Axionic gaugings in $N = 4$ supergravities

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based on work with Jean-Pierre Derendinger (Neuchâtel University)
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Highlights

Motivations, summary and conclusions

Gauged supergravities and the embedding tensor

The non-unimodular gaugings

Higher-dimensional origin: generalized Scherk–Schwarz reduction

Why gaugings and fluxes?

Usual caveats of string compactifications:

- Supersymmetry breaking: $N = 4$ or $8 \to N = 1$ and 0
- Many massless neutral scalars: moduli stabilization
- Cosmological constant (?)

Tool for a better control of the situation: give vev's to antisymmetric-tensor fields such as NS–NS, R–R, spin connection

From the low-energy viewpoint

- Toroidal compactifications: $N = 8$ or $N = 4$ *ungauged* sugras with neutral scalars without potential
- Flux compactifications: $N = 8$ or $N = 4$ (or less) *gauged* sugras with charged scalars under (non-)Abelian gauge groups and moduli-dependent superpotential (and potential)

Two complementary approaches

"Top-down" [many groups – see M. Graña's '05 review]

- Understand the *generalized geometrical tools* in the (semi-)fundamental 10-dim theory in presence of fluxes
- Find admissible compactifications
- Extract the low-energy properties

"Bottom-up" [Derendinger, Kounnas, Petropoulos, Zwirner '05]

- Start with phenomenologically relevant 4-dim gauged sugras
- Translate the gauging parameters into fluxes
- Reconstruct the fundamental theory

About the bottom-up programme

- No systematic oxidation recipe
- Not all 4-dim gauged $N = 4$ ($N = 8$) sugras are heterotic, type-I or type-II-orientifold (M-theory) vacua
- A good picture of the landscape of gauged sugras is nevertheless necessary
- Preferably accompanied by the appropriate dictionary *fluxes – gauging parameters*

The gauging procedure is a very powerful method, also useful for understanding the *non-geometric* string backgrounds

Here

We focus on 4-dim $N = 4$ theories (seeds for "realistic" vacua)

- Remind the basics on the gauging procedure using the *embedding tensor* – outstanding tool that
  - captures all consistency constraints
  - describes exhaustively the gauge algebras
  - allows for reconstructing the scalar potential
- Analyze the gauging of *axionic shifts and rescalings*
- Trace its 10-dim origin – not straightforward
  - requires a generalized Scherk–Schwarz with shift by the scaling symmetries
  - relies on a duality between massive vectors and massive two-forms due to the appearance of Stückelberg-like terms generated by the axionic gauging

The outcome

This analysis closes the chapter of characterizing a whole class of heterotic gaugings in terms of NS–NS and spin-connection fluxes

It calls for further investigation of other classes of gaugings related to the previous by duality transformations

- The embedding-tensor formalism is duality-covariant: it captures the effective dynamics of duality-related backgrounds
- The bottom-up approach
  - shows that 4-dim physics needs not delving into the microscopic details of the fundamental theory
  - is an appropriate tool for the study of non-geometric setups
- What are the geometrical features of the fundamental theory on the top that translate into the consistency constraints imposed to the embedding tensor from the bottom?

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Ungauged 4-dim $N = 4$ supergravity

Spectrum, interactions and symmetries

- 1 gravitational and n vector multiplets
- Bosonic content of the multiplets
  - gravitational multiplet: 1 graviton, 6 graviphotons, 2 real scalars combined into the *axion-dilaton* $\tau = \chi + i \exp -2\phi$
  - vector multiplet: 1 vector, 6 real scalars
- Gauge group: $U(1)^{6+n}$
- All scalars are neutral and non-minimally coupled to the vectors: interaction terms of the type $f(\text{scalars})\, F^2$
- There is *no* scalar potential
- The elimination of the auxiliary fields generates the *scalar manifold:*

$$\mathcal{M} = \frac{SL(2,\mathbb{R})}{U(1)} \times \frac{SO(6,n)}{SO(6) \times SO(n)}$$

- The $SL(2,\mathbb{R}) \times SO(6,n) \subset Sp(12+2n,\mathbb{R})$ is realized as a U-duality symmetry of the *full* theory [Gaillard, Zumino '81]
- The $SO(1,1) \times SO(6,n)$ is realized off-shell in heterotic
  - the $SO(1,1)$ does not mix electric and magnetic gauge fields
  - genuine electric–magnetic duality transformations relate different Lagrangians with equivalent field equations and each Lagrangian corresponds to a choice of "symplectic frame"

