

# Non-local SFT Tachyon and Cosmology

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*Patras, 14 June 2007*

Mainly based on JHEP **02** (2007) 041, [hep-th/0605085](#) by I.Ya. Aref'eva, A. K.  
and JHEP **04** (2007) 029, [hep-th/0701103](#) by A. K.

## Plan

- Overview of the problem
  - Cosmological motivations
  - Problems and challenge
  - Why String Field Theory?
- Tachyon spectroscopy
- Infinitely many scalars vs. the non-locality
- Emergence of a phantom
- Real Cosmology
- Comments, Summary and Outlook

## Cosmological motivation

- Data on Ia supernovae
  - Galaxy clusters measurements
  - WMAP
- } Universe exhibits an accelerated expansion

Equation of state:  $p = w\rho$ ,  $w < 0$  — Dark Energy

$$w = -1.06^{+0.13}_{-0.08}$$

Perlmutter et. al., 1999

Riess et. al., 2004

Spergel et. al., 2006

## Theoretical issues

- $w > -1$  — Quintessence models
- $w = -1$  — Cosmological constant
- $w < -1$  — Phantom models

Our universe is known to be homogeneous, isotropic and with high accuracy spatially flat.

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(1 + 3) dimensional spatially flat FRW universe,  
 $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

## Theoretical problems

- Just a cosmological constant has no theoretical explanation so far
- It is difficult to cook a Phantom divide ( $w = -1$ ) crossing.
- $w = \text{const} < -1 \Rightarrow$  Big Rip singularity.
- Phantoms (ghosts) being physical particles look harmful for the theory.
- There are signals that  $w$  changes with time.

## Challenge

We need a dynamical model of Dark Energy which might be able to cross the Phantom divide.

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## Proposal

Derive a scalar field model of Dark Energy starting from initially reliable theory with (probably) non-local interaction for this scalar field.



## SFT ( $p$ -adic) Tachyon

Tachyon effective action ( $\alpha' = 1$ )

$$S = \frac{1}{g_4^2} \int dx \sqrt{-\eta} \left( \frac{1}{2} \Phi \mathcal{F}(\square) \Phi - \frac{1}{p+1} \Phi^{p+1}(x) \right)$$

Cubic Fermionic SFT:  $\mathcal{F}(z) = (\xi^2 z + 1) e^{-\frac{1}{4}z}$ ,  $\xi^2 \approx 0.9556$ ,  $p = 3$

Aref'eva, Belov, A.K.

Medvedev, NP**638** (2002) 3

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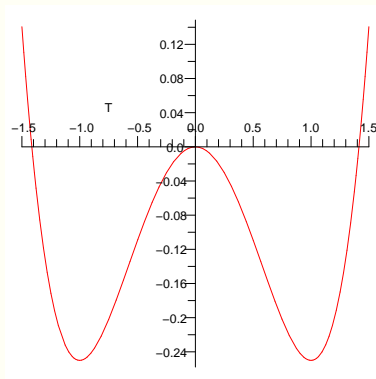
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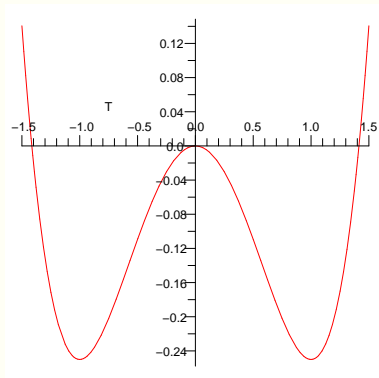
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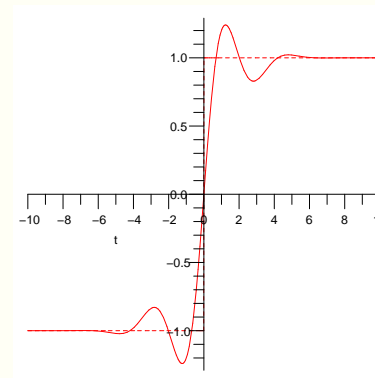
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Rolling solution

Aref'eva, Joukovskaya, A.K., JHEP **09** (2003) 012

## Good points of SFT

- The theory is UV complete
- Quantum computations in  $p$ -adic action ( $\xi = 0$ ) can be carried out analytically up to all orders and the resulting finite effective action can be constructed
- The interaction is non-local thus giving a chance for the Phantom divide crossing

