Plasmarings in Large N Gauge Theories

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Related Earlier Work:
hep-th 0507219 with O. Aharony, T. Wiseman

Outline of the Talk

- 1. Introduction
- 2. d=3: Plasmarings
- 3. d=4: Plasmadonughts
- 4. Validity of the fluid dynamic approximation.
- 5. Conclusions and Speculations

1. Introduction

The AdS/CFT correspondence relates the deconfined 'gluon plasma' phase to black branes and black holes in the bulk.

To see this recall that a nonzero Polyakov loop implies existence of a cigar shaped string world sheet about the time circle: the Euclidean signature of a horizon.

In this talk we will use computations in the field theory deconfined phase to make predictions for classical black hole physics in the bulk dual

In general, of course, we understand classical gravity better than strongly interacting deconfined dynamics.

However in the large N and long wavelength limit, the deconfined phase is well described, in a statistically averaged sense, by the relativistic Navier Stokes equations.

As we will see below, it is almost trivial to construct and systematically study simple classes of solutions to these fluid dynamic equations, and thereby obtain rather nontrivial information about the structure of bulk black holes.

2.a d=3. Thermodynamics

Consider $\mathcal{N} = 4$ Yang Mills theory at t'Hooft coupling $g_{YM}^2 N = \lambda$, compactified on a Scherk Schwarz circle of radius R. At distance $\gg R$ this system is described by a confining pure 3d Yang Mills theory (Witten).

At large λ this theory admits a dual description as IIB supergravity on a space that is asymptotically AdS_5 compactified on a Scherk Schwarz circle (Witten).

The background dual to the vacuum is the so called AdS soliton (double analytically continued non extremal brane background).

At finite temperature the bulk geometry is asymptotically AdS_5 on a Scherk Schwarz torus.

There exist an infinite number of such backgrounds. The two relevant for our discussion are (Euclidean) the thermal AdS-soliton and the Euclidean non extremal D3 brane.

Thermal AdS at $T = 1/\beta$ has lower free energy than the black brane when $\beta > 2\pi R$, but has larger free energy when $\beta < 2\pi R$. Consequently, the system undergoes a deconfining phase transition at $\beta = 2\pi R$

The low temperature phase or thermal soliton phase is dual to a gas of glueballs.

The high temperature or black brane phase has a free energy density of order N^2. It is dual to the deconfined gluon plasma. Its equation of state is

$$P = -f = \frac{N^2}{2^{10}\pi T_c}\left((2\pi T)^4 - (2\pi T_c)^4\right).$$

where $T_c = 1/2\pi R$.

Note that the free energy, and hence the pressure, of the deconfined phase vanishes at the phase transition temperature.

2.b: Domain Wall

In this talk we will be interested in finite lumps of fluid. An understanding of the the boundaries of these lumps is crucial to controling their dynamics.

Luckily, it turned out to be possible to construct a static gravitational solution dual to the simplest fluid configuration with a boundary. In this solution the plasma at $T = T_c$ fills the half space $x > 0$, smoothly interpolating to the vacuum across a domain wall at $x = 0$ whose thickness is of order Λ_{gap}^{-1}.

The domain wall gravitational solution has been found numerically. It may be used to read of the properties - e.g. the surface tension - of the domain wall. The latter turns out to be a positive number of order $\frac{N^2}{R^2}$

The Scherk Schwarz circle is of finite size at the horizon in the bulk of the plasma, but shrinks to zero size at the plasma's edge.

Consequently, the horizon topology of the bulk solution dual to a lump of plasma, is given by the Scherk Schwarz S^1 trivially fibered over the spatial regions occupied by fluid, subject to the constraint that the S^1 vanishes at boundaries.

