# Strings on RR Backgrounds

Yaron Oz (Tel-Aviv University)

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# Outline



With Ido Adam, Amit Dekel and Luca Mazzucato.

# RR Fields

- The massless spectrum of superstrings contains p-form fields $F_p$ called Ramond-Ramond (RR) fields.
- For instance, type IIA superstrings have one-form RR field potential $A_\mu$ (abelian gauge field) and a two-form RR field strength $F_{\mu\nu}$.
- type IIA superstrings have RR even-forms $F_{2n}$ and type IIB superstrings have RR odd-forms $F_{2n-1}$.
- Dp-branes are sources of RR $F_{p+2}$ fluxes. One has the coupling

$$\int_{Dp} A_{p+1} \tag{1}$$

# RR Fluxes

- In curved backgrounds one can have fluxes through p-cycles $C_p$

$$\int_{C_p} F_p \tag{2}$$

- For instance: in the AdS/CFT correspondence between superstrings on asymptotically $AdS_5 \times S^5$ spaces and $N = 4$ $SU(N_c)$ SYM,

$$\int_{S^5} F_5 = N_c \tag{3}$$

$N_c$ is the number of colors of the dual gauge theory.

# RR Fluxes - Compactifications

- RR fluxes can be used to lower the amount of supersymmetry of the compactification or break it completely.
- RR fluxes can be used to generate potentials for moduli and provide various cosmological scenarios.
- For instance: A compactification on a $d$-dimensional manifold of typical size $L$ with RR flux

$$\int_{p-cycle} F_p = N_p \tag{4}$$

yields a potential

$$V(L) \sim \frac{N_p^2}{L^{d+2p}} \tag{5}$$

# RR Fluxes - Holography

- Superstring theory on curved backgrounds with Ramond-Ramond flux has not been quantized yet.
- This is the reason why most of the AdS/CFT results have been obtained in the supergravity (low-energy) approximation of superstring theory $E \ll M_s = l_s^{-1}$.
- For conformal field theory, such as $N = 4$ SYM, the dual supergravity description is a good one at large t'Hooft coupling $\lambda_{'t\ Hooft} = g_{YM}^2 N_c \gg 1$.

# RR Fluxes - Holography

- This is generally not the case for confining theories such as QCD.
- In the supergravity approximation

$$m_{J\leq 2}^{glueballs} \ll m_{J>2}^{glueballs} \tag{6}$$

- In QCD one expects

$$m_J^{glueballs} \sim \Lambda_{QCD} \tag{7}$$

# Non-critical backgrounds

- In dimensions $d < 10$, the Liouville mode is dynamical and needs to be quantized as well.
- The total conformal anomaly vanishes for the non-critical superstrings due to the Liouville background charge.
- Alternative to compactification.
- Dual description of gauge theories (QCD):

$$ds^2 = d\varphi^2 + a^2(\varphi) d\vec{x}^2 \tag{8}$$

# Non-critical backgrounds

- A complication:

$$S \sim \int d^d x \sqrt{G} e^{-2\Phi} \left( \frac{d-10}{l_s^2} \right)$$

implies that the low energy approximation $E \ll l_s^{-1}$ is not valid when $d \neq 10$, and the higher order curvature terms of the form $\left(l_s^2 \mathcal{R}\right)^n$ cannot be discarded.

# Non-critical backgrounds

- Solutions of the $d$-dimensional non-critical supergravity equations have typically curvatures of the order of the string scale $l_s^2\mathcal{R} \sim O(1)$ when $d \neq 10$.
- $AdS_d$ backgrounds:

$$l_s^2\mathcal{R} = d - 10, \quad e^{2\phi} = \frac{1}{N_c^2}, \quad l_s^2 F_d^2 = 2(10 - d)d!N_c^2 \quad (9)$$

# Superstrings on RR Backgrounds

- The RNS formalism:
  $X : 2d \;\; susy \;\; worldsheet \rightarrow space - time$, is inadequate for the quantization of superstrings on RR backgrounds due to the complicated (non-polynomial) worldsheet coupling to the RR fields.
- The Green-Schwarz sigma-model on such backgrounds
  $X : 2d \;\; bosonic \;\; worldsheet \rightarrow susy \;\; space - time$, is an interacting two-dimensional conformal field theory. The Green-Schwarz formalism works (so far) only in the light-cone gauge.
- The pure spinor formalism ($\lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta = 0$) allows quantization of background with RR fluxes without the need for gauge fixing.

