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Inflationary de Sitter Solutions
from String Effective Theories

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In collaboration with:

- **Hervé Partouche**, hep-th : 0705.3206 and hep-th : 0706.0728,
[In effective supergravities from Strings]
- **Nicolaos Toumbas and Jan Troost**, hep-th : 0704.1996
[In Suprerstrings with broken SUSY]

1. Introduction

In the framework of superstring compactifications there are always **moduli - fields coupled in a very special way** to the gravitational and matter sector of the effective $N = 1$ **four-dimensional supergravity**.

The gravitational and the scalar field part of the $N = 1$ effective Lagrangian has the generic form

$$\mathcal{L} = \sqrt{-\det g} \left[\frac{1}{2} R - g^{\mu\nu} K_{i\bar{j}} \partial_\mu \phi_i \partial_\nu \bar{\phi}_{\bar{j}} - V(\phi_i) \right]$$

$K_{i\bar{j}}$ is the metric of the scalar manifold

V is the scalar potential of the $N = 1$ SUGRA.

We will work always in gravitational mass units with

$$M = \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18} \text{GeV}.$$

What will be crucial in this work is the **non-triviality** of the scalar kinetic terms $K_{i\bar{j}} \longrightarrow$ which will provide us with **new accelerating cosmological solutions** once the

i) Radiative

and

ii) Temperature

corrections are taken into account.

Superstring vacua with **spontaneously broken supersymmetry** and consistent at the classical level with **a flat space-time** define a very large class of **“no-scale” supergravity models**.

Among them, a class of potential candidates exist which **extend in low energies the physics of the standard model at $O(1)$ TeV energy scale**.

This class of models contains **an enormous number of consistent string vacua**, that are constructed via **freely acting orbifolds, “geometrical fluxes”**, in Heterotic or Type IIA,B orientifold compactification, or in type IIA or IIB compactification, **with geometrical or non-geometrical fluxes RR-fluxes or else**.

- Generalization of the Scherk–Schwarz gauging to superstring theory.

Rohm, 84; Kounnas, Porrati, 88

Ferrara, Kounnas, Porrati, Zwirner, 89

Kounnas, Rostand, 90

Kiritsis, Kounnas, Petropoulos, Rizos, 99

Antoniadis, Dudas, Sagnotti, 99

Antoniadis, Derendinger, Kounnas, 99

Derendinger, Kounnas, Petropoulos,

Zwirner, 04

Derendinger, Kounnas, Petropoulos, 05,06,

- Simultaneous presence of NS, RR H^3 , F^3 .

Frey, Polchinski, 02

Giddings, Kachru, Polchinski, 02

Kachru, Schulz, Trivedi, 03

Kachru, Schulz, Tripathy, Trivedi, 03

Derendinger, Kounnas, Petropoulos,

Zwirner,04, . . .

Thanks to the **supersymmetric “no-scale” structure**, and despite the plethora of this type of string vacua, **an interesting universal scaling property emerges**, that we will try to explore in this work.

Namely, we will study in more detail, the **non-trivial cosmological implications** due to the existence (at the classical level) of some **special moduli fields**.

One of these special moduli, Φ , is the **super-partner of the Goldstino**. It defines the **field-dependence of the gravitino mass-term** :

$$m(\Phi) = C e^{\alpha\Phi},$$

and couples to the trace of the energy momentum tensor of a sub-sector of the theory.

Other special moduli are those with **non-trivial Φ -dependent kinetic terms**. These moduli **appear naturally in all string compactifications**.

In order to be more explicit let us consider as example the **type IIB orientifold with D_3 -branes and non-trivial NS and RR three form fluxes H^3 and F^3** .

Due to the presence of **non trivial fluxes**, a **well known superpotential $W(S, U_A)$ is induced that stabilizes the complex structure moduli U_A and the coupling constant modulus S** .

The remaining $h_{1,1}$ - moduli T_A are still flat directions at the classical level.

