} \exp\left(\frac{ieB}{2}\epsilon^{ab} x^a y^b - \frac{|eB|}{4}\left(\mathbf{x}_{\perp} - \mathbf{y}_{\perp}\right)^2\right), \qquad a,b = 1,2$$

Full Photon propagator in LLL approximation

$$i\widetilde{\mathcal{D}}_{\mu\nu}(q) = \frac{g^{\parallel}_{\mu\nu}}{q^2 + \mathbf{q}_{\parallel}^2 \Pi\left(\mathbf{q}_{\perp}^2, \mathbf{q}_{\parallel}^2\right)}$$

$$\Pi(\mathbf{q}_{\perp}^2, \mathbf{q}_{\parallel}^2) = \frac{2\alpha|eB|N_f}{\mathbf{q}_{\parallel}^2} e^{-\frac{\mathbf{q}_{\perp}^2}{2|eB|}} \left(\frac{1}{2\sqrt{y(y-1)}} \ln\left(\frac{\sqrt{1-y} + \sqrt{-y}}{\sqrt{1-y} - \sqrt{-y}} \right) - 1 \right), \qquad y \equiv \frac{\mathbf{q}_{\parallel}^2}{4m_{dyn.}^2}$$

Expanding $\Pi(\mathbf{q}_{\perp}^2, \mathbf{q}_{\parallel}^2)$ for $\mathbf{y \ll 1}$ and $\mathbf{y \gg 1}$

(a) $\Pi(\mathbf{q}_{\perp}^2, \mathbf{q}_{\parallel}^2) \simeq +\frac{\alpha|eB|N_f}{3\pi m_{dyn.}^2} e^{-\frac{\mathbf{q}_{\perp}^2}{2|eB|}}$ for $|\mathbf{q}_{\parallel}^2| \ll m_{dyn.}^2 \ll |eB|$

(b) $\Pi(\mathbf{q}_{\perp}^2, \mathbf{q}_{\parallel}^2) \simeq -\frac{2\alpha|eB|N_f}{\pi\, \mathbf{q}_{\parallel}^2} e^{-\frac{\mathbf{q}_{\perp}^2}{2|eB|}}$ for $m_{dyn.}^2 \ll |\mathbf{q}_{\parallel}^2| \ll |eB|$

For regime (b) $\Pi(\mathbf{q}_{\perp}^2, \mathbf{q}_{\parallel}^2)$ has a pole at $\mathbf{q}_{\parallel}^2 = 0$ ▶▷▶ Finite photon mass $M_{\gamma}^2 = \frac{2\alpha|eB|N_f}{\pi}$

Part II: NC-QED vs. Strong QED in the LLL

$\hookrightarrow$ *Miransky et al., PRD (2004-05)*

$\hookrightarrow$ *A. Jafari Salim and N.S., PRD (2006)*

▶ **New Development**

The Claim

▶▷▶ *Effective action of strong QED in LLL* $\simeq$ *Action of noncommutative QED* ◀◁◀

Noncommutative (NC) QED

- Action of NC-QED

$$\mathcal{S}[A_\mu, \bar{\psi}, \psi] = -\frac{1}{4}\int d^4x \, F_{\mu\nu} \star F^{\mu\nu} + \int d^4x \, \bar{\psi}(x) \star (i\not{D} - m)\, \psi(x)$$

  - Noncommutative $\star$-product

$$f(x) \star g(x) \equiv f(x+\xi) \exp\left(\frac{i\Theta^{\mu\nu}}{2} \frac{\partial}{\partial \xi^\mu} \frac{\partial}{\partial \zeta^\nu}\right) g(x+\zeta)\Big|_{\xi=\zeta=0}$$

  ↬ Field strength tensor

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + ie\big[A_\mu, A_\nu\big]_\star$$

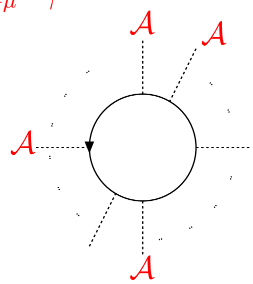
- To prove their claim:

