AdS boundary conditions, stability and M-theory instantons

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hep-th/0703152 [I. P.]
hep-th/0611315 [S. de Haro, I. P., A. C. Petkou]
hep-th/0606038 [I. P.]

Outline

Motivation

- In gravitational theories that contain matter fields, such as -but not only- supergravity theories, the boundary conditions are usually determined uniquely by the requirement of finite conserved charges at infinity.
- In such theories there are well known results on the (non-perturbative) stability of supersymmetric solutions [Witten '81, Gibbons-Hall-Warner '83], as well as on the stability of non-supersymmetric solutions of supersymmetric theories and even solutions of non-supersymmetric theories [Boucher '84, Townsend '84, Skenderis-Townsend '99].
- There are certain theories of gravity, however, admitting an AdS vacuum where the requirement of finiteness for the conserved charges does not uniquely determine the boundary conditions for some of the fields.
- Such theories have been collectively called 'designer gravities' [Hertog-Horowitz '04], but include very familiar theories such as $\mathcal{N}=8$ gauged supergravity in four dimensions, where all 70 scalars and 28 gauge fields admit generalized boundary conditions, as well as $\mathcal{N}=8$ gauged supergravity in five dimensions, where the 20 scalars parameterizing the Coulomb branch of the dual $\mathcal{N}=4$ Super-Yang-Mills admit generalized boundary conditions.

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- In gauged supergravities that arise from a dimensional reduction of string/M-theory and have a known CFT dual these generalized boundary conditions correspond to multi-trace deformations of the dual CFT [Witten '01, Berkooz-Sever-Shomer '01].
- The stability/instability of asymptotically AdS solutions of such theories with generalized (generically supersymmetry breaking) boundary conditions on the matter fields is holographically dual to the stability/instability of the dual CFT under the corresponding multi-trace deformations.
- The stability/instability of AdS under generalized boundary conditions has been recently addressed from the gravity point of view [Hertog-Horowitz '04, Hertog-Hollands '05, Amsel-Marolf '06, Amsel-Hertog-Hollands-Marolf '07] by generalizing the spinorial arguments of the old stability theorems. Here we will address this question -holographically- from the perspective of the dual CFT and find agreement with the spinorial arguments.

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Background independent variational problem on a non-compact space

- As a first step in constructing a background independent formulation of the variational problem of a diffeomorphism invariant theory on a non-compact manifold, $\mathcal{M}_{d+1}$, we write the metric on $\mathcal{M}_{d+1}$ in the form

$$ds^2 = dr^2 + \gamma_{ij}(r,x)dx^i dx^j, \quad i,j = 1,\dots d,$$

where r is the Gaussian normal to the boundary $\partial\mathcal{M}_{d+1}$ located at $r \to \infty$ and γ_{ij} is the induced metric on the hypersurfaces Σ_r of constant r.

- One then formulates the variational problem on a regulating surface Σ_{r_o}, $r_o < \infty$, and the limit $r_o \to \infty$ is taken in the end.
- However, since the regulator Σ_{r_o} breaks the diffemorphisms $r \to r + \delta r$, this procedure is ill defined! For example, for a scalar field ϕ the variation of the action takes the form

$$\delta S = \int_{\mathcal{M}_{r_o}} d^{d+1}x(eom) + \int_{\Sigma_{r_o}} d^d x \delta\phi\pi_\phi,$$

where π_ϕ is the scalar radial canonical momentum, the radial coordinate being the Hamiltonian 'time', and the boundary term has no well defined transformation under radial shifts.

- Suitable boundary terms must then be added to the action such that full diffeomorphism invariance -on the regulating surface- is restored (at least asymptotically as $r_o \to \infty$) and the limit $r_o \to \infty$ makes sense.