The Kähler potential is also well known and is given by the **intersection numbers** d_{abc} of the Calabi-Yau manifold:

$$K = -\log d_{abc}(T_a + \bar{T}_a)(T_b + \bar{T}_b)(T_c + \bar{T}_c)$$

The superpotential W is constant.

The potential of the effective $N = 1$ “no-scale” supergravity is identically zero with a non-trivial gravitino mass

$$m^2 = |W|^2 e^K$$

This generic “no-scale” structure emerges in all type IIB orientifold compactifications with fluxes.

Keeping for simplicity the direction $T_a = \gamma_a T$ and freezing all other directions the Kähler potential take the well known $SU(1, 1)$ form

$$K = -3 \log(T + \bar{T})$$

giving rise to kinetic term and gravitino mass term

$$g_{\mu\nu} = 3 \frac{\partial_\mu T \partial_\nu \bar{T}}{(T + \bar{T})^2}, \quad m^2 = C e^K = \frac{C}{(T + \bar{T})^3}$$

Freezing further the $\text{Im}T$ and defining the field Φ :

$$e^{2\alpha\Phi} = m^2 = \frac{c}{(T + \bar{T})^3}$$

$$g_{\mu\nu} = 3 \frac{\partial_\mu T \partial_\nu \bar{T}}{(T + \bar{T})^2} = g_{\mu\nu} \frac{\alpha^2}{3} \partial_\mu \Phi \partial_\nu \Phi .$$

The choice $\alpha^2 = 3/2$ normalize canonically the kinetic terms of the modulus Φ .

The other **extra moduli** we will consider here are those with kinetic terms that scale with the **inverse volume of the T -moduli** :

$$K_s = g_{\mu\nu} c_s \frac{\partial_\mu \Phi_s \partial_\nu \Phi_s}{(T + \bar{T})^3} = g_{\mu\nu} \frac{1}{2} e^{2\alpha\Phi} \partial_\mu \Phi_s \partial_\nu \Phi_s$$

Moduli with this scaling property appear in ***a very large class of string compactifications.***

i) All moduli fields leaving in the parallel space of D_3 -branes.

ii) All moduli coming from the twisted sectors in Z_3 - orbifold compactifications in heterotic string. (After the non-perturbative stabilization of S due to gaugino condensation and flux-corrections.)

Kounnas Porrati, 87

Ferrara Kounnas Porrati, 87

Derendinger, Kounnas, Petropoulos, 05, 06,

...

2. Gravitational, Moduli and Thermal Equations

Consider the system of the moduli fields Φ, Φ_s , taken together with **all other relevant degrees of freedom** of the theory which can be parameterized by an effective **pressure** $P(T)$ and **energy density** $\rho(T)$.

We are interested for the gravitational solutions that are based on **isotropic and homogeneous FRW space time metrics**:

$$ds^2 = -dt^2 + a(t)^2 d\Omega_3^2.$$

Ω_3 denotes a 3-dimensional closed space with constant curvature k , e.g 3-dimensional sphere.

The **two independent** gravitational equations of the system are:

i) The “Hubble” equation

$$3H^2 = -\frac{3k}{a^2} + \rho(T) + \frac{1}{2}\dot{\Phi}^2 + \frac{1}{2}e^{2\alpha\Phi}\dot{\Phi}_s^2 + V(\Phi), \quad H = \left(\frac{\dot{a}}{a}\right)$$

ii) The equation of the variation with respect to the scale factor a , modulo the “Hubble” equation:

$$\dot{H} + 3H^2 = -\frac{2k}{a^2} + \frac{1}{2}(\rho - P) + V + \frac{1}{2}a\frac{\partial V}{\partial a}$$

***** In the literature the term $a\frac{\partial V}{\partial a}$ is frequently neglected.**

This term will play a crucial role in the derivation of the inflationary solutions under investigation.