2.c: Equations of Fluid Dynamics

Lumps of plasma with boundaries obey the Relativistic Navier Stokes equations

$$\nabla_\mu T^{\mu\nu}$$

$$T^{\mu\nu} = T^{\mu\nu}_{\text{perfect}} + T^{\mu\nu}_{\text{dissipative}} + T^{\mu\nu}_{\text{surface}}$$

$$T^{\mu\nu}_{\text{perfect}} = (\rho + P)u^\mu u^\nu + P g^{\mu\nu}$$

$$T^{\mu\nu}_{\text{surface}} = \sigma f^\mu f^\nu - g^{\mu\nu}\sqrt{\partial f \ \dot{\partial} f}\,\delta(f)$$

Where the surface is $f = 0$ and the pressure P is the pressure given as a function of density from the equation of state.

The dissipative term is more complicated

$$T^{\mu\nu}_{\text{dissipative}} = -\zeta\theta P^{\mu\nu} - 2\eta\sigma^{\mu\nu} + q^\mu u^\nu + u^\mu q^\nu$$

where

$$\begin{aligned} a^\mu &= u^\nu \nabla_\nu u^\mu, \\ \theta &= \nabla_\mu u^\mu, \\ P^{\mu\nu} &= g^{\mu\nu} + u^\nu u^\mu, \\ \sigma^{\mu\nu} &= \frac{1}{2}\left(P^{\mu\lambda}\nabla_\lambda u^\nu + P^{\nu\lambda}\nabla_\lambda u^\mu\right) - \frac{1}{d-1}\theta P^{\mu\nu}, \\ \omega^{\mu\nu} &= \frac{1}{2}\left(P^{\mu\lambda}\nabla_\lambda u^\nu - P^{\nu\lambda}\nabla_\lambda u^\mu\right). \end{aligned}$$

The various viscosity and conductivity parameters in this equation have all be determined from gravity- however we will not need them in what follows.

2.d: Rigid Rotation

We say the fluid undergoes 'rigid rotation' if $u^r = 0$, $u^\theta = \omega r$, $\rho = \rho(r)$.

Perhaps unsurprisingly, it turns out such solutions to $\partial^\mu T_{\mu\nu} = 0$ at vanishing values of viscosities and conductivities, also satisfy the same equations at arbitrary values of these parameters

As a consequence all rigidly rotating solutions are easy to find explicitly. In the bulk

$$(\rho(r) - \rho_0)\left(1 - \omega^2 r^2\right)^2 = \text{constant.}$$

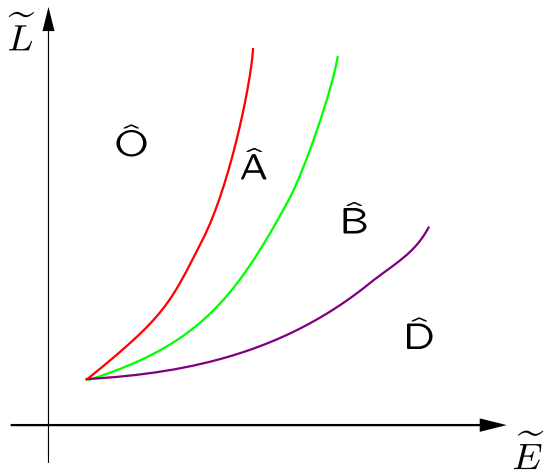
Matching the bulk solution above with boundary conditions yields two classes of solutions-spinning plasma balls and plasma rings.

Plasmaballs and plasmarings are, respectively, disk and annulus shaped configurations of rotating fluid, each of which appear in 2 parameter families. The two parameters may be thought of as the outer radius and angular velocity of the solutions.