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Constructing the boundary terms

- A systematic method to construct the relevant boundary terms for a general manifold $\mathcal{M}_{d+1}$ is an interesting open problem! (e.g. asymptotically flat manifolds, linear dilaton, KS)
- For asymptotically locally AdS manifolds -in which case the radial translations correspond to Weyl transformations on the boundary- and for a two-derivative action, it has been shown that full diffeomorphism invariance completely determines the boundary terms [I. P., K. Skenderis '05], which turn out to be precisely the standard counterterms necessary to remove the divergences of the on-shell action!
- The procedure [I. P., K. Skenderis '04, '05] can be summarized in the following steps:
  - The leading asymptotic form of the scalar, $\phi \sim e^{-\Delta_- r/l}\phi_-(x)$, implies that the generator of radial translations to leading asymptotic order is nothing but the dilatation operator $\delta_D \equiv -\Delta_- \int \phi\delta/\delta\phi$:

$$\partial_r = \int_{\Sigma_r} d^d x \dot{\phi}\frac{\delta}{\delta\phi} \sim \frac{1}{l}\delta_D .$$

  - A formal expansion of the canonical momentum ($= \sqrt{\gamma}\dot{\phi}$ for a minimally coupled scalar) in eigenfunctions of the dilatation operator:

$$\pi_\phi = \sqrt{\gamma}(\pi_{(\Delta_-)} + \cdots + \pi_{(\Delta_+)} + \cdots),$$

    where $\delta_D \pi_{(n)} = -n\pi_{(n)}$, is inserted into the (radial Hamiltonian) equations of motion, which determine all terms $\pi_{(n)}$ for $n < \Delta_+$ - but not $\pi_{(\Delta_+)}$- as *local* functions of ϕ.

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Constructing the boundary terms (cont.)

- Defining then

$$\delta S_{\rm ct} \equiv -\sum_{n<\Delta_+} \int_{\Sigma_r} d^dx\sqrt{\gamma}\delta\phi\pi_{(n)},$$

  and $\widehat{\pi}_{(\Delta_+)} = \lim_{r\to\infty} e^{\Delta_+ r/l}\pi_{(\Delta_+)}$, one has

$$\boxed{\delta(S+S_{\rm ct}) = \int_{\partial\mathcal{M}} d^dx\delta\phi_-\widehat{\pi}_{(\Delta_+)}.}$$

- The RHS is now a *class function* since

$$\delta_D(\sqrt{\gamma}\delta\phi\pi_{(\Delta_+)}) = (d-\Delta_- - \Delta_+)\sqrt{\gamma}\delta\phi\pi_{(\Delta_+)} = 0.$$

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Boundary conditions

- A boundary condition is a choice of a function $J(\phi_-, \widehat{\pi}_{(\Delta_+)})$ of the two independent modes that are kept fixed on the boundary.
- To impose the boundary condition $J(\phi_-, \widehat{\pi}_{(\Delta_+)}) = J(x)$ on the boundary, we need to add a suitable (finite) boundary term, $S_J[\phi_-, \widehat{\pi}_{(\Delta_+)}]$ such that $\delta(S + S_{\text{ct}} + S_J) \propto \delta J(\phi_-, \widehat{\pi}_{(\Delta_+)})$.
- The three inequivalent boundary conditions are:

	$J(\phi_-, \widehat{\pi}_{(\Delta_+)})$	$S_J[\phi_-, \widehat{\pi}_{(\Delta_+)}]$
Dirichlet	$J_+ = \phi_-$	$S_+ = 0$
Neumann	$J_- = -\widehat{\pi}_{(\Delta_+)}$	$S_- = -\int_{\partial\mathcal{M}} d^dx \sqrt{g_{(0)}}\phi_-\widehat{\pi}_{(\Delta_+)}$
Mixed	$J_{f_-} = -\widehat{\pi}_{(\Delta_+)} - f'(\phi_-)$	$S_{f_-} = S_- + \int_{\partial\mathcal{M}} d^dx \sqrt{g_{(0)}}(f(\phi_-) - \phi_- f'(\phi_-))$

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The AdS/CFT dictionary

- The (renormalized) on-shell action, $I[J]$ is a functional of the source $J(x)$ and involves three pieces: the bulk action, S, the covariant boundary counterterms, S_{ct}, and the boundary term, S_J, defining the boundary condition. i.e. $I[J] = (S + S_{\text{ct}} + S_J)|_{\phi}$.

