Induced Cosmology in No-Scale Effective Theories of Superstrings

Hervé Partouche
(Ecole Polytechnique, FRANCE)

Work done with

Costas Kounnas

arXiv: 0706.0728
arXiv: 0705.3206

Fourth Regional Meeting
in String Theory

Patras, 14 June 07

$N=1$ string Theory in 4D

complex Structure

Classically:
- S, U_a stabilized
- ~~Susy~~ spontaneously
- Flat
- $m = e^{\alpha \Phi}$

Gravitino Mass   Goldstino's Superpartner

Quantum + Thermal Corrections

$$S_{eff} = \frac{-1}{6}\int dt\, a^3\left[\frac{3}{N}\left(\frac{\dot a}{a}\right)^2 - \frac{3kN}{a^2} - \frac{1}{2N}\dot\Phi^2 - \frac{1}{2N}\dot\chi^2 + NV - \frac{1}{2N}(\rho+P) + \frac{N}{2}(\rho-P)\right]$$

$$ds^2 = -N^2 dt^2 + a^2 \underbrace{d\Omega_3^2}_{k}$$

$$3\left(\frac{\dot a}{a}\right)^2 = -\frac{3k}{a^2} + \frac{1}{2}\dot\Phi^2 + \frac{1}{2}\dot\chi^2 + V + \rho$$

V: m^4, $m^4 \log\frac{m^2}{\mu^2}$

m^2, $m^2 \log\frac{m^2}{\mu^2}$

m^0, $m^0 \log\frac{m^2}{\mu^2}$

ρ: $T^4 f\left(\frac{m_i}{T}\right)$

$T^2 M^2$

Critical Trajectory:

$$m \equiv e^{\alpha\Phi}$$

$$= \frac{1}{\gamma' a} = \frac{T}{\xi} = \frac{\gamma}{\gamma'}\mu$$

$$\Downarrow$$

$$\left(\frac{\dot a}{a}\right) = \alpha\dot\Phi$$

$$3\left(\frac{\dot a}{a}\right)^2 = \lambda - \frac{\hat k}{a^2} + \frac{C_R}{a^4} + \underbrace{\frac{C_M}{a^6}}_{\Uparrow\ \dot\chi^2}$$

- Constant $\boxed{\text{on Shell only}}$
- λ : de Sitter or Anti de Sitter

  $\hat k$, C_R: positive due to other Moduli

NB: We are more interested in

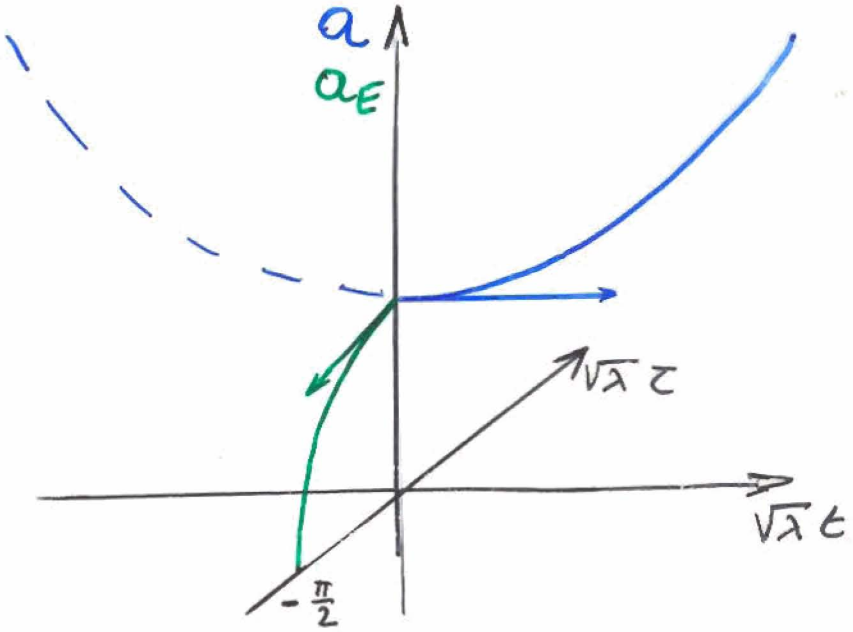
$\lambda, \hat k, C_R > 0$

Rescale $a \to \sqrt{\hat k}\, a \Rightarrow \hat k = 1$

Pure de Sitter Case :

$$\left(\frac{\dot a}{a}\right)^2 = \lambda - \frac{1}{a^2} \qquad \text{ie} \quad C_R = C_M = 0$$