  Compare action of NC-QED with the effective action of Strong QED in LLLA

Effective action of QED in LLL approximation

- Full effective action for $\mathcal{A} \equiv A_\mu \cdot \gamma^\mu$

$$\Gamma[\mathcal{A}] \equiv \sum_{n=0}^{\infty} \frac{1}{n!}\Gamma_n[\mathcal{A}];$$



$$= \Gamma_n[\mathcal{A}]$$

- n-point vertex function of $\mathcal{A}$

$$\Gamma_n[\mathcal{A}] = \int d^4x_1 \cdots d^4x_n \left[\mathcal{A}(x_1)\, \mathcal{S}_F(x_1, x_2)\mathcal{A}(x_2)\, \mathcal{S}_F(x_2, x_3) \cdots \mathcal{A}(x_n)\, \mathcal{S}_F(x_n, x_1)\right]$$

In LLL approximation (LLLA):

$\mathcal{S}_F(x_i, x_j)$ is the full fermion propagator in LLLA between the insertion points x_i and x_j

Integrating over $\mathbf{x}_\perp^i$ ►▷►

n-point vertex function of strong QED in the LLLA

$$\Gamma^{(n)}_{LLL} = i\frac{(ie)^n\ N_f|eB|}{2\pi n}\int d^2x_\perp d^2x_1^\parallel \cdots d^2x_n^\parallel \ \mathrm{tr}\left(S_\parallel(\mathbf{x}_1^\parallel - \mathbf{x}_2^\parallel)\widetilde{\mathcal{A}}(\mathbf{x}^\perp, \mathbf{x}_2^\parallel)\cdots S_\parallel(\mathbf{x}_n^\parallel - \mathbf{x}_1^\parallel)\widetilde{\mathcal{A}}(\mathbf{x}^\perp, \mathbf{x}_1^\parallel)\right)_\star$$

with the smeared gauge field

$$\widetilde{\mathcal{A}}(x) \equiv \gamma^\parallel \cdot \mathcal{A}_\parallel(x) \qquad \text{and} \qquad \mathcal{A}_\parallel(x) \equiv e^{\frac{\nabla_\perp^2}{4|eB|}} A_\parallel(x)$$

Full effective action

Adding up the n-point vertex functions ►▷► Full effective action $\Gamma = \Gamma^{(0)} + \Gamma^{(1)}$

- The tree level part

$$\Gamma^{(0)} = -\frac{1}{4}\int d^4x F_{\mu\nu}F^{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- The one-loop part

$$\Gamma^{(1)} = -\frac{i|eB|N_f}{2\pi}\int d^2x_\perp \mathrm{Tr}_\parallel\left(\mathcal{O}\ln\left(i\gamma^\parallel\cdot(\partial_\parallel - ie\mathcal{A}_\parallel)\right)\right)_\star$$

Properties of the effective action of strong QED in the LLLA

$$\Gamma^{(1)} = -\frac{i|eB|N_f}{2\pi}\int d^2x_\perp \mathrm{Tr}_\parallel \left(\mathcal{O}\ln\left(i\gamma^\parallel\cdot(\partial_\parallel - ie\mathcal{A}_\parallel)\right)\right)_\star$$

- The ordinary product of functions $\rightrightarrows$ Noncommutative $\star$ -product
- $\Gamma^{(1)}$ is invariant under NC-U(1) gauge symmetry

$$\mathcal{A}_\mu(x) \to U(x)\star\mathcal{A}_\mu(x)\star U^{-1}(x) + \frac{i}{e}U(x)\star\partial_\mu U^{-1}(x), \qquad \mu = 0,3$$

- Smeared gauge field

$$\mathcal{A}_\parallel(x) \equiv e^{\frac{\nabla_\perp^2}{4|eB|}} A_\parallel(x)$$

►▷► The additional form factor leads to a removal of NC UV / IR mixing

Here, in contrast to the ordinary NCFT a natural cutoff, $\Lambda_B \propto \sqrt{|eB|}$ appears ►▷►**NO** UV / IR mixing