# Integrability of RR Backgrounds

- In the case of the type IIB superstring on $AdS_5 \times S^5$ it has been shown that the sigma-model has infinite number of conserved charges and is classically integrable.
- This corresponds to the integrability of $N = 4$ susy gauge theory as $N_c \to \infty$.
- One can ask under what conditions superstrings on general curved backgrounds, and $AdS$ in particular, with RR flux are integrable.
- Using hologrpahy, this can be formulated as the question: under what conditions large $N_c$ field theories in general and conformal field theories in particular, are integrable.

# Integrability of RR Backgrounds

- We will address this question by looking at superstring theories on such backgrounds, both in the Green–Schwarz and the pure spinor formalisms.
- All our models are realized as nonlinear sigma-models on supercosets $G/H$, where the supergroup $G$ has a $\mathbb{Z}_4$ automorphism, whose invariant locus is $H$.
- For instance: $AdS_5 \times S^5 : PSU(2,2|4)/(SO(1,4) \times SO(5))$.
- In general in this discussion we will consider bosonic manifolds of the form $AdS_p \times S^q$.

# Integrability

- The first step in the construction of the charges is to find a one-parameter family of currents $a(\mu)$ satisfying the flatness condition

$$da(\mu) + a(\mu) \wedge a(\mu) = 0 \tag{10}$$

- One then constructs the Wilson line

$$U_{(\mu)}(x,t;y,t) = \mathrm{P}\exp\left(-\int_{(y,t)}^{(x,t)} a(\mu)\right) \tag{11}$$

- One obtains the infinite set of non-local charges $Q_n$ by expanding

$$U_{(\mu)}(\infty,t;-\infty,t) = 1 + \sum_{n=1}^{\infty} \mu^n Q_n \tag{12}$$

The conservation of $Q_n$ is implied by the flatness of $a(\mu)$

# Integrability

- The first two charges $Q_1$ and $Q_2$ generate the Yangian algebra, which we find to be a symmetry algebra underlying the type II superstrings propagating on the AdS backgrounds with Ramond-Ramond fluxes in various dimensions.

$$[Q_1^a, Q_1^b] = f_c^{ab} Q_1^c, \qquad [Q_1^a, Q_2^b] = f_c^{ab} Q_2^c \tag{13}$$

- In the case of type IIB superstrings propagating on $AdS_5 \times S^5$, a similar Yangian algebra has been identified in the free field theory limit of $\mathcal{N} = 4$ SYM at large $N_c$. We expect that a similar structure underlies the field theory duals in various dimensions.

# Integrability

- The Yangian algebra suggests the existence of an affine Kac-Moody algebra. This is to be contrasted with NS-NS backgrounds, where the affine algebra comes in two copies, one left- and one right-moving, while in the case of RR backgrounds there would be only a single copy of such an algebra.
- The question arises whether this symmetry is sufficient for solving for the spectrum of the superstring.
- For $N = 4$ SYM, there is by now much evidence that this is indeed the case.