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σ	$\widehat{\pi}_{(\Delta_+)}$	ϕ_-	ϕ_-
$W[J]$	$I_+[J_+]$	$I_-[J_-]$	$I_{f_-}[J_{f_-}]$
$\Gamma[\sigma]$	$I_-[-\widehat{\pi}_{(\Delta_+)}]$	$I_+[\phi_-]$	$I_+[\phi_-] + \int_{\partial\mathcal{M}} d^dx\sqrt{g_{(0)}}f(\phi_-)$
$\langle T_i^i \rangle$	$-(d-\Delta_+)J\sigma$	$-(d-\Delta_-)J\sigma$	$-(d-\Delta_-)J\sigma - d\left(f(\sigma) - \frac{\Delta_-}{d}\sigma f'(\sigma)\right)$

- Conformal mixed boundary conditions: $f(\phi_-) \propto \phi_-^{d/\Delta_-}$

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- The (renormalized) on-shell action, $I[J]$ is a functional of the source $J(x)$ and involves three pieces: the bulk action, S, the covariant boundary counterterms, S_{ct}, and the boundary term, S_J, defining the boundary condition. i.e. $I[J] = (S + S_{\text{ct}} + S_J)|_{\phi}$.

	Dirichlet	Neumann	Mixed
J	$J_+ \equiv \phi_-$	$J_- \equiv -\widehat{\pi}_{(\Delta_+)}$	$J_{f_-} \equiv -\widehat{\pi}_{(\Delta_+)} - f'(\phi_-)$
σ	$\widehat{\pi}_{(\Delta_+)}$	ϕ_-	ϕ_-
$W[J]$	$I_+[J_+]$	$I_-[J_-]$	$I_{f_-}[J_{f_-}]$
$\Gamma[\sigma]$	$I_-[-\widehat{\pi}_{(\Delta_+)}]$	$I_+[\phi_-]$	$I_+[\phi_-] + \int_{\partial\mathcal{M}} d^dx\sqrt{g_{(0)}}f(\phi_-)$
$\langle T^i_i \rangle$	$-(d-\Delta_+)J\sigma$	$-(d-\Delta_-)J\sigma$	$-(d-\Delta_-)J\sigma - d\left(f(\sigma) - \frac{\Delta_-}{d}\sigma f'(\sigma)\right)$

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- Large N factorization implies that under the deformation

$$S_f[\phi] = S[\phi] + N^2 \int d^d x f(\mathcal{O})$$

of the CFT action by a function $f(\mathcal{O})$ of the local, single-trace and gauge-invariant operator $\mathcal{O}(x)$, the generating functional $w[J] \equiv N^{-2}W[J]$ and the effective action $\bar{\Gamma}[\sigma] = N^{-2}\Gamma[\sigma]$ transform as:

	Undeformed	Deformed
Source	$J = -\frac{\delta\bar{\Gamma}[\sigma]}{\delta\sigma}$	$J_f = J - f'(\sigma)$
VEV	$\sigma \equiv \langle\mathcal{O}\rangle_J = \frac{\delta w[J]}{\delta J}$	$\sigma_f = \sigma$
Gen. functional	$w[J]$	$w_f[J_f] = w[J] + \int d^d x \, \left(f(\sigma) - \sigma f'(\sigma)\right)\Big\vert_{\sigma=\delta w/\delta J}$
Eff. action	$\bar{\Gamma}[\sigma]$	$\bar{\Gamma}_f[\sigma] = \bar{\Gamma}[\sigma] + \int d^d x f(\sigma)$

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Stability (perturbative)

- The mass of scalar fluctuations around the AdS vacuum must satisfy the BF bound $-(d/2)^2 \leq m^2 l^2$ [Breitenlohner-Freedman '82].