Putting electric and magnetic duals together

- The $2 \times (6+n)$ fields $\left(\left\{\mathbf{A}^{M+}\right\}, \left\{\mathbf{A}^{M-}\right\}\right)$, $M = 1, \ldots, 6+n$ form a $(\mathbf{2}, \mathbf{Vec})$ of $SL(2,\mathbb{R}) \times SO(6,n)$
- A Lagrangian exists that captures their dynamics without altering the number of propagating degrees of freedom – extra 2-form auxiliary fields [de Wit, Samtleben, Trigiante '02 –]

Gauging: deformation compatible with supersymmetry

Promotion of a subgroup of the U-duality group to a local gauge symmetry supported by (part of) the existing $U(1)^{n+6}$ vectors

- The generators of the duality group are
  1. $T^{MN} = -T^{NM}$, $M, \ldots = 1, \ldots, 6+n$ generate the $SO(6,n)$
  2. $S^{\beta\gamma} = S^{\gamma\beta}$, $\beta, \ldots = +, -$ generate the $SL(2,\mathbb{R})$
- The generators of the *gauge algebra* are

$$\Xi_{\alpha L} = \frac{1}{2}\left(\Theta_{\alpha LMN}\, T^{MN} + \Theta_{\alpha L\beta\gamma}\, S^{\beta\gamma}\right)$$

  where $\{\Theta_{\alpha LMN}, \Theta_{\alpha L\beta\gamma}\} \in (\mathbf{2}, \mathbf{Vec} \times \mathbf{Adj}) + (\mathbf{2} \times \mathbf{3}, \mathbf{Vec})$ of $SL(2,\mathbb{R}) \times SO(6,n)$ is the *embedding tensor*
- At most $6+n$ Ξ's are independent and the Θ's are subject to *constraints*

Consistency constraints for the embedding tensor

Gauge invariance and supersymmetry [linear constraints]

This reduces the embedding tensor to $(\mathbf{2}, \mathbf{Ant}_{[3]}) + (\mathbf{2}, \mathbf{Vec})$:

$$\Xi_{\alpha L} = \frac{1}{2}\left(f_{\alpha LMN}\, T^{MN} + \eta_{LQ}\, \xi_{\alpha P}\, T^{QP} + \epsilon^{\gamma\beta}\, \xi_{\beta L}\, S_{\gamma\alpha}\right)$$

The fundamental of $Sp(12+2n, \mathbb{R})$ must contain the adjoint of the gauge algebra and the latter must close [quadratic constraints]

(i) $\eta^{MN}\, \xi_{\alpha M}\, \xi_{\beta N} = 0$

(ii) $\eta^{MN}\, \xi_{(\alpha M}\, f_{\beta) NIJ} = 0$

(iii) $\epsilon^{\alpha\beta}\left(\xi_{\alpha I}\, \xi_{\beta J} + \eta^{MN}\, \xi_{\alpha M}\, f_{\beta NIJ}\right) = 0$

(iv) $\eta^{MN}\, f_{\alpha MI[J}\, f_{\beta KL]N} - \frac{1}{2}\xi_{\alpha[J}\, f_{\beta KL]I} - \frac{1}{6}\epsilon_{\alpha\beta}\, \epsilon^{\gamma\delta}\, \xi_{\gamma I}\, f_{\delta JKL} + \frac{1}{2}\eta^{MN}\, \xi_{\alpha M}\, f_{\beta N[JK}\, \eta_{L]I} + \frac{1}{6} f_{\alpha JKL}\, \xi_{\beta I} = 0$ (Jacobi-like)

Important remarks

- f's and ξ's are the *gauging parameters* which determine
  - the algebra and its commutators
  - the scalar potential
  - the mass matrices
  - ...
- $f_{\alpha JKL}$ *are not* necessarily structure constants

Examples

Gaugings with non-vanishing $f_{\alpha LMN}$ only

- Pure $SO(6, n)$ gaugings, extensively studied in the literature
- Large variety of gauge algebras as e.g. *flat algebras* related to unimodular Scherk–Schwarz reductions

Gaugings with non-vanishing $\xi_{\alpha L}$

- Only some isolated examples have been studied that fall in this class [Villadoro, Zwirner '04; Schön and Weidner '06]
- Their systematic analysis is the subject of the next chapters

Lagrangian formulation – including electric and magnetic

An explicit Lagrangian is associated with any consistent gauging and its bosonic sector has three parts [Schön and Weidner '06]