The moduli field equations:

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{\partial}{\partial\Phi} \left(V - P - \frac{1}{2} e^{2\alpha\Phi} \dot{\Phi}_s^2 \right) = 0$$

$$\ddot{\Phi}_s + (3H + 2\alpha\dot{\Phi}) \dot{\Phi}_s = 0$$

The Φ_s equation can be solved immediately

$$K_s \equiv \frac{1}{2} e^{2\alpha\Phi} \dot{\Phi}_s^2 = c_s \frac{e^{-2\alpha\Phi}}{a^6}$$

The **total energy conservation of the system** is the remaining independent equation of the system

$$\frac{d}{dt} \left(\rho + \frac{1}{2} \dot{\Phi}^2 + K_s + V(\Phi) \right) + 3H \left(\rho + P + \dot{\Phi}^2 + 2K_s \right) = 0.$$

Some useful relations that are valid for the thermal quantities $\rho(T)$ and $P(T)$:

i) The entropy equation

$$T \frac{\partial P}{\partial T} = \rho + P$$

ii) Scaling equations

$$\left(m_i \frac{\partial}{\partial m_i} + T \frac{\partial}{\partial T} \right) P \equiv 4P, \quad \left(m_i \frac{\partial}{\partial m_i} + T \frac{\partial}{\partial T} \right) \rho \equiv 4\rho$$

i) and ii) imply:

$$m_i \frac{\partial}{\partial m_i} P = -(\rho - 3P).$$

In the special case of Φ -dependent masses,

$$m_i = C_i e^{\alpha\Phi},$$

we obtain a very fundamental equation involving the modulus field Φ ,

$$-\frac{\partial P}{\partial\Phi} = \alpha (\rho - 3P)$$

The above equation shows clearly that Φ couples to the (sub-)trace of the energy momentum tensor of the thermal system ρ, P .

3) Radiative and the Thermal Corrections

To proceed further, it is necessary to analyze the structure of the scalar potential V and the thermal functions ρ, P . More precisely, we had to **specify their dependence on Φ, T, a** .

Although this analysis looks hopeless in a generic field theory, **it is perfectly under control in string effective no-scale supergravity theories.**

Classically, the potential V is zero along the moduli directions Φ and Φ_s .

At the quantum level V receives **radiative and thermal corrections** that are given in terms of the **effective potential** $V(\Phi, a; m_i)$ and in terms of the **thermal function** $-P(\Phi, T; m_i)$.

i) Effective Potential

$$V = V_0 + \frac{1}{64\pi^2} \text{Str}\mathcal{M}^0 \Lambda^4 \log \frac{\Lambda^2}{\mu^2} \\ + \frac{1}{32\pi^2} \text{Str}\mathcal{M}^2 \Lambda^2 + \frac{1}{64\pi^2} \text{Str}\mathcal{M}^4 \log \frac{\mathcal{M}^2}{\mu^2} + \dots$$

Λ is an ultraviolet cut-off

μ stands for the renormalization scale

$$\text{Str}\mathcal{M}^n \equiv \sum_i (-)^{2J_i} (2J_i + 1) m_i^n$$

Weighted sum over the field-dependent masses m_i , and the **spin-statistic** of particles J_i .

$V_0 \equiv 0$ in “no-scale” supergravity.

$$\text{Str}\mathcal{M}^0 = (n_B - n_F) \equiv 0$$

The Λ^4 term is absent in SUSY theories.

The Λ^2 term is always proportional to the gravitino mass-term $m(\Phi)^2$

$$\text{Str}\mathcal{M}^2 = C_2 m(\Phi)^2$$

C_2 is a field independent number. It depends only on the geometry of the kinetic terms but not on the details of the superpotential.

The $\text{Str}\mathcal{M}^4$ term has logarithmic μ infrared behavior.

In the **infrared regularization method** adapted in string theory, the scale μ^2 turns out to be proportional to the **curvature of the three dimensional space**.

$$\mu = \frac{1}{\gamma a}$$

The numerical coefficient γ is chosen according to the Renormalization Group Equation arguments.

Another choice for the infrared scale μ would be the **temperature scale**, $\mu = \eta T$.

Both choices are physically equivalent. The **curvature** choice looks more natural and has the advantage to be **valid even in the absence of the thermal bath**.