## Minimal coupling to gravity

$$S = \int dx \sqrt{-g} \left( \frac{R}{2\kappa^2} + \frac{1}{g_4^2} \left( \frac{1}{2} \Phi \mathcal{F}(\square_g) \Phi - \frac{1}{p+1} \Phi^{p+1}(x) - \frac{p-1}{2(p+1)} - \tau \right) \right)$$

- $\kappa^2 = 8\pi G = \frac{1}{M_P^2}$
- $\tau$  is a correction to the brane tension dictated by an existence of the rolling solution

Aref'eva, [astro-ph/0410443](#); Aref'eva, A.K., Vernov, [astro-ph/0412619](#)

- $\tau$  is expected to be generated through coupling to closed string excitations
- We introduce  $\Lambda = \frac{\tau}{g_4^2}$  (to be discussed later)

## Late time tachyon spectroscopy

We consider a generalization:

- $\mathcal{F}(z)$  is analytic in  $\mathbb{C}$ , i.e.  $\mathcal{F}(z) = c_n z^n$ ,  $\mathcal{F}(0) = 1$ ,  $c_n \in \mathbb{R}$
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Our expectation:

Tachyon rolls down to the minimum and is expected to stop at the bottom in infinite time

$$\Phi = 1 - \psi \Rightarrow S_\psi = \frac{1}{g_4^2} \int dx \sqrt{-g} \left( \frac{1}{2} \psi \mathcal{F}(\square_g) \psi - \frac{p}{2} \psi^2 - \tau \right)$$

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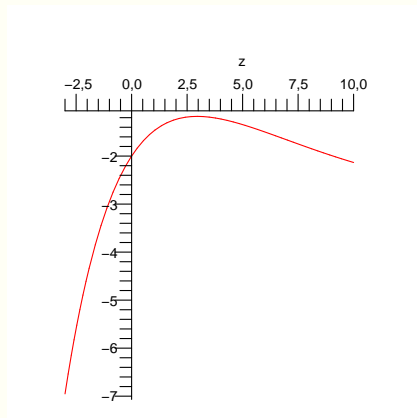
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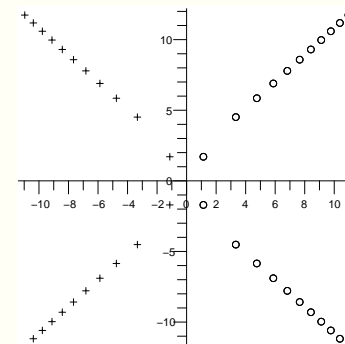
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$\mathcal{F}(z) - p$  in CSSFT

EOM:  $(\mathcal{F} - p)\psi = 0$

Characteristic equation:  
 $\mathcal{F}(\omega^2) = p$



Roots  $\omega$  in CSSFT



## Infinitely many scalars vs. the non-locality

New action

$$S = \frac{1}{g_4^2} \int dx \sqrt{-g} \frac{1}{2} \sum_k (\mathcal{F}'(\omega_k^2) \psi_k (\square_g - \omega_k^2) \psi_k + \mathcal{F}'(\omega_k^{2*}) \bar{\psi}_k (\square_g - \omega_k^{2*}) \bar{\psi}_k)$$

- EOMs are manifestly local and linear.
- Sum over  $k$  is indefinite until  $\mathcal{F}$  is not specified explicitly
- On the solution  $\psi_k = \psi_{k+} + \psi_{k-}$  because  $\square_g$  is the second order differential operator

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- Sum over  $k$  is indefinite until  $\mathcal{F}$  is not specified explicitly
- On the solution  $\psi_k = \psi_{k+} + \psi_{k-}$  because  $\square_g$  is the second order differential operator
- Spectrum and Energy momentum tensor are reproduced in this way
- The construction does not depend on a particular metric
- It is consistent to keep only one mode, say  $\psi_{k+}$  afterwards

## Phantom emergence

Simplest consistent possibility: only single  $\psi_{k+} \neq 0$

We put  $\psi = \alpha + i\beta$ ,  $\omega^2 = M + iN$ ,  $\mathcal{F}'(\omega^2) = x + iy$

Action for fields  $\alpha$  and  $\beta$  becomes

$$S = \frac{1}{g_4^2} \int dx \sqrt{-g} (\alpha(x\mathcal{D} - xM + yN)\alpha - \beta(x\mathcal{D} - xM + yN)\beta - 2\alpha(y\mathcal{D} - yM - xN)\beta).$$

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- For any signs of parameters one normal and one phantom field present in the system
- Only field  $\alpha$  is physical one since  $\alpha = \frac{\psi + \psi^*}{2}$
- $N \neq 0$  because there are no real roots
- $M$ ,  $x$ ,  $y$  are not restricted but at least one of  $x$  or  $y$  is non-zero

This action may serve as a toy model for the tachyon around its vacuum.