- $\mathcal{L}_{\text{kin}}$ – Einstein term and kinetic terms for vectors and scalars
- $\mathcal{L}_{\text{top}}$ – auxiliary-field contributions necessary to maintain the correct number of propagating vectors
- $\mathcal{L}_{\text{pot}} = -\frac{e}{16}\Big(f_{\alpha MNP}\, f_{\beta QRS}\, M^{\alpha\beta}\big(\frac{1}{3}M^{MQ}\, M^{NR}\, M^{PS} + \big(\frac{2}{3}\eta^{MQ} - M^{MQ}\big)\eta^{NR}\eta^{PS}\big) - \frac{4}{9} f_{\alpha MNP}\, f_{\beta QRS}\, \epsilon^{\alpha\beta}\, M^{MNPQRS} + 3\xi^M_\alpha\, \xi^N_\beta\, M^{\alpha\beta} M_{MN}\Big)$ – the scalar potential
  - $M^{\alpha\beta}$ are the components of $\frac{1}{\text{Im}\tau}\begin{pmatrix} 1 & -\text{Re}\tau \\ -\text{Re}\tau & |\tau|^2 \end{pmatrix}$
  - M^{MQ} and M^{MNPQRS} are constructed similarly with the remaining $6n$ scalars

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The axionic transformations and their gaugings

The axionic transformations are generated by the $SL(2,\mathbb{R})$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{R})$ acts on the axion-dilaton as $\tau \longrightarrow \frac{a\tau+b}{c\tau+d}$

- $S^{--} = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$ generates the *electric–magnetic duality*
- $S^{++} = \begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix}$ generates the *axionic shifts* $\tau \longrightarrow \tau + b$
- $S^{+-} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ generates the *axionic rescalings* $\tau \longrightarrow a^2\tau$

Gauging the axionic symmetries [see the embedding tensor]

- requires an embedding tensor with $\xi_{\alpha M} \neq 0$
- is necessarily accompanied by a partial gauging of $SO(6,n)$

Aim: gauge the axionic shifts S^{++} and rescalings S^{+-} but not the electric–magnetic duality transformation S^{--} – "electric gaugings"

- We must set $\xi_{-I} = 0$ [see embedding tensor]
- Our further choice: $f_{-LMN} = 0$ (not compulsory)
- The constraints are ("+" index dropped)

$$\begin{aligned} (i) \quad & \eta^{MN} \xi_M \xi_N = 0 \\ (ii) \quad & \eta^{MN} \xi_M f_{NIJ} = 0 \\ (iv) \quad & \eta^{MN} f_{MI[J} f_{KL]N} = \tfrac{2}{3} f_{[IJK} \xi_{L]} \end{aligned}$$

Solution: non-unimodular gaugings captured by $\{\lambda_i, i = 1, \ldots, 6\}$

- We focus on $n = 6$: 12 vectors in total
- Light-cone-like convention: $\{I\} \equiv \{i, i'\}$, $\eta = \begin{pmatrix} 0 & \mathbb{I}_6 \\ \mathbb{I}_6 & 0 \end{pmatrix}$
- $\xi_i = \lambda_i$, $\xi_{i'} = 0$ and $f_{iji'} = -\lambda_{[i} \delta_{j]i'}$

Remarks

- Here *all* other f's vanish: $f_{ijk}, f_{ii'j'}, f_{i'j'k'}$
- The f's are *not* Lie-algebra structure constants

The gauge algebra [see embedding tensor] has 8 independent generators out of 2×12*:* $\{\Upsilon, \Xi, \Xi_{i'}\}$

- $\Xi_{-i} = -\frac{\lambda_i}{2} S^{++} \equiv \lambda_i \Upsilon$
- $\Xi_{-i'} = 0$
- $\Xi_{+i} = -\frac{\lambda_i}{2} \left(T^j{}_j + S^{+-} \right) \equiv \lambda_i \Xi$
- $\Xi_{+i'} = -\lambda_j T^j{}_{i'} \equiv \Xi_{i'}$

Axionic symmetries are gauged along with 6 $\{\Xi_{i'}\} \subset SO(6,6)$

Commutation relations for $\{\Upsilon, \Xi, \Xi_{i'}\} \subset SL(2,\mathbb{R}) \times SO(6,6)$

- $[\Xi_{i'}, \Xi_{j'}] = 0$
- $[\Upsilon, \Xi_{j'}] = 0$
- $[\Xi_{i'}, \Xi] = \Xi_{i'}$
- $[\Upsilon, \Xi] = -\Upsilon$

More remarks and summary

- $\{\Upsilon, \Xi, \Xi_{i'}\}$ is *non-flat* in contrast to the algebras obtained by standard Scherk–Schwarz reductions [see latter]
- $\{\Upsilon, \Xi\}$ is the non-semi-simple subalgebra $A_{2,2} \subset SL(2,\mathbb{R})$ of axionic rescalings and axionic shifts
- $\{\Upsilon, \Xi_{i'}\}$ spans an Abelian 7-dim ideal
- More general electric gaugings with $f_{ijk} \neq 0$ allow for non-Abelian extensions