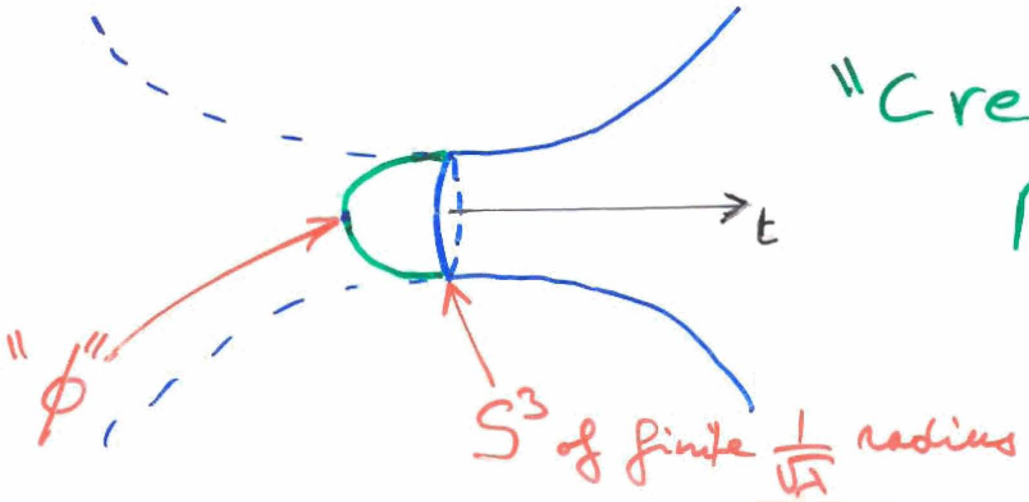
$$a(t) = \frac{1}{\sqrt{\lambda}} \cosh(\sqrt{\lambda}\, t)$$



$$a_E(\tau) = \frac{1}{\sqrt{\lambda}} \cos(\sqrt{\lambda}\,\tau)$$

$$ds_E^2 = \frac{1}{\lambda}\left[d(\sqrt{\lambda}\,\tau)^2 + \cos^2(\sqrt{\lambda}\,\tau)\, d\Omega_3^2 \right]$$

Vilenkin's Scenario (82') =



"Creation from Nothing"

Switch on Radiation :

$$\left(\frac{\dot{a}}{a}\right)^2 = \lambda - \frac{1}{a^2} + \frac{\delta_T^2}{4\lambda a^4}$$

$$a_c(t) = \mathcal{N}\sqrt{\mathcal{E} + \cosh^2(\sqrt{\lambda}\,t)}$$

$$\mathcal{N} \to \frac{\left(1-\delta_T^2\right)^{1/4}}{\sqrt{\lambda}} \qquad \mathcal{E} \to \frac{1}{2}\left(\frac{1}{\sqrt{1-\delta_T^2}} - 1\right)$$