# Algebraic Structure

- We will be interested in sigma-models whose target space is the coset $G/H$, where $G$ is a supergroup with a $\mathbb{Z}_4$ automorphism and the subgroup $H$ is the invariant locus of this automorphism.
- The super Lie algebra $\mathcal{G}$ of $G$ can be decomposed into the $\mathbb{Z}_4$ automorphism invariant spaces $\mathcal{G} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3$
- The decomposition satisfies the algebra ($i = 1, \ldots, 3$)

$$[\mathcal{H}_0, \mathcal{H}_0] \subset \mathcal{H}_0 \ , \quad [\mathcal{H}_0, \mathcal{H}_i] \subset \mathcal{H}_i \ , \quad [\mathcal{H}_i, \mathcal{H}_j] \subset \mathcal{H}_{i+j \mod 4} \tag{14}$$

# Green-Schwarz Superstrings

- The worldsheet fields are the maps $g : \Sigma \rightarrow G$ and dividing by the subgroup $H$ is done by gauging the subgroup $H$.
- The left-invariant current is defined as

$$J = g^{-1} dg \tag{15}$$

- This current can be decomposed according to the $\mathbb{Z}_4$ grading of the algebra $J = J_0 + J_1 + J_2 + J_3$
- Using the above properties of the algebra $\mathcal{G}$ and the requirement of gauge invariance leads to the GS action

$$S_{\rm GS} = \frac{1}{4} \int \langle J_2 \wedge *J_2 + J_1 \wedge J_3 \rangle \tag{16}$$

# Integrability of Green-Schwarz Superstrings

- We are looking for a one parameter family of flat connections $D = d + a(\mu)$, satisfying the zero curvature condition $D^2 = 0$.
- In other words

$$da(\mu) + a(\mu) \wedge a(\mu) = 0 \tag{17}$$

where the gauge invariant current $a(\mu)$ is usually referred to as the Lax connection and $\mu$ as the spectral parameter.
- Define: $A = g^{-1}ag$ and expand

$$A = \alpha J_2 + \beta {*J_2} + \gamma J_1 + \delta J_3 \ . \tag{18}$$

- It satisfies the equation

$$dA + A \wedge A + J \wedge A + A \wedge J = 0 \tag{19}$$

which allows us to solve for $(\alpha, \beta, \gamma, \delta)$.

# Integrability of Green-Schwarz Superstrings

- An infinite set of conserved charges can be obtained using the expansion of the solution about $\mu = 0$

$$a = \mu(j_1 - j_3 - 2{*}j_2) + \mu^2\left(2j_2 + \frac{1}{2}j_1 + \frac{1}{2}j_3\right) + O(\mu^3) \quad (20)$$

where the $j_i = gJ_ig^{-1}$.

- The monodromy matrix is the Wilson line of the flat connection

$$U_C = \mathrm{P}\exp\left(-\int_C a\right) = 1 + \sum_{n=1}^{\infty}\mu^n Q_n \quad (21)$$

- Its expansion around $\mu = 0$ leads to the conserved charges $Q_n$.

# Integrability of Green-Schwarz Superstrings

- The first two conserved charges are

$$
\begin{aligned}
Q_1 &= -\int_C (j_1 - j_3 - 2*j_2) \qquad (22)\\
Q_2 &= -\int_C \left(2j_2 + \frac{1}{2}j_1 + \frac{1}{2}j_3\right) + \\
&\quad + \int_C [j_1(x) - j_3(x) - 2*j_2(x)] \int_o^x (j_1 - j_3 - 2*j_2)
\end{aligned}
$$

- The former is local, while the latter is non-local. The other charges can be generated by repetitive Poisson brackets of $Q_2$ and together they form a classical Yangian.
- In the Green-Schwarz formalism the currents are $\kappa$-invariant.

# Example: $AdS_2$

- $AdS_2 \sim \frac{OSp(1|2)}{SO(2)}$ or $AdS_2 \sim \frac{OSp(2|2)}{SO(2)\times SO(2)}$.
- The supergroup $OSp(2|2)$ is a subgroup of $GL(2|2)$ such that for each $M \in OSp(2|2)$ $M^{st}HM = H$, where

$$\mathbf{H} = \left(\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

- It implies that its algebra satisfies $m^{st}H + Hm = 0$.