- Unitarity (well-defined symplectic form) implies that Neumann or Mixed boundary conditions can be imposed only if the mass satisfies $-(d/2)^2 \leq m^2 l^2 \leq -(d/2)^2 + 1$ [Breitenlohner-Freedman '82, Balasubramanian-Kraus-Lawrence '98, Klebanov-Witten '99].

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Stability (non-perturbative)

- When only Dirichlet boundary conditions are allowed, non-perturbative stability follows from the old positivity theorems [Gibbons-Hull-Warner '83 (true supergravity), Boucher '84 (D=4, no supersymmetry), Townsend '84 (general D, no supersymmetry)].
- When Mixed boundary conditions are perturbatively allowed, non-perturbative stability for the unique supersymmetric boundary conditions (generically a particular conformal Mixed boundary condition) still follows from the old stability theorems.
- When Mixed boundary conditions are perturbatively allowed, the recent analysis in the context of designer gravity has shown that non-perturbative stability for a generic Mixed boundary condition defined by the function $f(\phi_-)$ requires (for minimally coupled scalars):
  1. The bulk scalar potential $V(\phi)$ should be globally expressible in terms of an 'auxiliary' function ('superpotential') $W_-(\phi)$ as

     $$V(\phi) = \frac{1}{2}\left(W_-'^2 - \frac{d\kappa^2}{d-1}W_-^2\right).$$

  2. $f(\phi_-)$ must be bounded from below.

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Dual field theory perspective

- In theories with a known field theory dual, the question of stability/instability with generalized boundary conditions can be addressed by studying the quantum effective action for the dual deforming operator.
- This effective action admits a derivative expansion away from the conformal vacuum of zero VEV. For conformal boundary conditions this can be seen as follows:
  - For $|p| << \sigma \equiv \langle \mathcal{O}_{\Delta_-} \rangle$, the two-point function of the dual operator $\mathcal{O}_{\Delta_-}$ is dominated by a massless Goldstone pole due to the spontaneously broken scale invariance:

$$\langle \mathcal{O}_{\Delta_-}(p)\mathcal{O}_{\Delta_-}(-p)\rangle \sim \frac{1}{p^2}.$$

  - For $|p| >> \sigma$, conformal invariance is restored and hence

$$\langle \mathcal{O}_{\Delta_-}(p)\mathcal{O}_{\Delta_-}(-p)\rangle \sim |p|^{2\Delta_- - d}.$$

  - This small momentum behavior of the two-point function implies, via the Legendre transform of the generating functional, that the effective action admits a derivative expansion provided $\sigma \neq 0$. Near $\sigma = 0$, however, this derivative expansion breaks down.
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The holographic effective action

- The holographic effective action is just the on-shell action with the appropriate boundary terms.
- To determine the regularized on-shell action I_r one solves the Hamilton-Jacobi equation that is obtained by inserting the canonical momenta

$$\pi^{ij} = \frac{\delta I_r}{\delta \gamma_{ij}}, \quad \pi_\phi = \frac{\delta I_r}{\delta \phi},$$

into the Hamiltonian and momentum constraints

$$\mathcal{H} = 0, \qquad 2D_i \pi^i_j = \pi_\phi \partial_j \phi.$$

- Since the effective action admits a derivative expansion away from the conformal point, we can insert the ansatz for the 2-derivative regularized effective action:

$$I_r = \int_{\Sigma_r} d^d x \sqrt{\gamma} \left(W(\phi) + Z(\phi) R[\gamma] + \frac{1}{2} M(\phi) \gamma^{ij} \partial_i \phi \partial_j \phi \right),$$

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The holographic effective action (cont.)