$$\delta_T^2 \leq 1$$

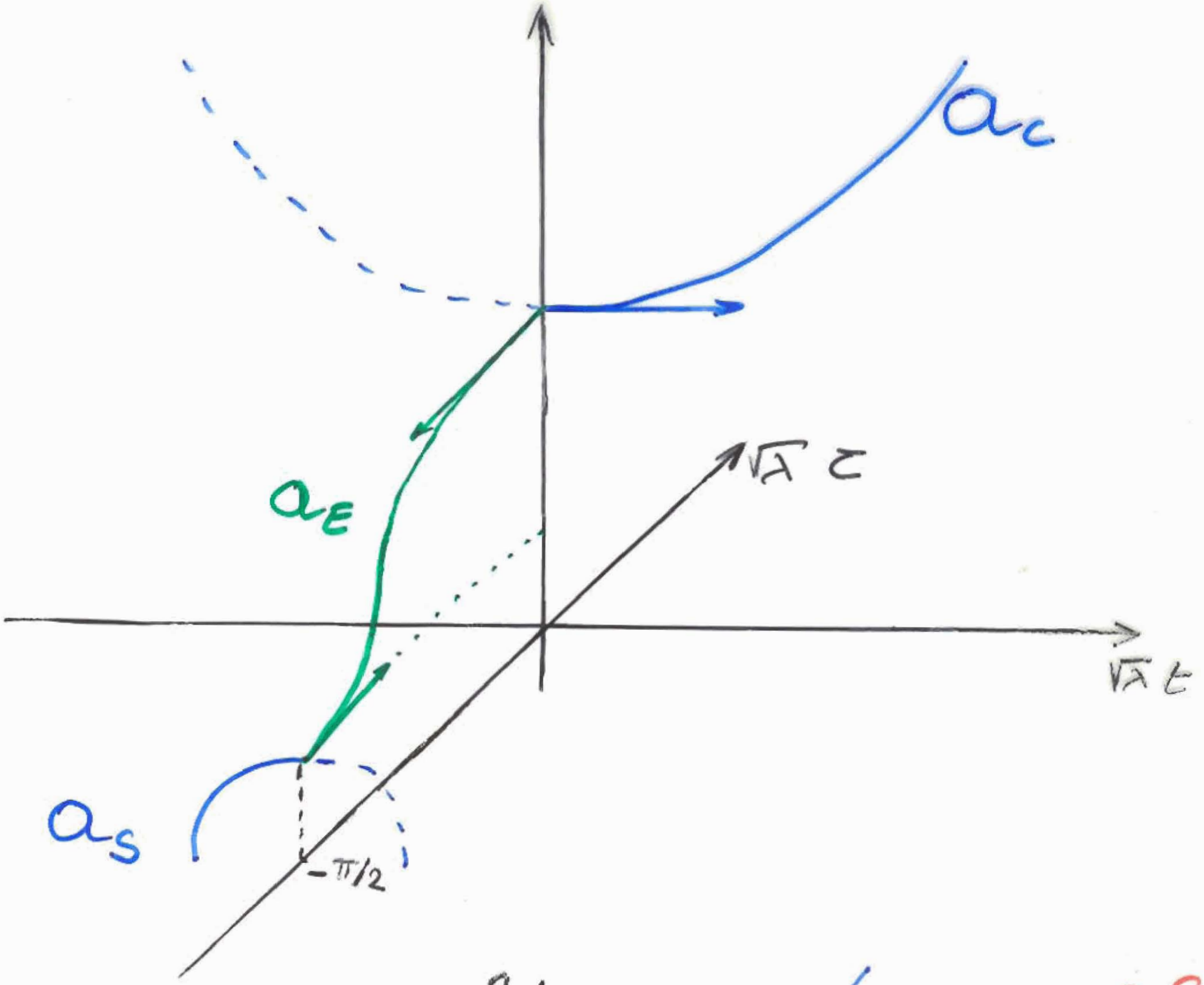
- Sarangi and Tye
- Brustein and Alwis

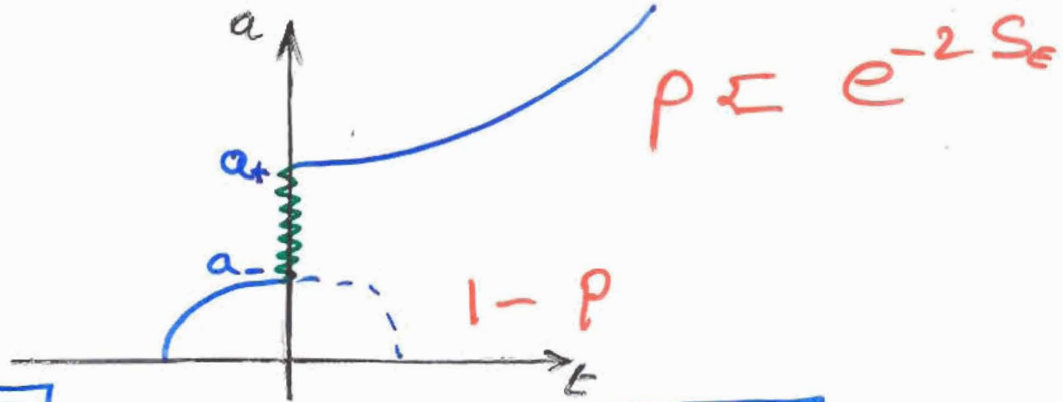
- At $t=0$, $t = -i\tau$

$$a_E(\tau) = \mathcal{N}\sqrt{\mathcal{E} + \underbrace{\cos^2(\sqrt{\lambda}\,\tau)}_{\sin^2\left(\sqrt{\lambda}\,\tau + \frac{\pi}{2}\right)}}$$