# Example: $AdS_2$

- The general solution of this equation is :

$$\mathbf{m} = \left(\begin{array}{cc|cc} \multicolumn{2}{c|}{sl(2)} & a & b \\ & & c & d \\ \hline -c & a & & \\ -d & b & \multicolumn{2}{c}{so(2)} \end{array}\right)$$

# Example: $AdS_2$

- The bosonic generators:

$$H = \frac{1}{2}\left(\begin{array}{cc|c} 1 & 0 & \\ 0 & -1 & \\ \hline & & \\ & & \end{array}\right) \qquad E^+ = \left(\begin{array}{cc|c} 0 & 1 & \\ 0 & 0 & \\ \hline & & \\ & & \end{array}\right)$$

$$E^- = \left(\begin{array}{cc|c} 0 & 0 & \\ 1 & 0 & \\ \hline & & \\ & & \end{array}\right) \qquad \widetilde{H} = \frac{1}{2}\left(\begin{array}{c|cc} & & \\ & & \\ \hline & 0 & 1 \\ & -1 & 0 \end{array}\right)$$

# Example: $AdS_2$

- The fermionic generators:

$$F^+ = \frac{1}{2}\left(\begin{array}{cc|cc} & & 0 & 1 \\ & & 0 & 0 \\ \hline 0 & 0 & & \\ 0 & 1 & & \end{array}\right) \qquad F^- = \frac{1}{2}\left(\begin{array}{cc|cc} & & 0 & 0 \\ & & 0 & -1 \\ \hline 0 & 0 & & \\ 1 & 0 & & \end{array}\right)$$

$$\widetilde{F}^+ = \frac{1}{2}\left(\begin{array}{cc|cc} & & 1 & 0 \\ & & 0 & 0 \\ \hline 0 & 1 & & \\ 0 & 0 & & \end{array}\right) \qquad \widetilde{F}^- = \frac{1}{2}\left(\begin{array}{cc|cc} & & 0 & 0 \\ & & -1 & 0 \\ \hline 1 & 0 & & \\ 0 & 0 & & \end{array}\right)$$

# $AdS_2$: Example

- The left invariant one form is given by

$$J = G^{-1} dG = J_0 + J_1 + J_2 + J_3 \tag{23}$$

$$J_0 = H + \tilde{H}, \quad J_1 = F^+ + \tilde{F}^+, \quad J_2 = E^+ + E^-, \quad J_3 = F^- + \tilde{F}^- \tag{24}$$

# Pure Spinor Variables

- These are bosonic spinor ghosts $\lambda^\alpha$, which satisfy the (complex) pure spinor constraint (Berkovits)

$$\lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta = 0 \qquad (25)$$

and their conjugate momenta $w_\alpha$.

# Pure Spinor Sigma Model

- The worldsheet action in the pure spinor formulation of the superstring consists of a matter and a ghost sector.
- The matter fields are written in terms of the left-invariant currents $J = g^{-1}\partial g$, $\bar{J} = g^{-1}\bar{\partial} g$, where $g : \Sigma \to G$.
- They are decomposed according to the invariant spaces of the $\mathbb{Z}_4$ automorphism:

$$J = J_0 + J_1 + J_2 + J_3 \tag{26}$$

and similarly for the anti-holomorphic component $\bar{J}$,

# Pure Spinor Sigma Model

- Pure spinor currents are defined by

$$N = -\{w, \lambda\} \, , \quad \bar{N} = -\{\bar{w}, \bar{\lambda}\} \tag{27}$$

- The pure spinor sigma-model action reads $S = S_1 + S_2$:

$$S_1 = \int d^2z \left\langle \frac{1}{2} J_2 \bar{J}_2 + \frac{1}{4} J_1 \bar{J}_3 + \frac{3}{4} J_3 \bar{J}_1 \right\rangle \tag{28}$$

$$S_2 = \int d^2z \left\langle w \bar{\partial} \lambda + \bar{w} \partial \bar{\lambda} + N \bar{J}_0 + \bar{N} J_0 - N \bar{N} \right\rangle \tag{29}$$

- It is the same for all dimensions and it matches the critical $AdS_5 \times S^5$.