QED in the regime of LLL dominance $\equiv$ Modified Noncommutative QED

Part III: NC-anomalies and LLL anomaly

↪ *F. Ardalan, H. Arfaei and N.S., IJMPA (2006)*

↪ *N.S. and A. Jafari Salim, PRD (2006)*

▸ **NC-Anomalies**

- Noether Theorem
  - Two currents corresponding to one global $U_A(1)$ transformation

$$J^{\mu,5} = \psi^\beta \star \bar{\psi}^\alpha \left(\gamma^\mu \gamma^5\right)_{\alpha\beta} \qquad j^{\mu,5} = \bar{\psi}^\alpha \star \left(\gamma^\mu \gamma^5\right)_{\alpha\beta} \psi^\beta$$

Two classical conservation laws ► Two anomaly equations at the quantum level

$$\begin{array}{rclcrcl} D_\mu J^{\mu,5} & = & 0 & & \partial_\mu j^{\mu,5} & = & 0 \\ D_\mu J^{\mu,5} & = & \mathcal{A}_{cov.} & \neq & \partial_\mu j^{\mu,5} & = & \mathcal{A}_{inv.} \end{array}$$

  - The integrated form of these two anomalies are equal

    Due to the properties of $\star$- product

$$\int dx_\perp \, \mathcal{A}_{cov.} = \int dx_\perp \, \mathcal{A}_{inv.}$$

Integration over NC-coordinates

Noncommutative ABJ Anomalies

- Covariant anomaly ◀◁◀ Planar diagrams (↪ *F. Ardalan and N.S. 2001*)

$$D_\mu J^{\mu,5} = -\frac{e^2}{16\pi^2} F_{\mu\nu} \star \tilde{F}^{\mu\nu}$$

- Invariant anomaly ◀◁◀ Nonplanar diagrams (↪ *F. Ardalan and N.S. 2002*)

Using *e.g.* Pauli-Villars Regularization

$$q_\mu \langle j_5^\mu(q) \rangle = \lim_{M\to\infty} -\frac{e^2}{16\pi^2} e^{-\frac{(M\theta)^2 \mathbf{q}_\perp^2}{4}} \int \frac{d^4p}{(2\pi)^4} F_{\mu\nu}(q-p) \frac{\sin(q \wedge p)}{q \wedge p} \tilde{F}^{\mu\nu}(p) + \cdots$$

Noncommutative ABJ Anomalies (Nonplanar anomaly)

$$q_\mu \langle j_5^\mu(q)\rangle = \lim_{M\to\infty} -\frac{e^2}{16\pi^2} e^{-\frac{(M\theta)^2 \mathbf{q}_\perp^2}{4}} \int \frac{d^4p}{(2\pi)^4} F_{\mu\nu}(q-p) \frac{\sin(q\wedge p)}{q\wedge p} \tilde{F}^{\mu\nu}(p) + \cdots$$

For large but finite cutoff M:

- The result is affected by NC UV / IR -Mixing
  ↪ *F. Ardalan and N.S. (2001-02)*

$$\partial_\mu j^{\mu,5} = \begin{cases} 0 & (\Theta q)^2 \gg \frac{1}{M^2} \quad \text{UV limit} \\ -\frac{e^2}{16\pi^2} F_{\mu\nu} \star' \tilde{F}^{\mu\nu} + \cdots & (\Theta q)^2 \ll \frac{1}{M^2} \quad \text{IR limit} \end{cases}$$

with generalized $\star'$-product (H. Liu *et al.*, Garousi, $\cdots$ 2001-2003)

$$f(x) \star' g(x) \equiv f(x+\xi) \frac{\sin\left(\frac{\Theta_{\mu\nu}}{2}\frac{\partial}{\partial\xi_\mu}\frac{\partial}{\partial\zeta_\nu}\right)}{\left(\frac{\Theta_{\mu\nu}}{2}\frac{\partial}{\partial\xi_\mu}\frac{\partial}{\partial\zeta_\nu}\right)} g(x+\zeta)\Big|_{\xi=\zeta=0}$$