- At $\sqrt{\lambda}\,\tau = -\frac{\pi}{2}$, $\sqrt{\lambda}\,\tau + \frac{\pi}{2} = i\sqrt{\lambda}\,t$

$$a_s(t) = \mathcal{N}\sqrt{\mathcal{E} - \sinh^2(\sqrt{\lambda}\,t)}$$

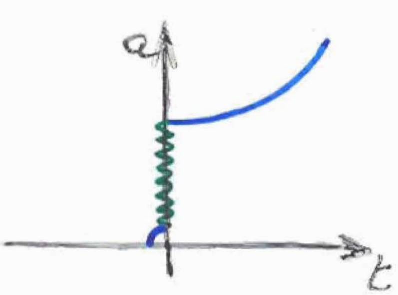


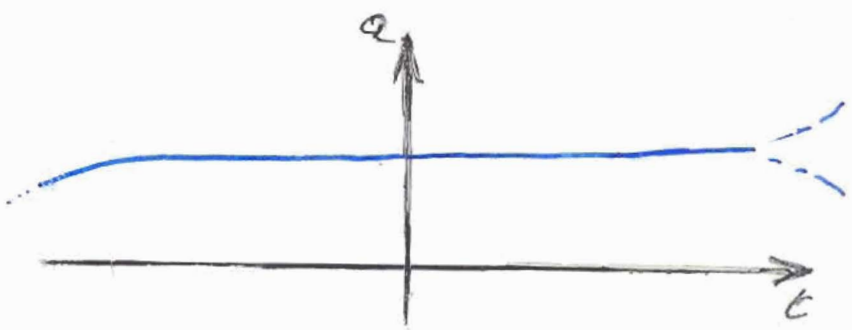


$$a_+ = \sqrt{\frac{1+\sqrt{1-\delta_T^2}}{2\lambda}} \qquad a_- = \sqrt{\frac{1-\sqrt{1-\delta_T^2}}{2\lambda}} \sim \frac{\delta_T}{2\sqrt{\lambda}}$$

$\delta_T^2 \simeq 0$: Creation from "almost nothing"

$\delta_T^2 \to 1$: Static





Switch on Moduli

Kinetic Energy:

$$\left(\frac{\dot{a}}{a}\right)^2 = N^2\left[\lambda - \frac{1}{a^2} + \frac{\delta_T^2}{4\lambda a^4} + \frac{4}{27}\frac{\delta_M^2}{\lambda^2 a^6}\right]$$

$$ds^2 = -N^2\, d\tilde{t}^2 + a^2\, d\Omega_3^2$$

$$N^2(\tilde{t}) = \frac{1}{1+\frac{\kappa}{\lambda a^2}} = 1 - \frac{\kappa}{\lambda a^2} + \frac{\kappa^2}{\lambda^2 a^4} - \cdots$$

$$\kappa^3 + \kappa^2 + \frac{\delta_T^2}{4}\kappa - \frac{4}{27}\delta_M^2 = 0$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \lambda - \frac{1+\kappa}{a^2} + \frac{\delta_T^2 + 4\kappa + 4\kappa^2}{4\lambda a^4}$$