# Integrability

- There is an infinite number of conserved charges.
- In the pure spinor formalism the currents are BRST invariant.
- In the pure spinor formalism we can prove that there is an infinite number of conserved charges also quantum mechanically.

# Open Questions and Outlook

- Use integrability to solve sigma-models on RR backgrounds: critical and non-critical.
- Generalize the discussion to confining backgrounds.

# Pure Spinor Variables

- Flat ten-dimensional open superstrings: Consider the supermanifold $(x^m, \theta^\alpha)$, where $x^m, m = 0, ..., 9$ are commuting coordinates and $\theta^\alpha, \alpha = 1, ..., 16$ are anti-commuting coordinates.
- One introduces $p_\alpha$ as the conjugate momenta to $\theta^\alpha$ with the OPE

$$p_\alpha(z)\theta^\beta(0) \sim \frac{\delta_\alpha^\beta}{z} \quad (30)$$

- Next we add bosonic spinor ghosts $\lambda^\alpha$, which satisfy the (complex) pure spinor constraint (Berkovits)

$$\lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta = 0 \quad (31)$$

and their conjugate momenta $w_\alpha$.

# Pure spinor variables

- The system $(w_\alpha, \lambda^\alpha)$ is a $(\beta, \gamma)$ system of weights $(1, 0)$.
- The pure spinor set of constraints (31) is reducible. It defines a complex eleven-dimensional space $\mathcal{M}$, which is a cone over $\mathcal{Q} = \frac{SO(10)}{U(5)}$.
- $w_\alpha$ are defined up to the gauge transformation

$$\delta w_\alpha = \Lambda^m (\gamma_m \lambda)_\alpha \qquad (32)$$

It appears only in gauge invariant combinations:

$$M_{mn} = \frac{1}{2} w \gamma_{mn} \lambda, \qquad J_{(w,\lambda)} = w_\alpha \lambda^\alpha \qquad (33)$$

and the pure spinor stress-energy tensor $T_{(w,\lambda)}$.

# Pure spinor action

- The gauge fixed worldsheet action is $S = S_0 + S_1$, where

$$S_0 = \int d^2z \left( \frac{1}{2} \partial x^m \bar{\partial} x_m + p_\alpha \bar{\partial} \theta^\alpha - w_\alpha \bar{\partial} \lambda^\alpha \right) \tag{34}$$

and

$$S_1 = \int d^2z \left( \frac{1}{4} R^{(2)} \log \Omega(\lambda) \right) \tag{35}$$

$S_1$ is a coupling of the worldsheet curvature $R^{(2)}$ to the holomorphic top form $\Omega$ of the pure spinor space $\mathcal{M}$

$$\Omega = \Omega(\lambda) d\lambda^1 \wedge \ldots \wedge d\lambda^{11} \tag{36}$$

# The stress tensor

- The stress tensor of the $(w, \lambda)$ system reads

$$T_{(w,\lambda)} = w_\alpha \partial \lambda^\alpha - \frac{1}{2} \partial^2 \log \Omega(\lambda) \qquad (37)$$

- The system $(w_\alpha, \lambda^\alpha)$ is interacting due to the pure spinor constraint. It has the central charge $c_{(w,\lambda)} = 22$, which is twice the complex dimension of the pure spinor space

$$T_{(w,\lambda)}(z) T_{(w,\lambda)}(0) \sim \frac{dim_{\mathbb{C}}(\mathcal{M})}{z^4} + \dots \qquad (38)$$

# The total central charge

$$c^{total} = c(x^m) + c(p_\alpha, \theta^\alpha) + c(w_\alpha, \lambda^\alpha) = 10 - 32 + 22 = 0$$

# The BRST operator

- The physical states are defined as the ghost number one cohomology of the nilpotent BRST operator

$$Q = \oint dz\, \lambda^\alpha d_\alpha \ , \tag{39}$$

where

$$d_\alpha = p_\alpha - \frac{1}{2}\gamma^m_{\alpha\beta}\theta^\beta \partial x_m - \frac{1}{8}\gamma^m_{\alpha\beta}\gamma_{m\gamma\delta}\theta^\beta\theta^\gamma\partial\theta^\delta \tag{40}$$

- This BRST operator is an essential ingredient of the pure spinor formalism but it is not precisely clear how to derive its form by a gauge fixing procedure.