The dynamics of non-unimodular gaugings

The 12 vectors

4 inert and $2+6$ embedded in $SL(2,\mathbb{R}) \times SO(6,6)$ as generators of local symmetries – enter in covariant derivatives acting on scalars

The $36 = 21 + 15$ *scalars of* $\frac{SO(6,6)}{SO(6)\times SO(6)}$

- The usual coset parameterization is

$$M^{MN} = \begin{pmatrix} h^{ij} & -h^{ik}\, b_{kj} \\ b_{ik}\, h^{kj} & h_{ij} - b_{ik}\, h^{k\ell}\, b_{\ell j} \end{pmatrix}$$

- The gauging at hand generates $\mathcal{L}_{\text{pot}}$

$$\frac{1}{16} e^{2\phi} \lambda_i \left(8h^{ij} - h^{ij}\, h^{k\ell}\, b_{\ell m}\, h^{mn}\, b_{nk} + 2h^{ik}\, b_{km}\, h^{mn}\, b_{nr}\, h^{rj} \right) \lambda_j$$

The axion-dilaton

- The kinetic term is

$$e^{-1}\mathcal{L}_{\text{kin:axion-dilaton}} = -D_\mu \phi D^\mu \phi - \frac{1}{4}\mathrm{e}^{4\phi} D_\mu \chi \, D^\mu \chi$$

  - $D_\mu \phi = \partial_\mu \phi - \frac{1}{2} Y_\mu$
  - $D_\mu \chi = \partial_\mu \chi + X_\mu + Y_\mu \chi$

- Physical vectors involve electric *and* magnetic potentials:

$$Y_\mu = \lambda_i \, A_\mu^{i+} \quad X_\mu = \lambda_i \, A_\mu^{i-}$$

  - $Y_\mu \leftrightarrow \Upsilon$ (axion rescalings: $\chi \to a^2 \chi$, $\phi \to \phi - \log a$)
  - $X_\mu \leftrightarrow \Xi$ (axion shifts: $\chi \to \chi + b$)

- The axion can be gauged away – X_μ acquires a mass in this process via its Stückelberg coupling to χ

Trading massive X_μ for massive $C_{\nu\rho}$ [Townsend, Pilch, van Niewenhuizen '84; Quevedo '95]

- Start with a massive vector:

$$\mathcal{L} = -\frac{1}{4g^2}\left(\partial_\mu X_\nu - \partial_\nu X_\mu\right)\left(\partial^\mu X^\nu - \partial^\nu X^\mu\right) + \frac{1}{2}m^2 X_\mu X^\mu$$

- $\mathcal{L}$ is obtained by integrating the auxiliary two-form in

$$\tilde{\tilde{\mathcal{L}}} = 2C_{\mu\nu}\,\partial^\mu X^\nu + g^2 C_{\mu\nu}\,C^{\mu\nu} + \frac{1}{2}m^2 X_\mu X^\mu$$

- Integrating by parts $\tilde{\tilde{\mathcal{L}}}$ makes X_μ auxiliary and $C_{\mu\nu}$ dynamical
- Integrating out X_μ yields a massive-two-form Lagrangian:

$$\tilde{\mathcal{L}} = -\frac{2}{m^2}\partial_\mu C^{\mu\nu}\,\partial^\lambda C_{\lambda\nu} + g^2 C_{\mu\nu}\,C^{\mu\nu}$$

The final bosonic content of the non-unimodular gauging

- The dilaton
- $4 + 1 + 6$ vectors with Abelian algebra
  - 4 inert
  - 1 associated with the axionic rescalings of $SL(2, \mathbb{R})$
  - 6 associated with MASA transformations of $SO(6, 6)$
- 1 massive two-form
- 36 scalars
  - with scalar potential
  - and minimal couplings to the $1 + 6$ vectors

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Heterotic 10-dim pure supergravity

The $SO(1,1)$ symmetry

- Action for the bosonic sector ($H = \mathrm{d}B$)

$$\int_{M_4} \mathrm{d}x \int_{K_6} \mathrm{d}y \sqrt{-G}\, \mathrm{e}^{-\Phi} \left(R + G^{MN}\, \partial_M \Phi\, \partial_N \Phi - \frac{1}{12} H_{MNK} H^{MNK} \right)$$