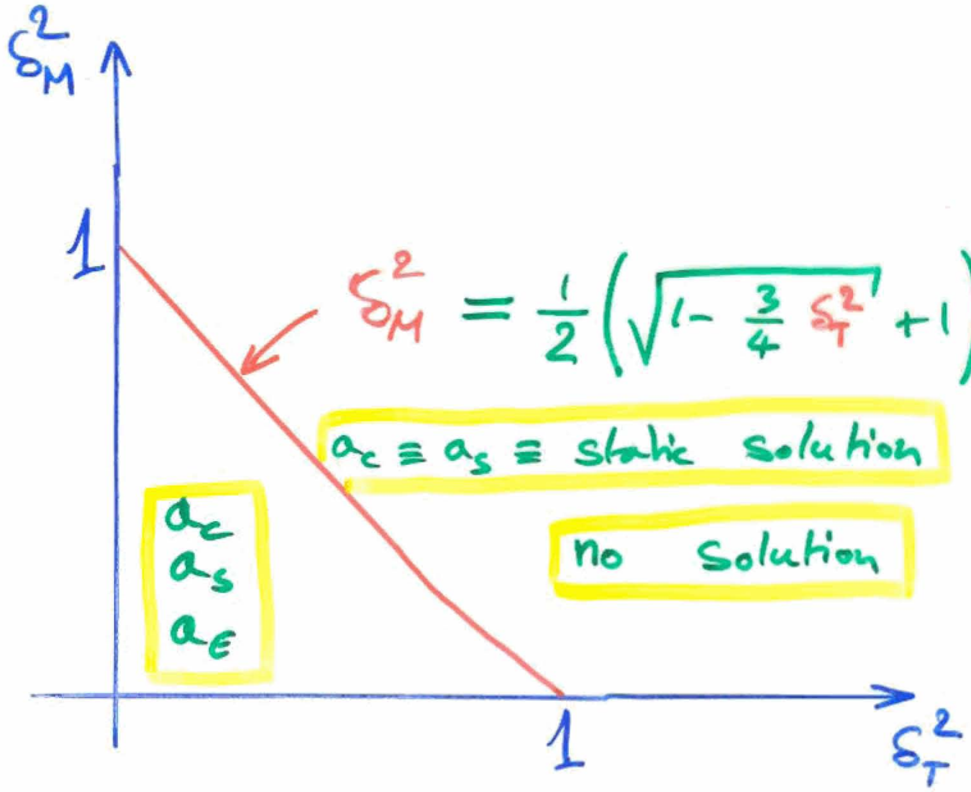
$$a_c(\tilde{t}) = \mathcal{N}\sqrt{\mathcal{E} + \cosh^2(\sqrt{\lambda}\,\tilde{t})}$$

$$a_s(\tilde{t}) = \mathcal{N}\sqrt{\mathcal{E} - \sinh^2(\sqrt{\lambda}\,\tilde{t})}$$

$$a_E(\tilde{t}) = \mathcal{N}\sqrt{\mathcal{E} + \cos^2(\sqrt{\lambda}\,\tilde{t})}$$

$$\sqrt{\frac{1+\kappa}{\lambda}}\,(1-\Delta)^{1/4} \qquad\qquad \frac{1}{2}\left(\frac{1}{\sqrt{1-\Delta}} - 1\right)$$

$$\Delta = \frac{4\kappa^2 + \kappa + \delta_T^2}{(1+\kappa)^2} \leq 1$$



$$\delta_M^2 = \frac{1}{2}\left(\sqrt{1-\frac{3}{4}\delta_T^2}+1\right)^2\left(\sqrt{1-\frac{3}{4}\delta_T^2}-\frac{1}{2}\right)$$

We may preffer :

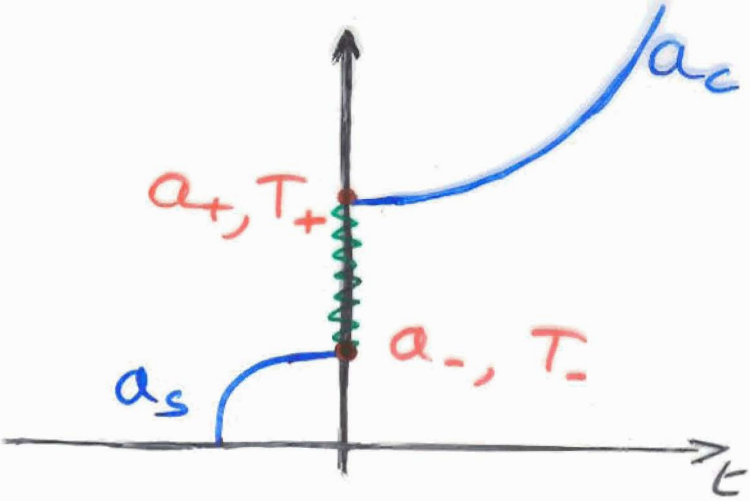
$$ds^2 = -\underbrace{dt^2}_{\equiv\, N^2(\tilde{t})\, d\tilde{t}^2} + a^2\, d\Omega_3^2$$