Noncommutative ABJ Anomalies (Nonplanar anomaly)

$$q_\mu \langle j_5^\mu(q) \rangle = \lim_{M \to \infty} -\frac{e^2}{16\pi^2} e^{-\frac{(M\theta)^2 \mathbf{q}_\perp^2}{4}} \int \frac{d^4 p}{(2\pi)^4} F_{\mu\nu}(q-p) \frac{\sin(q \wedge p)}{q \wedge p} \tilde{F}^{\mu\nu}(p) + \cdots$$

- In the limit $M \to \infty$, anomaly $q_\mu \langle j_5^\mu(q) \rangle = 0$ except when $\mathbf{q}_\perp = \mathbf{0}$

$$e^{-\frac{(M\theta)^2 \mathbf{q}_\perp^2}{4}} = 1, \qquad \text{for} \qquad \mathbf{q}_\perp = \mathbf{0} \rightrightarrows \qquad \underline{\text{Finite nonplanar anomaly}}$$

$\hookrightarrow$ *A. Armoni, S. Theisen and E. Lopez (2003)*

Noncommutative ABJ Anomalies (Nonplanar anomaly)

- According to the above results

$$\text{Whereas} \qquad \mathcal{A}_{cov.} \neq 0, \qquad \curvearrowright \qquad \mathcal{A}_{inv.} \begin{cases} = 0 & \text{UV limit} \\ \neq 0 & \text{IR limit} \end{cases}$$

This result seems to be in conflict with our previous statement

$$\int dx_{\perp}\, \mathcal{A}_{cov.} = \int dx_{\perp}\, \mathcal{A}_{inv.}$$

Noncommutative ABJ Anomalies (Nonplanar anomaly)

- To solve this apparent puzzle, perform UV + IR regularization

  This makes the role played by $\mathbf{q}_\perp = \mathbf{0}$ more precise

  $\hookrightarrow$ *F. Ardalan, H. Arfaei and N.S. (2005)*

  - UV: Point splitting ▶ UV cutoff: the point-split parameter ϵ
  - IR: Compactified NC coordinates around a circle with radius R ▶ IR cutoff: R

$$\langle \partial_\mu j^{\mu,5} \rangle = \lim_{\epsilon \to 0} \lim_{R \to \infty} \cdots$$

▶ After taking the UV limit ◀

Unintegrated nonplanar anomaly includes only the zero modes of the FT of $\mathcal{F} \equiv F\tilde{F}$

$$\mathcal{A}_{inv.} \equiv \langle \partial_\mu j^{\mu,5}(x) \rangle \sim \widetilde{\mathcal{F}}^{(0)}\left(\mathbf{q}_\perp = 0, \mathbf{x}_\parallel\right) \equiv \frac{1}{(2R)^2} \int_{-R}^{+R} d^2 y_\perp \mathcal{F}\left(\vec{y}_\perp, \vec{x}_\parallel\right)$$

It vanishes in the limit $R \to \infty$

$$\mathcal{A}_{inv.} = 0$$

▶ The integrated form of the anomaly remains finite in the limit $R \to \infty$ ◀

$$\int d^2 x_\perp \; \mathcal{A}_{inv.} = \int d^2 x_\perp \; \mathcal{A}_{cov.}$$

Zero charge density ◀◁◀▶▷▶ *Finite total charge*

Part III: NC-anomalies and LLL anomaly

$\hookrightarrow$ *N.S. and A. Jafari Salim, PRD (2006)*

▶ **LLL Anomaly**

LLL Anomaly

- Consider the axial vector current $\mathcal{J}_5^\mu(x) = \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x)$ of the original QED
  - Calculate $\langle\mathcal{J}_5^\mu\rangle = \langle\bar{\psi}(x)\gamma^\mu\gamma^5\psi(x)\rangle$ *perturbatively* by replacing

Fermion Propagators $\rightrightarrows$ Fermion propagators in LLL

- We arrive first at

$$\langle\mathcal{J}_5^\mu(q)\rangle = -e\int d^4x\, d^4y\, e^{-iqx}\,\mathrm{tr}\left(\gamma^\mu\gamma^5 S_\parallel(\mathbf{x}_\parallel - \mathbf{y}_\parallel)P(\mathbf{x}_\perp,\mathbf{y}_\perp)\, \not{A}(y)\, S_\parallel(\mathbf{y}_\parallel - \mathbf{x}_\parallel)P(\mathbf{y}_\perp,\mathbf{x}_\perp)\right)$$