# The Green-Schwarz constraints

- The $d_\alpha$ are the supersymmetric Green-Schwarz constraints

$$d_\alpha(z) d_\beta(0) \sim -\frac{\gamma^m_{\alpha\beta}\Pi_m(0)}{z} \tag{41}$$

$$d_\alpha(z)\Pi^m(0) \sim \frac{\gamma^m_{\alpha\beta}\partial\theta^\beta(0)}{z} \tag{42}$$

$$\Pi_m = \partial x_m + \frac{1}{2}\theta\gamma_m\partial\theta \tag{43}$$

- $d_\alpha$ acts on function on superspace $F(x^m, \theta^\alpha)$ as the ten-dimensional supersymmetric derivative

$$d_\alpha(z) F(x^m(0), \theta^\beta(0)) \sim \frac{D_\alpha F(x^m(0), \theta^\beta(0))}{z} \,, \tag{44}$$

# Massless states

- Massless states are described by the ghost number one weight zero vertex operators

$$\mathcal{V} = \lambda^\alpha A_\alpha(x, \theta) \tag{45}$$

  $A_\alpha(x, \theta)$ is an unconstrained spinor superfield.
- The BRST conditions:

$$Q\mathcal{V} = 0, \quad \mathcal{V} \simeq \mathcal{V} + Q\Omega \tag{46}$$

  imply that

$$\gamma^{\alpha\beta}_{mnpqr} D_\alpha A_\beta = 0, \quad \delta A_\alpha = D_\alpha \Omega \tag{47}$$

# Massless states

- $A_\alpha$ is the on-shell super Maxwell spinor superfield in ten dimensions

$$A_\alpha(x,\theta) = \frac{1}{2}(\gamma\theta)_\alpha a_m(x) + \frac{1}{3}(\gamma\theta)_\alpha(\gamma\theta)_\beta\psi^\gamma(x) + O(\theta^3) \quad (48)$$

where $a_m(x)$ is the gauge field and $\psi^\gamma(x)$ is the gluino.

- It is related to the gauge field $A_m$ by

$$A_m = \gamma_m^{\alpha\beta} D_\alpha A_\beta, \quad A_m(x,\theta) = a_m(x) + O(\theta) \quad (49)$$

- Only in ten dimensions do these conditions give an on-shell vector multiplet. In lower dimensions they describe an off-shell vector multiplet.

# Massive states

- The analysis of the massive states proceeds in a similar way. At the first massive level the ghost number one vertex operator has the expansion

$$\mathcal{U} = \partial\lambda^\alpha A_\alpha + \lambda^\alpha \partial\theta^\beta B_{\alpha\beta} + ... \tag{50}$$

It describes a massive spin two multiplet with 128 bosons and 128 fermions.

# Closed pure spinor superstrings

- One introduces the right moving superspace variables $(\bar{p}_{\hat{\alpha}}, \bar{\theta}^{\hat{\alpha}})$, the pure spinor system $(\bar{w}_{\hat{\alpha}}, \bar{\lambda}^{\hat{\alpha}})$.
- We add the nilpotent BRST operator

$$\bar{Q} = \oint d\bar{z}\, \bar{\lambda}^{\hat{\alpha}} \bar{d}_{\hat{\alpha}} \qquad (51)$$

The analysis of the spectrum proceeds by combining the left and right sectors.