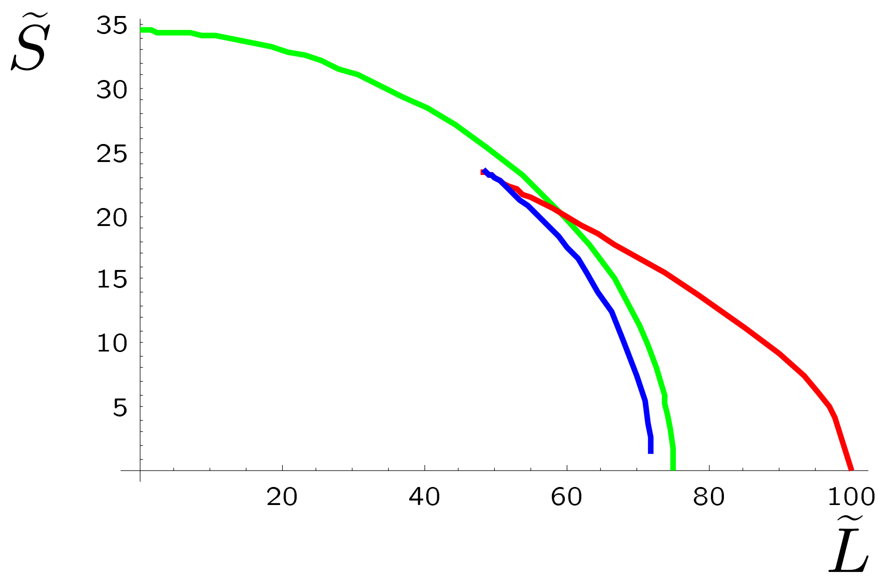
The pressure in each of these solutions is positive at the outer boundary (to balance the surface tension) but decreases due to the centrifugal force upon moving radially inwards towards the center.

In plasmaring solutions, the pressure is negative at inner boundary, to balance the negative surface tension. This second boundary condition determines $r_{in} = r_{in}(r_{out}, \omega)$.

Upon varying parameters over all allowed values we find



In the region B above we have 3 distinct solutions. We plot the entropy versus angular momentum, at a particular fixed value of energy, for these solutions in colours : Ball, Small Ring, Large Ring.



Thus the ‘small ring’ is always entropically disfavoured: upon increasing angular momentum we have a ‘phase transition’ from the ball to the big ring

This suggests that the small ring is dynamically unstable towards rotationally invariant density fluctuations, while the big ring and ball are stable to the same fluctuations.

We suspect that the big ring solution is also unstable to density fluctuations that break rotational invariance. The end point of this ‘Rayleigh’ instability could be a necklace of rotating droplets.

2.e Bulk Duals

It is easy to compute the horizon topologies of the bulk black objects dual to our solutions.

Trivially fibering an S^1 over a disk, subject to the condition that the S^1 vanishes at the boundary of the disk, yields an S^3. Thus plasmaballs are dual to rotating black holes.

Fibering the S^1 over an annuls yields $S^1 \times S^2$. Consequently, the plasmaring is dual to a black ring.

Unfortunately, finite energy black hole and black ring solutions have not yet been constructed in Scherk Schwarz compactified AdS_5.

However the corresponding solutions have been constructed and studied in detail in flat 5 dimensional space, which form a useful comparison point for our results.

The existence and stability curves for black holes and black rings in flat space turn out to be qualitatively - and in some respects quantitatively - strikingly similar to our graphs above.

3: d=4

While rotating black holes and black rings have been very well studied in 5 dimensional gravitational theories, $d = 6, 7$... is relatively uncharted territory.

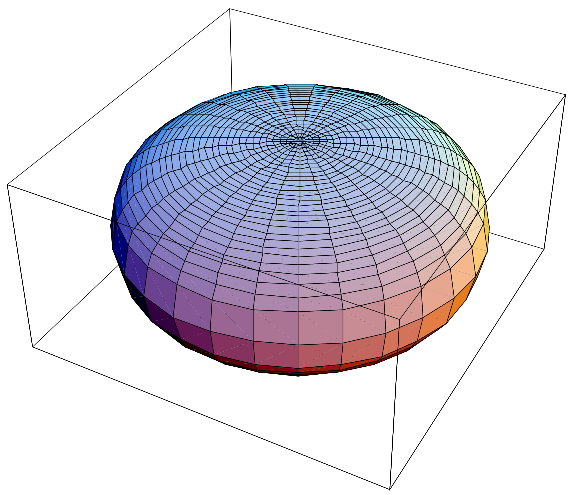
The only gravitational results that I am aware of, regarding the possible allowed topologies in black solutions to Einstein's equations, take the form of a complicated mathematical condition on the horizon topology. This condition turns out to be satisfied by all 'product of sphere' topologies

In, e.g. $d = 6$, this theorem is consistent with the existence of the horizon topologies S^4, $S^3 \times S^1$ and $S^2 \times S^2$ ($S^1 \times S^1 \times S^1 \times S^1$ is a marginal case of the theorem).