$$a_c = \mathcal{N}\sqrt{\mathcal{E} + \cosh^2\left(\sqrt{\lambda}\,\tilde{t}(t)\right)}$$

where $t = \displaystyle\int_0^{\tilde{t}} dv \sqrt{\frac{\cosh^2(\sqrt{\lambda}\,v) + \mathcal{E}}{\cosh^2(\sqrt{\lambda}\,v) + \mathcal{E} + \tilde{\mathcal{E}}}}$

$$\frac{\kappa}{(1+\kappa)\sqrt{1-\Delta}}$$

Idem for a_s and $a_\mathcal{E}$.



Parameterize with $T_\pm$ instead of δ_T^2, δ_M^2 : $a_+ T_+ = a_- T_-$

$= \xi/\gamma'$

$$T_\pm = T_m \frac{\sqrt{\delta_T}}{\sqrt{(1+\kappa)(1 \pm \sqrt{1-\Delta})}}$$

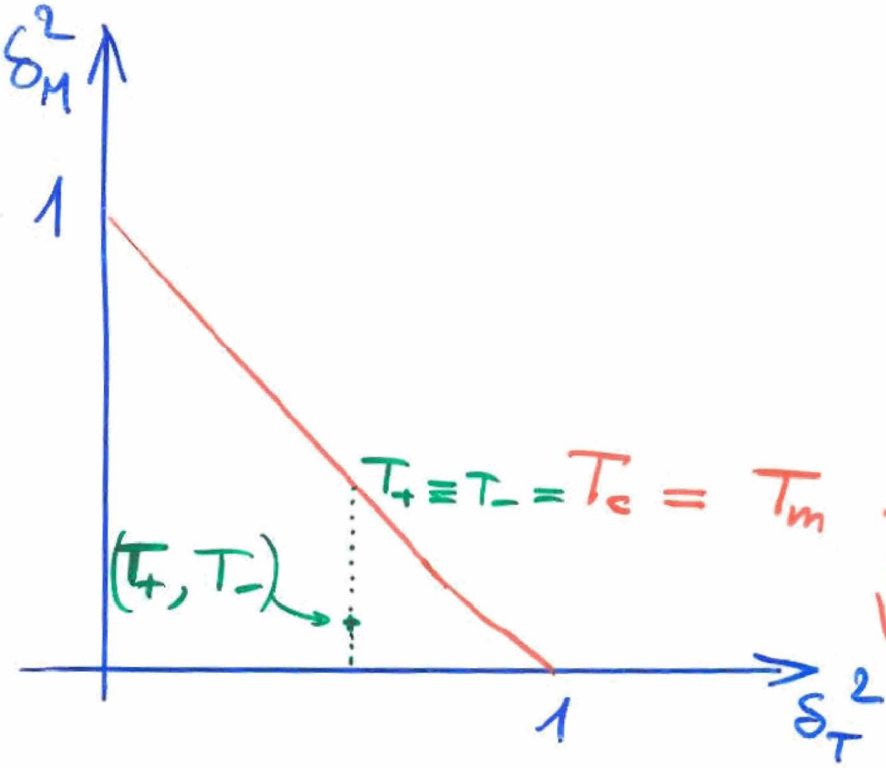
$$\frac{\xi}{\gamma'} \sqrt{\frac{2\lambda}{\delta_T}}$$

$$a_c = \frac{1}{\sqrt{2\lambda(1-\alpha t)}} \sqrt{1 + \frac{T_-^2 - T_+^2}{T_-^2 + T_+^2} \cosh(2\sqrt{\lambda}\, \tilde{t}(t))}$$