# Closed pure spinor superstrings

- Massless states: the ghost number one vertex operator

$$\mathcal{V} = \lambda^\alpha \bar{\lambda}^\beta A_{\alpha\beta}(x, \theta, \bar{\theta}) \tag{52}$$

where

$$A_{\alpha\beta}(x,\theta,\bar{\theta}) = h_{mn}(\gamma^m\theta)_\alpha(\gamma^n\bar{\theta})_\beta + \psi_n^\gamma(\gamma^m\theta)_\alpha(\gamma_m\theta)_\gamma(\gamma^n\bar{\theta})_\beta + \ldots$$
$$+F^{\gamma\delta}(\gamma^m\theta)_\alpha(\gamma_m\theta)_\gamma(\gamma^n\bar{\theta})_\beta(\gamma_n\bar{\theta})_\delta + \ldots$$

- $A_{\alpha\beta}(x,\theta,\bar{\theta})$ describes the on-shell supergravity multiplet in ten dimensions.
- $F^{\gamma\delta} = \oplus F^{k_1,\ldots,k_n}\left(\gamma_{k_1,\ldots,k_n}\right)^{\gamma\delta}$ are the RR fields.

# Anomalies

- There are global obstructions to define the pure spinor system $(\lambda^\alpha, w_\alpha)$ on the worldsheet and on target space associated with the need for holomorphic transition functions relating $(\lambda^\alpha, w_\alpha)$ on different patches of the pure spinor space. They are reflected by quantum anomalies in the worldsheet and target space (pure spinor space) diffeomorphisms (Witten).
- The conditions for the vanishing of these anomalies are the vanishing of the integral characteristic classes

$$\frac{1}{2}c_1(\Sigma)c_1(\mathcal{M}) = 0, \qquad \frac{1}{2}p_1(\mathcal{M}) = 0 \tag{53}$$

# The singularity

- The pure spinor space has a singularity at $\lambda^\alpha = 0$. Blowing up the singularity results in an anomalous theory. However, simply removing the origin leaves a non-anomalous theory (Nekrasov).
- This means that one should consider the pure spinor variables as twistor-like variables.

# Mapping RNS variables to pure spinor variables

- The RNS variables are:

$$x^m, \psi^m, b, c, \beta, \gamma \tag{54}$$

  The pure spinor variables are:

$$x^m, p_\alpha, \theta^\alpha, w_\alpha, \lambda^\alpha \tag{55}$$

  The RNS variables map to a patch of the pure spinor space. Consider part of the map.

- Under $SU(5) \times U(1)$ decomposition: $\lambda^\alpha = (\lambda^+, \lambda^a, \lambda_{ab})$. Consider a patch $\lambda^+ \neq 0$.

# The singularity

- We define:

$$\eta = e^{\tilde{\phi}+\tilde{\kappa}} p_+ \;, \qquad b = \quad e^{(\tilde{\phi}-\tilde{\kappa})/2} p_+ \tag{56}$$

$$\lambda^+ = e^{\tilde{\phi}+\tilde{\kappa}} \;, \quad w_+ = \partial\tilde{\kappa} e^{-\tilde{\phi}-\tilde{\kappa}} \tag{57}$$

- Recall the bosonization of the superghosts $\beta = \partial\xi e^{-\phi}$ and $\gamma = e^{\phi}\eta$.
- The spinor stress tensor we get is not $w_+\partial\lambda^+$ but

$$T_\lambda = w_+\partial\lambda^+ - \frac{1}{2}\partial^2 \log\Omega(\lambda) \;, \tag{58}$$

$$\Omega = e^{-3(\tilde{\phi}+\tilde{\kappa})} = (\lambda^+)^{-3} \tag{59}$$

# Mapping RNS variables to pure spinor variables

- The pure spinor formalism is equivalent to the RNS formalism when we take into account all the different pictures at the same time, which is achieved by working in the large Hilbert space, that is including the zero modes of the ghost $\xi$.
- The usual cohomology of the RNS BRST charge $Q_{RNS}$ in the small Hilbert space is equivalent to the cohomology of $Q_{RNS} + \oint \eta$ in the large Hilbert space.
- We are then mapping the $\oint \eta$ term of this extended BRST charge directly to the part $\oint \lambda^+ d_+$ of the pure spinor BRST operator.