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QUATERNIONIC GEOMETRY, BLACK HOLES, + TOPOLOGICAL STRINGS

Work (in progress) with:
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Quaternionic geometry appears as the structure of the moduli spaces upon dimensional reduction of SUGRA with 8 supercharges from $d=4 \to d=3$.

Why study this reduction?

- If study stationary BH in $d=4$, then reduce along timelike Killing vector $\Longrightarrow$ configurations in $d=3$ (Euclidean) theory.

— To define SUSY indices (e.g. for OSV), might like to compactify Euclidean time, compute a partition f^n in $d=3$ (use enhanced duality symmetries)

— To study higher-derivative couplings
(generalized top. string)

- Dim red $4 \to 3$ (c-map from special Kähler manifolds to QK manifolds)
- QK geometry
- SUSY configurations in $d=3$ (spher sym BPS BH)
- Naive quantization of BPS sector (wavefunction for BPS BH w/ fixed charges)
- Geometry of hol. anomalies + their extension

C-map

$(d=4,\ 8\ Q\text{'s})$

Begin with $\mathcal{N}=2$ SUGRA coupled to $n-1$ abelian vector multiplets, e.g. Type IIA on $Y \times \mathbb{R}^{3,1}$

(CY 3-fold)

$h^{1,1}(Y) = n-1$

Totally described (2-deriv) by prepotential $\mathcal{F}_0$.

Has $2n-2$ scalar moduli
n gauge fields
graviton

} bosonic field content

After reducing on the 0-th direction, have

$2n-2$ scalars from $d=4$
$2n$ scalars from gauge fields (A_0, A_0^D)
2 scalars from gravity sector $(g_{00}=e^U,\ \sigma = \text{dual of } g_{0\mu})$

$4n$ scalars

$2n+2$ isometries (gauge sym in $d=4$)

SUSY $\Rightarrow \mathcal{M}$ is QK.

QK: $\mathcal{M}$ has $Sp(1)\cdot Sp(n) \subset SO(4n)$

$\mathcal{M}$ is not Kähler!

equiv, $\exists$ decomposition

$$\underbrace{T_{\mathbb{C}}\mathcal{M}}_{\text{rank } 4n} \simeq \underbrace{E}_{\text{rank } 2n} \otimes \underbrace{H}_{\text{rank } 2}$$

$\mu = 1..4n \qquad A = 1..2n \qquad A' = 1..2$

$$\mu \longrightarrow AA'$$

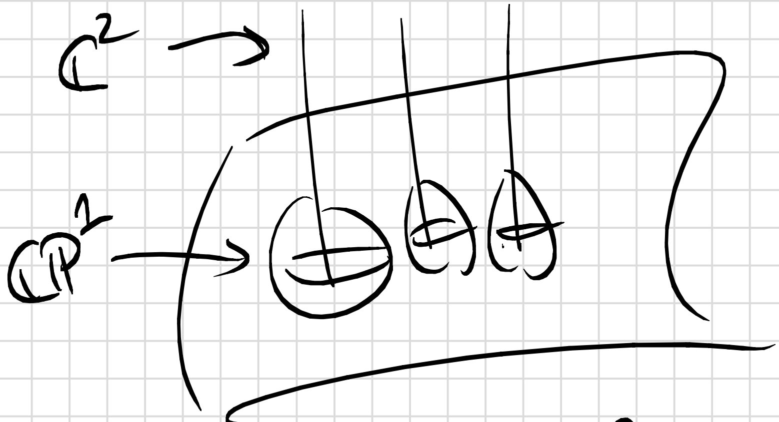
For σ-model into $\mathcal{M}$, fermions carry A index

superchages carry A' index

$R = 4n(n+2)\nu$ <u>constant</u>.

$\exists$ a complex manifold closely related to $\mathcal{M}$:

Look at the total space of $H \to \mathcal{M}$



$\mathbb{P}(H) \to \mathcal{M}$ is a complex manifold

Called twistor space $Z(\mathcal{M})$

Or $H \to \mathcal{M}$ itself is a hyperkähler manifold ("hyperkähler cone") S.

SUSY configurations

SUSY BH in $d=4$ have timelike K.v. $\Longrightarrow$ reduce them to $d=3$.

Study <u>spherically</u> <u>symmetric</u> configurations:

EOM $\longrightarrow$ geodesics on $\mathcal{M}$, parameterized by $t = \frac{1}{r}$.

Which ones preserve some SUSY?

Look at the <u>momentum</u> $P_\mu \to P_{AA'}$

$$P_{AA'} = e_A h_{A'}$$

$\Updownarrow$

SUSY.

$A' = 1,2$

$A = 1 \ldots 2n$

has rank 2

$$P^{[AA'} P_{A'}{}^{B]} = 0$$

Complicated set of quadratic constraints!

But SUSY configurations have a natural <u>lift</u> to $S = H \to \mathcal{M}$

Lift is $\quad (x(t)) \longrightarrow (h(t), x(t))$

$$p_{AA'} = h_{A'} e_A$$

Lifted trajectories obey a simpler constraint:

choose complex coordinates $(u^I, \bar{u}^{\bar{I}})$ on S

Then $\bar{u}^{\bar{I}}$ = constant along the BPS flows. $\leftrightarrow p_I = 0$

If $\mathcal{M}$ has $2n+2$ isometries preserving QK structure (as in c-map case) then also

$$p_{\bar{I}} = c_{\bar{I}} = g_{I\bar{I}} \frac{du^I}{dt}$$

$$= (\partial_I \partial_{\bar{I}} \chi) \frac{du^I}{dt}$$

$$\longrightarrow \boxed{\partial_{\bar{I}} \chi = c_{\bar{I}} t + d_{\bar{I}}}$$

Now, use this QK description to solve a quantum (naive) version of the BPS constraint.

Quantization of geodesic flow $\Longrightarrow \mathcal{H} = L^2(\mathcal{M}) \ni \varphi$ wavefunc.

BPS flows $\Longrightarrow$

$$\left(\nabla^{AA'}\nabla_{A'}^{\;B} + \nu\,\varepsilon^{AB}\right)\varphi = 0 \qquad (\star)$$

Difficult — solve them by lifting to twistor space

So consider a holomorphic function on Z, and $\oint$ over the $\mathbb{CP}^1$ fibers. $\xrightarrow{\text{details}}$ solutions of $(\star)$.

Suppose we fix electric, magnetic, NUT charges.

$q \qquad p \qquad k=0$

Then get

$$\varphi(U, X, \zeta^1, \tilde{\zeta}_1, \sigma) = \boxed{e^{2U + ip^1\tilde{\zeta}_1 - iq_1\zeta^1}\, J_0\!\left(2e^U|Z|\right)}$$

moduli   gauge fields

$e^U = g_{00}$