The $\text{Str}\mathcal{M}^4$ can be expanded in powers of gravitino mass $m(\Phi)$

$$\frac{1}{64\pi^2}\text{Str}\mathcal{M}^4 = C_4 m^4 + C_2 m^2 + C_0$$

Including the logarithmic terms and the contribution from $\text{Str}\mathcal{M}^2$, **the total effective potential is organized in powers of $m(\Phi)$:**

$$V = V_4(\Phi, a) + V_2(\Phi, a) + V_0(\Phi, a)$$

$$V_n(\Phi, a) = m^n(\Phi) [C_n + Q_n \log(m(\Phi)\gamma a)]$$

$$V'_n \equiv \frac{\partial V_n(\phi, a)}{\partial \Phi} = \alpha(nV_n + m^n Q_n),$$

$$a \frac{\partial V_n(\phi, a)}{\partial a} = \alpha m^n Q_n.$$

ii) Thermal Potential

At finite temperature the effective potential receives **extra contributions from the thermal fluctuations** of the bosonic and fermionic states

$$V_{\text{Total}} = V(\Phi, a; m_i) - P(T, m_i)$$

The general expressions of the energy density $\rho(T)$ and pressure $P(T)$ are

$$\rho(T) = \sum_i T^4 f_\rho\left(\frac{m_i}{T}\right) \quad P(T) = \sum_i T^4 f_P\left(\frac{m_i}{T}\right)$$

For massless degrees of freedom,

$$\rho = 3P = \frac{\pi^4}{15} \left(n_B + \frac{7}{8} n_F \right) T^4.$$

There are three distinct sub-sectors of states:

i) A sub-sector of **massless states** with.

$$\rho - 3P = 0 \quad \longrightarrow \quad \partial P / \partial \phi = 0$$

ii) A sub-sector of states with **non vanishing masses independent of $m(\Phi)$** $\rightarrow \partial P / \partial \phi = 0$

• If m_i is below T

$$P(T) = P(T, m_i = 0) + m_i \frac{\partial P}{\partial m_i} = c_p T^4 - \sum c_i m_i^2 T^2$$

• If m_i is above T , then the contribution of the particular degrees of freedom is exponentially suppressed and **decouples from the thermal system.**

iii) A sub-sector with **non vanishing masses proportional to $m(\Phi)$** .

$$\frac{\partial P}{\partial \Phi} = -\alpha(\rho - 3P).$$

According to the scaling with respect to T and $m(\Phi)$, we can separate

$$\rho = \rho_4 + \rho_2, \quad P = P_4 + P_2,$$

$$\left(m(\Phi) \frac{\partial}{\partial m(\Phi)} + T \frac{\partial}{\partial T} \right) (\rho_n, P_n) = n (\rho_n, P_n).$$

ρ_4, P_4 receive contributions from the **massless states** of the *i*)-sector, **the T^4 part of the *ii*)-sector** and from **all states of the *iii*)-sector**.

$$\rho_4 = T^4 \left(\frac{\pi^4}{8} n^* + \sum_i f_\rho \left(\frac{m_i(\Phi)}{T} \right) \right)$$

$$P_4 = T^4 \left(\frac{\pi^4}{24} n^* + \sum_i f_P \left(\frac{m_i(\Phi)}{T} \right) \right)$$

ρ_2 and P_2 arise from the T^2 part of ρ , P of the ii)-sector:

$$\rho_2 = P_2 = - \sum c_i m_i^2 T^2 \equiv - M^2 T^2 .$$

4) Critical Solution

The fundamental ingredients in our analysis are **the scaling properties** of the total effective potential at finite temperature.