**Tachyon at large times must have phantom properties**

## Cosmological scenarios

$$S = \int dx \sqrt{-g} \left( \frac{R}{2\kappa^2} + \frac{1}{g_4^2} \left( \frac{1}{2} \psi \mathcal{F}(\square_g) \psi - \frac{p}{2} \psi^2(x) \right) - \Lambda \right)$$

Using the developed machinery we pass to a local theory with many scalars and under an assumption that only one specific mode  $\psi_{k+} \neq 0$  one has as first approximation

$$\begin{aligned} \psi &= \alpha e^{-rt} \cos(\nu t + \varphi) \\ a &= a_0 e^{H_0 t} + \frac{e^{(H_0 - 2r)t}}{g_4^2 M_P^2} (s \sin(2\nu t) + c \cos(2\nu t)) \end{aligned}$$

$$\text{where } r + i\nu = \frac{3}{2}H_0 \pm \sqrt{\frac{9}{4}H_0^2 - \omega_k^2} \text{ and } H_0 = \sqrt{\frac{\Lambda}{3M_P^2}}$$

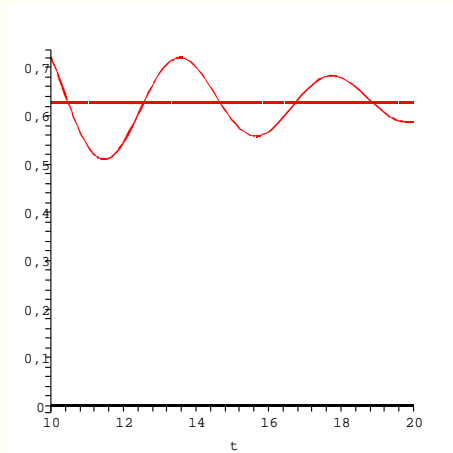
For  $r = H_0/2$  oscillations in  $a(t)$  will not die despite the fact that oscillations in  $\Phi$  vanish.

## Cosmological properties

Generic parameters, i.e. not necessarily  $r = H_0/2$ .

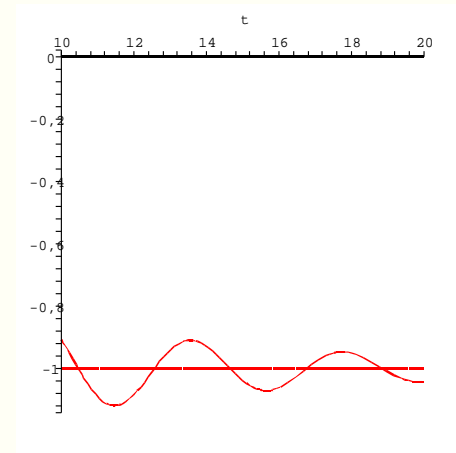
Hubble parameter

$$H = \frac{\dot{a}}{a}$$



Total effective state parameter

$$w = -1 - \frac{2 \dot{H}}{3 H^2}$$



Quintessence and Phantom phases change one each other.

No Big Rip singularity

Crossing of the phantom divide

## Comments on $\Lambda$

- $\Lambda$  is important to have non-trivial solutions.
- On the other hand,  $\Lambda \neq 0$  is the must for the existence of a rolling solution.
- $\Lambda$  is expected to be generated dynamically through the open-closed strings coupling.
- Moreover, its value is estimated to be a realistic one, e.g. giving the Hubble parameter  $\sim 10^{-60} M_P$ .

## Summary

- Non-local action with a general operator  $\mathcal{F}$  is analyzed and a local formulation for a linearization near a non-perturbative vacuum is given.
- The energy and pressure can be easily computed for a general function  $\mathcal{F}$  without specifying its explicit form as well as an arbitrary metric.
- It is shown that tachyon scalar field generates a crossing of the phantom divide in the cosmological constant background. This crossing is periodic one and a condition of non-vanishing oscillations is formulated.
- There is no Big Rip singularity in the model.



## Further directions

- Coupling to dilaton. A.K., in progress
- Coupling to vector field etc.
- Proof of stability of found solutions
- Cosmological perturbations of theories with infinitely many derivatives
- Numeric and may be analytic solution to full equations
- As well as many other questions

**Thank you!**