It is natural to ask if all these topologies, or only some of them, appear as stationary solutions to Einstein's equations in Scherk-Shwarz compactified AdS_6, where our fluid dynamic picture makes the analysis easy.

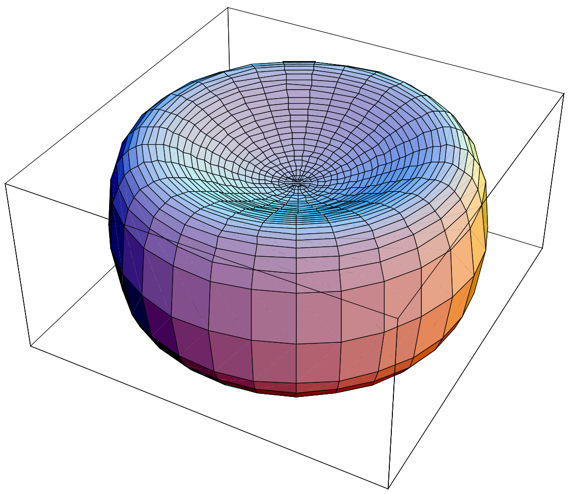
The computations may be performed in direct analogy with those described above. I present the results.

Fluid solutions that preserve the rotational invariance about an axis, to the relevant 4 dimensional Navier Stokes equations, occur in 2 different topological classes - balls and dough-



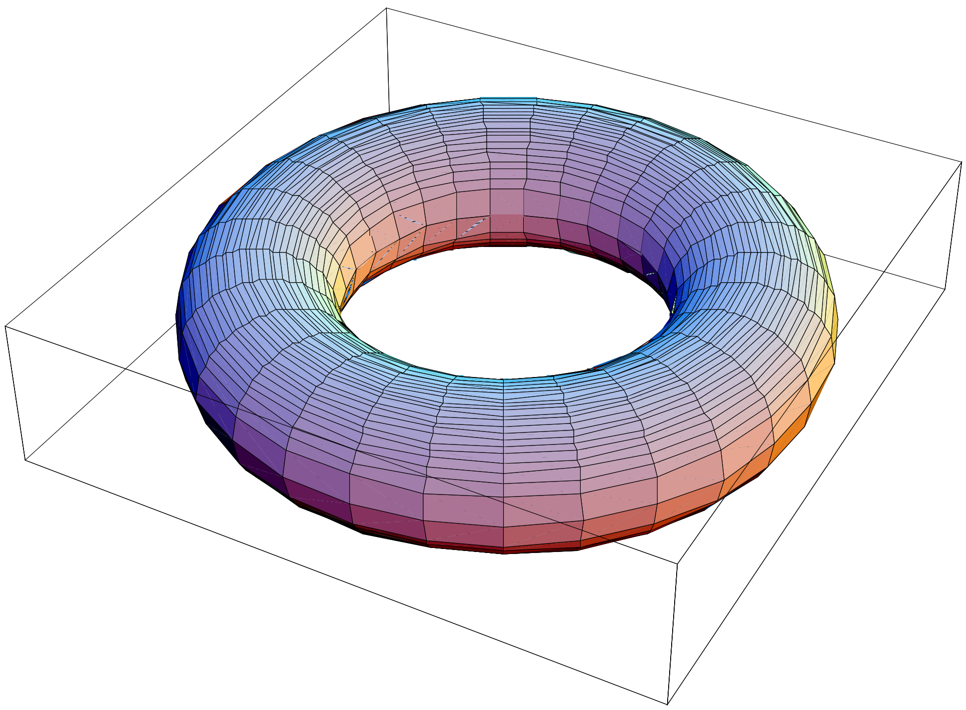
nuts or rings. Upon increasing the angular velocity, ordinary balls turn into...