The renormalized effective action then takes the form

$$\Gamma_{f_-}[\phi_-] = \int d^d x \sqrt{g_{(0)}} \left\{ c_- \phi_-^{\frac{d-2}{\Delta_-}} \left(R[g_{(0)}] + \frac{(d-1)(d-2)}{\Delta_-^2} \phi_-^{-2} \partial^i \phi_- \partial_j \phi_- \right) + V_{\text{eff}}(\phi_-) \right\},$$

where the effective potential is $V_{\text{eff}}(\phi_-) = \xi \phi_-^{d/\Delta_-} + f(\phi_-)$ and c_-, ξ are constants.

- To determine c_- and ξ one needs to compute exactly the effective potential on S^d and $\mathbb{R}^d$ respectively.
- This can be done by solving the Hamilton-Jacobi equation exactly in the 'minisuperspace approximation' where the scalar is independent of the transverse coordinates x^i and the induced metric on Σ_r takes the form $\gamma_{ij} = \exp(2A(r))g_{(0)ij}(x)$, where $g_{(0)ij}(x)$ is the metric either on S^d or on $\mathbb{R}^d$.
- This 'minisuperspace approximation' is equivalent to looking for S^d- or $\mathbb{R}^d$-sliced domain walls. Requiring that these domain walls are regular in the interior generically determines the parameters c_- and ξ.
- Finding these exact domain wall solutions - which are required to determine the effective action - is in general a rather difficult task. It is only for very special bulk scalar potentials that one is able to solve the relevant equations exactly.

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How does this reproduce the stability conditions?

- Existence of a real solution $W(\phi)$ implies the BF bound.
- Global existence of such a solution coincides (in the case of minimally coupled scalars) with the condition of [Amsel-Hertog-Hollands-Marolf '07] that the potential be expressible in terms of the auxiliary function $W_-(\phi)$.
- The condition that the derivative expansion of the effective action breaks down for small VEV, where conformal invariance is restored, implies the unitarity bound $\Delta_- \geq (d-2)/2$ or $m^2l^2 \leq -(d/2)^2 + 1$, which is precisely the condition for mixed boundary conditions to be admissible.
- For mixed boundary conditions, where the dual operator has dimension Δ_-, the freedom of adding further local finite counterterms corresponds to the freedom of defining what one calls the 'undeformed' theory. Since ξ is determined dynamically, one can always define the undeformed theory such that the term $\xi\phi_-^{d/\Delta_-}$ is removed from the effective potential. It is then clear that the boundedness from below of the effective potential is equivalent to the boundedness from below of $f(\phi_-)$, which is precisely the condition for stability discussed by [Hertog-Hollands '05].

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The conformal scalar

- A system for which all domain walls with constant curvature slicings can be found exactly -and hence the exact effective potential be computed- is a scalar conformally coupled to gravity and with a conformally invariant self interaction:

$$S = \int d^{d+1}x\sqrt{g}\left(-\frac{1}{2\kappa^2}R - \frac{d(d-1)}{2\kappa^2 l^2} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{d-1}{8d}R\phi^2 + \frac{\lambda}{2}\phi^{\frac{2(d+1)}{(d-1)}}\right),$$

  where λ is an arbitrary dimensionless coupling.

- There is a field redefinition [Henneaux-Martinez-Troncoso-Zanelli '02, Martinez-Troncoso-Zanelli '04, I.P. '07] which brings this action into that of a minimally coupled scalar with a rather complicated scalar potential.
- For $d = 3$ and $\lambda = \kappa^2/6l^2$ this potential takes the simple form

$$V(\tilde{\phi}) = -\frac{3}{\kappa^2 l^2}\cosh\left(\sqrt{2/3}\kappa\tilde{\phi}\right),$$

  which arises from a consistent truncation of $D = 4$ $\mathcal{N} = 8$ gauged supergravity [Duff-Liu '99, I.P. '06] and hence it can be uplifted to 11-dimensional supergravity [I.P. '06].