$\blacktriangleright\triangleright\blacktriangleright\cdots$ and then at LLL Anomaly

$$q_\mu\langle\mathcal{J}_5^\mu(q)\rangle = \frac{ieN_f|eB|\mathrm{sign}(eB)}{2\pi^2}\, e^{-\frac{\mathbf{q}_\perp^2}{2|eB|}}\epsilon^{12ab}q_a A_b(\mathbf{q}_\parallel,\mathbf{q}_\perp), \qquad a,b = 0,3$$

- Here, the factor $e^{-\frac{\mathbf{q}_\perp^2}{2|eB|}}$ is the expected form factor of $A_\parallel$

$$e^{-\frac{\mathbf{q}_\perp^2}{2|eB|}}A_b(q) \longrightarrow e^{\frac{\nabla_\perp^2}{2|eB|}}A_b(x)$$

Comparing LLL anomaly and NC nonplanar anomaly

Observations:

I. LLL anomaly is comparable with nonplanar anomaly !!!

▶▷▶ LLL Anomaly

$$q_\mu \langle \mathcal{J}_5^\mu(q)\rangle = \frac{ieN_f|eB|\text{sign}(eB)}{2\pi^2}\, e^{-\frac{\mathbf{q}_\perp^2}{2|eB|}} \epsilon^{12ab} q_a A_b(\mathbf{q}_\parallel, \mathbf{q}_\perp), \qquad a,b = 0,3$$

▶▷▶ Nonplanar anomaly

$$q_\mu \langle j_5^\mu(q)\rangle = \lim_{M\to\infty} -\frac{ie^2}{16\pi^2}\, e^{-\frac{(M\theta)^2 \mathbf{q}_\perp^2}{4}} \int \frac{d^4p}{(2\pi)^4}\, F_{\mu\nu}(q-p)\frac{\sin(q\wedge p)}{q\wedge p}\tilde{F}^{\mu\nu}(p) + \cdots$$

II. QED in a strong magnetic field has a natural cutoff $\Lambda_B \equiv \sqrt{eB} \sim (M\theta)^{-1}$ in NCFT

III. LLL anomaly is first order in A_μ ▶▶ Anomaly of a 2-dim Schwinger model

$$q_a \langle j_5^a(q)\rangle = \frac{g}{\pi}\epsilon^{ab} q_{ab} A_b(q), \qquad a,b = 0,1$$

This is the consequence of dimensional reduction $D \to D-2$ in LLL

Comparing LLL anomaly and NC nonplanar anomaly

- LLL anomaly is comparable with nonplanar anomaly !!!

  Remember: The unintegrated form of the nonplanar NC anomaly vanishes everywhere, but its integrated form is non-vanishing

  Questions

  - Integrated and unintegrated LLL anomalies?
  - What is the role played by $\mathbf{q}_{\perp} = 0$ in LLL anomaly?

Compactification of $\mathbf{x}_{\perp}$ around a circle with radius R

Results

- Unintegrated form
  - Nonplanar (NC) anomaly, for finite IR cutoff R, involves only the zero modes of $\mathcal{F} \equiv \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F^{\rho\sigma}$ in NC-directions and vanishes in the decompactification limit $R \to \infty$
  - LLL anomaly, for finite IR cutoff R, involves nonzero and zero modes of $\mathcal{F} \equiv \epsilon^{12ab} F_{ab}$ in transverse directions to $\mathbf{B}$, but in the decompactification limit $R \to \infty$ only the nonzero modes survive
- Integrated form
  - Nonplanar (NC) anomaly involves only the zero modes is finite
  - LLL anomaly involves nonzero and zero modes is finite

Summary

- Effective action of QED in the LLL corresponds to a modified NC-QED
  - No UV/IR mixing appears in this modified NC theory
- The LLL anomaly is comparable with the nonplanar anomaly of ordinary NC-QED
- The LLL anomaly is also comparable with the anomaly of a 2-dim Schwinger model