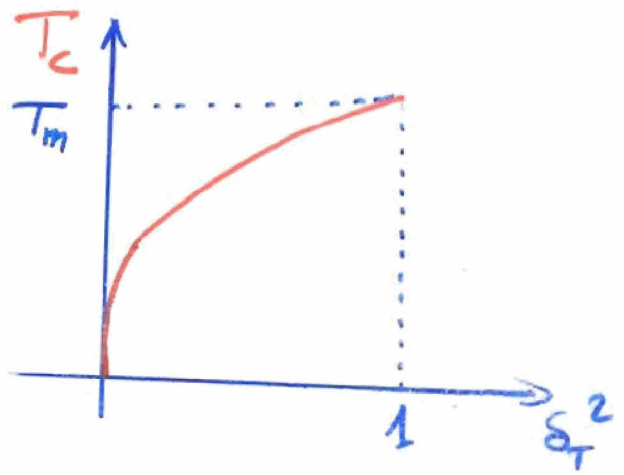
$$a_s = \frac{1}{\sqrt{2\lambda(1-\alpha t)}} \sqrt{1 + \frac{T_+^2 - T_-^2}{T_+^2 + T_-^2} \cosh(2\sqrt{\lambda}\, \tilde{t}(t))}$$

$$\frac{1 - T_+^2 T_-^2 / T_m^4}{\left(\frac{T_+}{T_-} + \frac{T_-}{T_+}\right)^2}$$

$$(T_+ \leftrightarrow T_-) \Longleftrightarrow (a_c \leftrightarrow a_s) \Longleftrightarrow (t>0 \leftrightarrow t<0)$$



$$T_+ \equiv T_- = T_c = T_m \frac{\sqrt{\frac{3}{2}\delta_T}}{\sqrt{1+\sqrt{1-\frac{3}{4}\delta_T^2}}}$$



$$\rho_{Radiation} = \frac{\delta_T^2}{4\lambda a^4} = \frac{\pi^4}{15}\left(n^B_{eff} + \frac{7}{8} n^F_{eff}\right) T^4$$

$$T^4 \equiv \frac{T_+^4 a_+^4}{a^4}$$

$$T_m = \frac{45}{\pi^4}\left(\frac{\lambda}{n^B_{eff} + \frac{7}{8} n^F_{eff}}\right)^{1/4}$$

Transition Probability:

$$P \simeq e^{-2 S_{E\,eff}}$$

$$S_{E\,eff} = \frac{-1}{2}\int d\tilde{\tau}\; N_E a_E^3 \left[\frac{1}{N_E^2}\left(\frac{\dot{a}_E}{a_E}\right)^2 + \frac{1}{a_E^2} - \lambda - \frac{\delta_T^2}{4\lambda a^4} - \frac{\dot{\chi}_E^2}{6N_E^2}\right]$$

$$= -\int_{-\frac{\pi}{2\sqrt{\lambda}}}^{0} d\tilde{\tau}\; N_E a_E^3 \left[\frac{1}{a_E^2} - \lambda - \frac{\delta_T^2}{4\lambda a^4}\right]$$