- Invariance under $SO(1,1)$

$$\Phi \to \Phi + 4\lambda, \quad G_{MN} \to \mathrm{e}^{\lambda} G_{MN}, \quad B_{MN} \to \mathrm{e}^{\lambda} B_{MN}$$

Dimensional reduction

- K_6 is compact: infinitude of modes
- Reduction: effective theory on M_4 for a *finite* subset
- Data: K_6 plus an ansatz for the y-dependance of all fields
- Necessary consistency: $\mathcal{L}$ independent of y

Ordinary vs. Scherk–Schwarz reduction

Standard reduction on flat torus

- Ansatz: no y-dependance
- Bosonic spectrum: 1 graviton, $6 + 6$ Abelian vectors, 36 scalars, 1 dilaton, 1 axion (dual to the NS–NS form) – all massless

Ordinary vs. Scherk–Schwarz reduction

Scherk–Schwarz reduction *[Scherk, Schwarz '79; long literature]*

- Ansatz: y-dependance compatible with internal symmetries
- Introduction of geometric (spin connection) fluxes $\gamma^i{}_{jk}$
  - $\mathrm{d}\theta^i = -\gamma^i{}_{jk}\,\theta^j \wedge \theta^k$
  - Bianchi–Jacobi $\gamma^i{}_{j[k}\,\gamma^j{}_{\ell m]} = 0$
  - $f_{jk}{}^i = 2\gamma^i{}_{jk}$ structure constants of a locally group manifold
  - $\gamma^i{}_{ij} = 0$: *unimodularity* property
- Results: non-Abelian vectors, massive scalars and vectors, spontaneous breaking of supersymmetry
- Example: *twisted tori* leading to gaugings in $SO(6,6)$

External Scherk–Schwarz reduction

Using the "duality" $SO(1,1)$ 10-dim symmetry

- Ansatz: $\Phi(x,y) = \Phi(x) + 4\lambda_i y^i$ ($SO(1,1)$-shift)
  $G_{MN}(x,y) = e^{\lambda_i y^i} G_{MN}(x)$ $B_{MN}(x,y) = e^{\lambda_i y^i} B_{MN}(x)$
- Decomposition:
  - $G_{MN} \to g_{\mu\nu}, A_{\mu k}, h_{ij}$
  - $B_{MN} \to B_{\mu\nu}, B_{\mu k}, b_{ij}$
  - $\phi = \Phi - \frac{1}{2} \log \det \mathbf{h}$

The ansatz is consistent and various couplings emerge

- $A_{\mu k}$ and $B_{\mu k}$ carry Abelian gauge symmetry
- h_{ij} charged under $A_{\mu k}$
- b_{ij} charged under $B_{\mu k}$ and Stückelberg-coupled to $A_{\mu k}$
- ϕ Stückelberg-coupled to $A_{\mu k}$
- scalar potential for h_{ij} and b_{ij}

Contact with axionic gaugings

After field redefinitions and integrations one vector drops and the two-form becomes massive due to the Stückelberg couplings

- Indicative of the gauging of a shift symmetry
- The reduced theory *is* the gauged $N = 4$ supergravity studied in the last chapter: exact matching of the Lagrangians
- The specific choice of generalized Scherk–Schwarz allows to
  1. turn on the gauging parameters ξ_i as $SO(1,1)$-shift parameters λ_i along the torus one-cycles
  2. gauge the $SL(2,\mathbb{R})$ axionic shifts and rescalings in 4-dim
  3. evade unimodularity (here $f_{ij}{}^j = -\frac{5}{2}\lambda_i$): non-unimodular geometric fluxes

All this elegantly demonstrates the power of the gauging procedure for describing diverse flux compactifications

Last slide

Further gaugings further fluxes

- f_{+IJK}, ξ_{+L}: 232 electric parameters
  - f_{+ijk} NS–NS, $f_{+ijk'}$ spin-connection [studied here in relation with axionic symmetries; Kaloper, Myers '99 in the unimodular case; ...]
  - $f_{+ij'k'}$ T-dual NS–NS, $f_{+i'j'k'}$ T-dual spin-connection: "non-geometric" [Hull *et al.* '05; Shelton, Taylor, Wecht '05; ...]
- f_{-IJK}, ξ_{-L}: 232 magnetic-dual parameters
  - f_{-ijk} NS–NS, $f_{-ijk'}$ spin-connection
  - $f_{-ij'k'}$ T-dual NS–NS, $f_{-i'j'k'}$ T-dual spin-connection

The number of degrees of freedom does not change – the algebra, its $SL(2,\mathbb{R}) \times SO(6,n)$ embedding and the higher-dimensional setup do