Pinched balls



Upon further increasing ω, our solutions pinch off (undergoing a smooth topology changng trainsition into...

Plasmadongughts



It is easy to check that the balls and rings are dual to bulk solutions with horizon topology S^4 and $S^3 \times S^1$ respectively.

A hollow ball of fluid - which would have mapped to horizon topology $S^2 \times S^2$ - never occurs.

This seems intuitively reasonable. The rotational centrifugal force pushes fluid away from an axis rather than a point - favouring a doughnut over a hollow ball

The intuition of the previous slide suggests that we might a qualitatively kind of solution in $d = 5$. In this dimension the most general rotation has two orthogonal rotational axes.

Spinning up this solution along any one of these axes should produce a solution whose with dual horizon topology $S^4 \times S^1$, while spinning the fluid up along both axes might produce a fluid lump with dual horizon topology $S^3 \times S^1 \times S^1$

We are in the process of performing a detailed analysis to check if this intuition is borne out

2. Validity of the Fluid Dynamics

The equations of fluid dynamics, applied to our system, are approximate in three distinct ways.

First they keep track only of average values of the local density, velocity etc, but ignore fluctuations.

Fluctuations however turn out to be of order $1/N^2$. This is dual to the fact that quantum fluctuations of the metric are of order $\frac{1}{N^2}$. Ignoring fluid fluctuations is the same as the classical approximation in bulk physics, and is justified at large N.

Second they ignore glueball production in 'gluon' collisions.

However glueball production rates are $\mathcal{O}(\frac{1}{N^2})$. Intuitively this is because gluon collisions produce glueballs only when a gluon meets its colour antipartner.

Glueball production in gluon collisions is dual to the production of gravitons via Hawking radiation. The effect of each of these processes on, respectively, fluid dynamics and classical black hole physics, may be ignored over time scales of unit order, in the large N limit.

Third, the Navier Stokes equations (and our treatment of domain walls) are simply the first terms in a derivative expansion.

Higher derivative terms suppressed by powers of the mean free path $\sim \mathcal{O}(\Lambda_{gap}^{-1})$, and are negligible only on solutions that vary on scales large compared to this number. These conditions are met for our solutions provided all charges are large.

In the bulk dual, it is clear that it is permissable to integrate out all other fields of 10d supergravity to obtain an approximately local theory of quasinormal modes only for wavelengths much larger than Λ_{gap}.

Fourth, we have modeled the fluid surface simply by a constant surface tension, that we have read off from the domain wall gravity solution.

This approximation is valid only when the fluid temperature at every boundary is near to T_c, and when the 'curvature' of the boundary is small in units of Λ_{gap}.

These conditions are also met on our solutions, provided all charges are large.

5. Concluding comments

Dual geometries to ‘confining’ theories are ‘capped off’ in the IR. A point in the IR casts a shadow of size Λ_{gap}^{-1} on the boundary. In particular an IR black hole of size $R \gg \Lambda_{gap}^{-1}$ maps to lump of deconfined fluid of size $\sim R$ on the boundary.

Thus our use of the AdS/CFT correspondence provides an approximately local fluid dynamical description of black hole horizon physics. This is reminicent of the ‘membrane paradigm’ and may be the precise version of this claim for black holes in asymptotically AdS like spaces.

In this talk I have only applied the fluid picture to the study of stationary gravitational black holes.

However more general fluid processes will also have bulk duals. It would be really fun - and may be possible - to study, for example, black hole collisions, using fluid dynamics.

Of course one might also consider trying to reverse the flow of information. Could plasmarings appear in a modified RHIC experiment? Will they absorb particles like black rings will.... There may be fun things in store for us.