Independently of the complication appearing in the radiative and temperature corrected effective potential **the scaling violating terms are under control**. Their structure suggests to search for a solution where all the scales of the system $m(\Phi)$, T and $\mu = (1/\gamma a)$ have similar evolution in time.

$$m(\Phi) = \frac{1}{\gamma a} \quad \rightarrow \quad H = -\alpha \dot{\Phi}, \quad \xi m(\Phi) = T.$$

The Φ -equation

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{\partial}{\partial\Phi} \left(V - P - \frac{1}{2} e^{2\alpha\Phi} \dot{\Phi}_s^2 \right) = 0,$$

on the critical trajectory becomes

$$\begin{aligned} \dot{H} + 3H^2 &= \alpha^2 \left((4C_4 + Q_4)m^4 + (2C_2 + Q_2)m^2 + Q_0 \right) \\ &+ \alpha^2 \left((r_4 - 3p_4)\xi^4 m^4 - 2C_s \gamma^6 m^4 \right). \end{aligned}$$

$$\frac{\partial V_n}{\partial\Phi} = \alpha m^n (nC_n + Q_n),$$

$$P_4 = p_4 T^4, \quad \rho_4 = r_4 T^4 \quad K_s = C_s \frac{\gamma^2}{a^4}$$

The gravity equation

$$\dot{H} + 3H^2 = -\frac{2k}{a^2} + \frac{1}{2}(\rho - P) + V + \frac{1}{2} a \frac{\partial V}{\partial a},$$

on the critical trajectory takes the form

$$\begin{aligned} \dot{H} + 3H^2 = & -2k\gamma^2 m^2 + \frac{1}{2}(r_4 - p_4)\xi^4 m^4 \\ & + (C_4 m^4 + C_2 m^2 + C_0) + \frac{1}{2}(Q_4 m^4 + Q_2 m^2 + Q_0). \end{aligned}$$

The compatibility of the Φ -equation and the gravity equation along the critical trajectory implies an identification of the coefficients of the monomials in m .

i) The constant terms

$$C_0 = \frac{2\alpha^2 - 1}{2} Q_0,$$

→ determination of γ

ii) The m^2 terms, \rightarrow determination of k :

$$k = -\frac{2\alpha^2 - 1}{4\gamma^2}(2C_2 + Q_2).$$

iii) The m^4 terms relate ξ to the constant C_s appearing in K_s

$$C_s = \frac{1}{\gamma^6} \left(\frac{4\alpha^2 - 1}{2\alpha^2} C_4 + \frac{2\alpha^2 - 1}{4\alpha^2} (Q_4 + r_4\xi^4) \right) - \frac{1}{\gamma^6} \left(\frac{6\alpha^2 - 1}{4\alpha^2} p_4\xi^4 \right).$$

The Hubble equation,

$$3H^2 = -\frac{3k}{a^2} + \rho + \frac{1}{2}\dot{\Phi}^2 + \frac{1}{2} e^{2\alpha\Phi}\dot{\Phi}_s^2 + V.$$

in the background of the **critical solution :**

$$\left(\frac{6\alpha^2 - 1}{6\alpha^2}\right) 3H^2 = \rho + V + \frac{1}{2} e^{2\alpha\Phi} \dot{\Phi}_s^2 - \frac{3k}{a^2}$$

The dilatation factor in front of $3H^2$,
can be absorbed by a redefinition of :
 C_R , λ and $\hat{k} \longrightarrow$

$$3H^2 = 3\lambda - \frac{3\hat{k}}{a^2} + \frac{C_R}{a^4},$$

$$3\lambda = \frac{2\alpha^2 - 1}{6\alpha^2 - 1} 3\alpha^2 Q_0,$$

$$\hat{k} = \frac{\alpha^2}{\gamma^2} \left(\frac{2}{6\alpha^2 - 1} \xi^2 M^2 - C_2 - \frac{3}{2} \frac{2\alpha^2 - 1}{6\alpha^2 - 1} Q_2 \right),$$

$$C_R = \frac{3}{2\gamma^4} \left((r_4 - p_4) \xi^4 + 2C_4 + \frac{2\alpha^2 - 1}{6\alpha^2 - 1} Q_4 \right).$$

The lesson of this exercise is that :

- the cosmological constant scale, λ
- the curvature scale, \hat{k}

are both generated by “the scaling violating terms” of the thermal effective potential.

- In the **absence** of the scaling violating terms (or when these terms are negligible), the “no-scale modulus Φ couples to the total trace of the energy momentum tensor.