on Shell

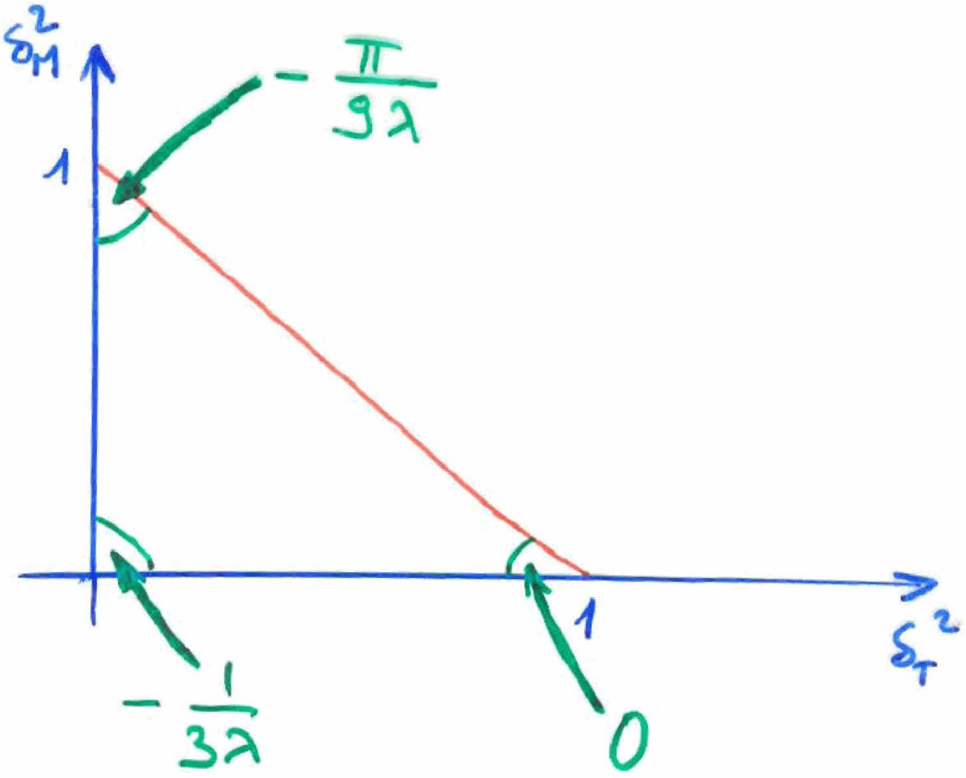
$$N_E = \sqrt{\frac{\cos^2(\sqrt{\lambda}\,\tilde{\tau}) + \varepsilon}{\cos^2(\sqrt{\lambda}\,\tilde{\tau}) + \varepsilon + \tilde{\varepsilon}}}$$

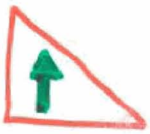
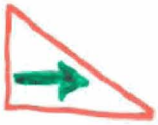
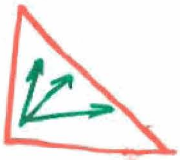
non trivial if $\dfrac{\dot{\chi}_E^2}{6N_E^2} = \dfrac{4}{27\lambda^2}\dfrac{\delta_M^2}{a_E^6} \neq 0$

$$= -\frac{1}{3^{5/4}\lambda}(4-3\delta_T^2)^{1/4}\sqrt{\sin\frac{\theta+\pi}{3}}\left(E(\mu) - \frac{\sqrt{4-3\delta_T^2}\cos\frac{\theta}{3} - 1 + \frac{3}{2}\delta_T^2}{\sqrt{3}\sqrt{4-3\delta_T^2}\sin\frac{\theta+\pi}{3}}K(\mu)\right)$$

where $\mu = \sqrt{\dfrac{\sin\frac{\theta}{3}}{\sin\frac{\theta+\pi}{3}}}$, $\theta = \arccos\left(\dfrac{16\,\delta_M^2 + 9\,\delta_T^2 - 8}{(4-3\,\delta_T^2)^{3/2}}\right)$

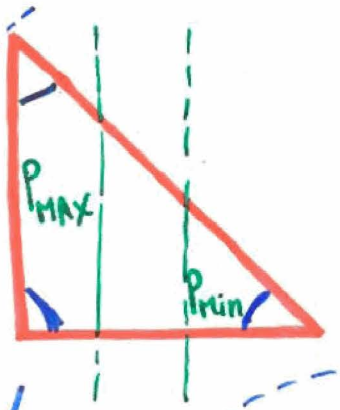
$\boxed{\text{Brustein and Alwis for Pure Thermal case}}$



-  $\rho \simeq$ constant
-  $\rho \searrow$: Big Crunch favored
-  Big Bang Branch grows up

cl:

Viable "Big Bang Branch" followed by Inflation.



Viable "Big Bang / Big Crunch" Cosmology

Inflationary Space created from "almost nothing"

Conclusions :

- Superstrings with
  $\mathcal{N}=1$ ~~Susy~~ spontaneously
  Flat background
- One-loop radiative corrections
  Thermal effects

  induce (for $\lambda, \hat{k}, c_R > 0$)

  i) Inflation created from "almost nothing"

  ii) Big Bang $\xrightarrow[\text{transition}]{\text{1st order}}$ Inflation

  iii) Big Bang / Big Crunch evolution
- Generic.
  No fine-tuning