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A non-perturbative instability

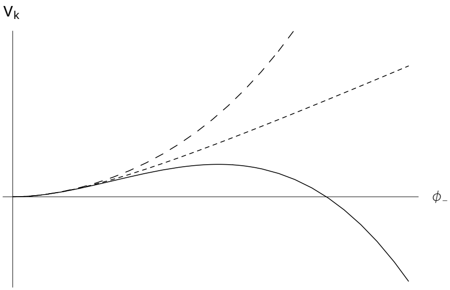
- From the exact effective potential we read off the parameters

$$c_- = \frac{(d-1)^2}{16d(d-2)\sqrt{\lambda}}, \quad \xi = \frac{(d-1)}{2d}\sqrt{\lambda}.$$

- For conformal boundary conditions defined by

$$f(\phi_-) = -\alpha \frac{(d-1)}{2d} \phi_-^{\frac{2d}{(d-1)}},$$

where α is a dimensionless parameter, the effective potential on S^d, depending on whether $\alpha < \sqrt{\lambda}$ (long dashes), $\alpha = \sqrt{\lambda}$ (short dashes), or $\alpha > \sqrt{\lambda}$, looks like:



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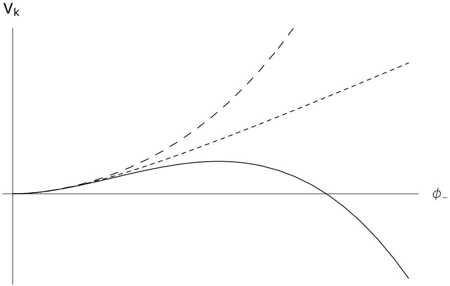

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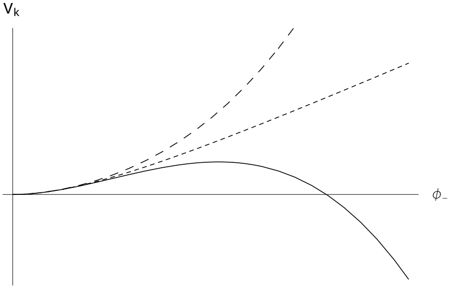

Instantons

- This non-perturbative instability is manifested by the existence of instanton solutions satisfying these conformal boundary conditions [S. de Haro, I. P., A. C. Petkou '06, I. P. '07].
- These instantons are the most general solutions of the equations of motion such that the metric is exact AdS:

$$ds^2 = \frac{l^2}{z^2}(dz^2 + d\vec{z}^2).$$

- The instanton solutions take the form

$$\phi^{2/(d-1)} = \frac{(d-1)}{l\sqrt{|\lambda|}}\left(\frac{bz}{-\mathrm{sgn}(\lambda)b^2 + (z+a)^2 + (\vec{z}-\vec{z}_0)^2}\right),$$

where $a, b, \vec{z}_o$ are constants of integration.
- Smoothness of the solution requires $a > b > 0$, or $\alpha = \sqrt{|\lambda|}a/b > \sqrt{|\lambda|}$, which is precisely the condition that the effective potential is unbounded from below!
- The vacuum decay rate can be computed by evaluating the on-shell action.

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Summary

- The stability properties of ‘designer gravities’ can be studied systematically and non-perturbatively by holographic methods.
- We applied these methods to a single scalar field conformally coupled to gravity and with a conformal self-interaction and demonstrated the existence of a non-perturbative instability that occurs along a line conformal boundary conditions.
- The onset of this instability coincides with the appearance of instanton solutions which mediate the decay of the unstable vacuum.
- We have shown that in four dimensions the conformal scalar can be embedded in $\mathcal{N} = 8$ gauged supergravity and hence embedded into 11-dimensional supergravity. In this case the instability we have found occurs along a line of marginal multi-trace deformations of the dual $\mathcal{N} = 8$ SCFT in three dimensions.

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