- This special case was studied in 1986 by I. Antoniadis and C. Kounnas. They found that the critical trajectory is the only stable solution under any field fluctuation.

In that case, $\lambda = \hat{k} = 0$, the critical trajectory is an **attractor at late times** giving rise to a radiation evolving universe with :

$$a^2 \sim t, \quad T^2 \sim m_{\Phi}^2 \sim 1/t, \quad V \sim 1/t^2.$$

In the general case $\lambda \neq 0, \hat{k} \neq 0$, the time evolution of a, T and Φ is similar to

the radiation-deformed de Sitter Solutions!

H.Firouzjahi, S.Sarangi, S.H.H.Tye, 04 ;

S.Sarangi, S.H.H.Tye, 05;

R. Brustein, S.P. de Alvis, 06;

C. Kounnas, H. Partouche, 07; ...

$$a_+^2 = \frac{1}{\lambda} \left(ch^2(\sqrt{\lambda} t) + \epsilon_\rho^2 \right)$$

$$a_-^2 = \frac{1}{\lambda} \left(-sh^2(\sqrt{\lambda} t) + \epsilon_\rho^2 \right)$$

The cosmological solutions a_- and a_+ are connected by a Φ -Gravitational Instanton

$$a_E^2 = \frac{1}{\lambda} \left(\cos^2(\sqrt{\lambda} \tau) + \epsilon_\rho^2 \right)$$

with a transition probability ($a_- \rightarrow a_+$)

$$\langle \Psi_- || \Psi_+ \rangle = P \sim e^{\left(\frac{2}{3\lambda} - \frac{\chi_\rho}{\lambda^2} \right)}$$

- χ_ρ is proportional to the number of the effective marginal and/or thermal degrees of freedom at the temperature scale T_0 , defined at the transition point.

C. Kounnas, H. Partouche, 07

- Ψ_{\pm} is the wave-function of the universe.

J.B. Hartle, S.W. Hawking, 83;

A. Vilenkin, 82, 83;

A.D. Linde, 84;

H.Firouzjahi, S.Sarangi, S.H.H.Tye, 04;

S.Sarangi, S.H.H.Tye, 05;

R. Brustein, S.P. de Alvis, 06;

C. Kounnas, H. Partouche, 07;;

5. String Perspectives and Conclusion

At classical string level it seems **difficult** to construct **exact cosmological string solutions** and is even more difficult to obtain **de Sitter like inflationary** solutions **even in lower than four dimensions**.

- Consider for instance the euclidian version of S^3 which is an exact conformal field theory base on $SU(2)_k$ WZW model.

However, the $SU(2)_k$ WZW model does not admit any real-time analytic continuation due to the existence of a non-trivial torsion H_{ijk} which becomes imaginary!!

I.Antoniadis, C.Bachas, A.Sagnotti, 90;
P.K Townsend 01;
J. Sonner, P.K Townsend 06;
C.Bachas, C.Kounnas,
D.Orlando, M.Petropoulos 07;

...

- The only known cosmological solution based on an exact conformal field theory is that of $SL(2, R)/U(1)_{-|k|} \times K$.

Its euclidian version is also well defined by the parafermionic T-fold.

C.Kounnas, D.Luest 92;

L.Coralba, M.S.Costa, C.Kounnas 02;

C.Kounnas, N.Toumbas, J.Troost 07;

...

• In all String cosmological models with a well define euclidian version,

(like for instance the $SL(2, R)/U(1)_{-|k|} \times K$),

the super-string analog of:

“the Stringy wave function of the universe”
can be unambiguously defined.

Furthermore the transition probabilities
can be evaluated at the string level.

C.Kounnas, N.Toumbas, J.Troost 07.

Concluding Remark

- Our proposal however goes even beyond the scope of the above statement; namely *towards to* :

The plausible existence of cosmological super-string solutions (Inflationary or not) which are generated dynamically at the quantum string level from a flat classical space-time and spontaneously broken supersymmetry (no-scale radiative-induced cosmology).