A string theory realization of Kramers-Wannier duality

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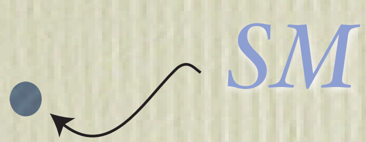
$$\Lambda(\varphi^i) \neq 0$$

φ^i *are the "**moduli**"*

A successful mechanism of supersymmetry breaking is expected to

- *result in a stable (enough) configuration*
- *reproduce the physics of the Standard Model*
- *not generate a (sizeable) cosmological constant*



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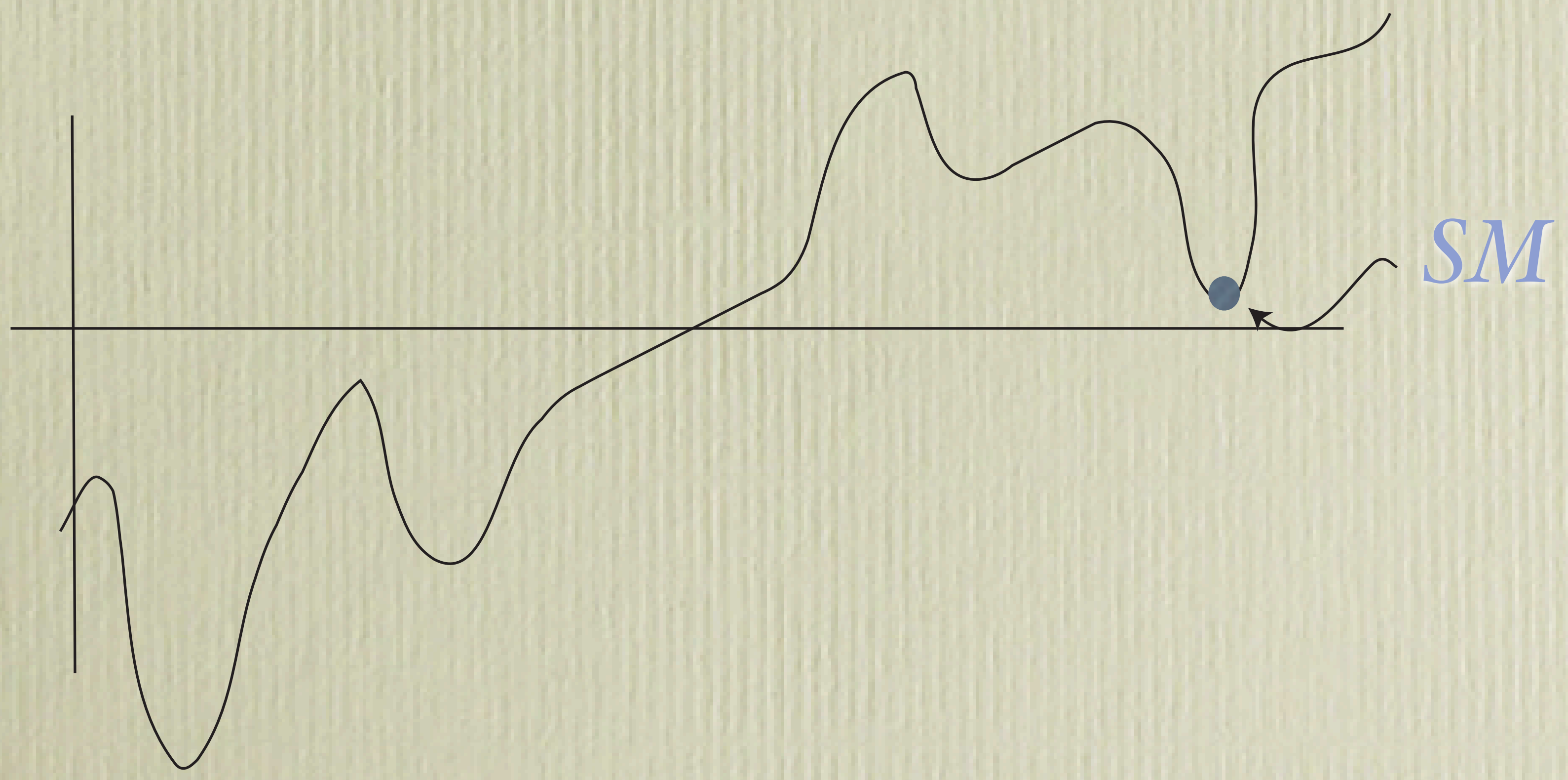

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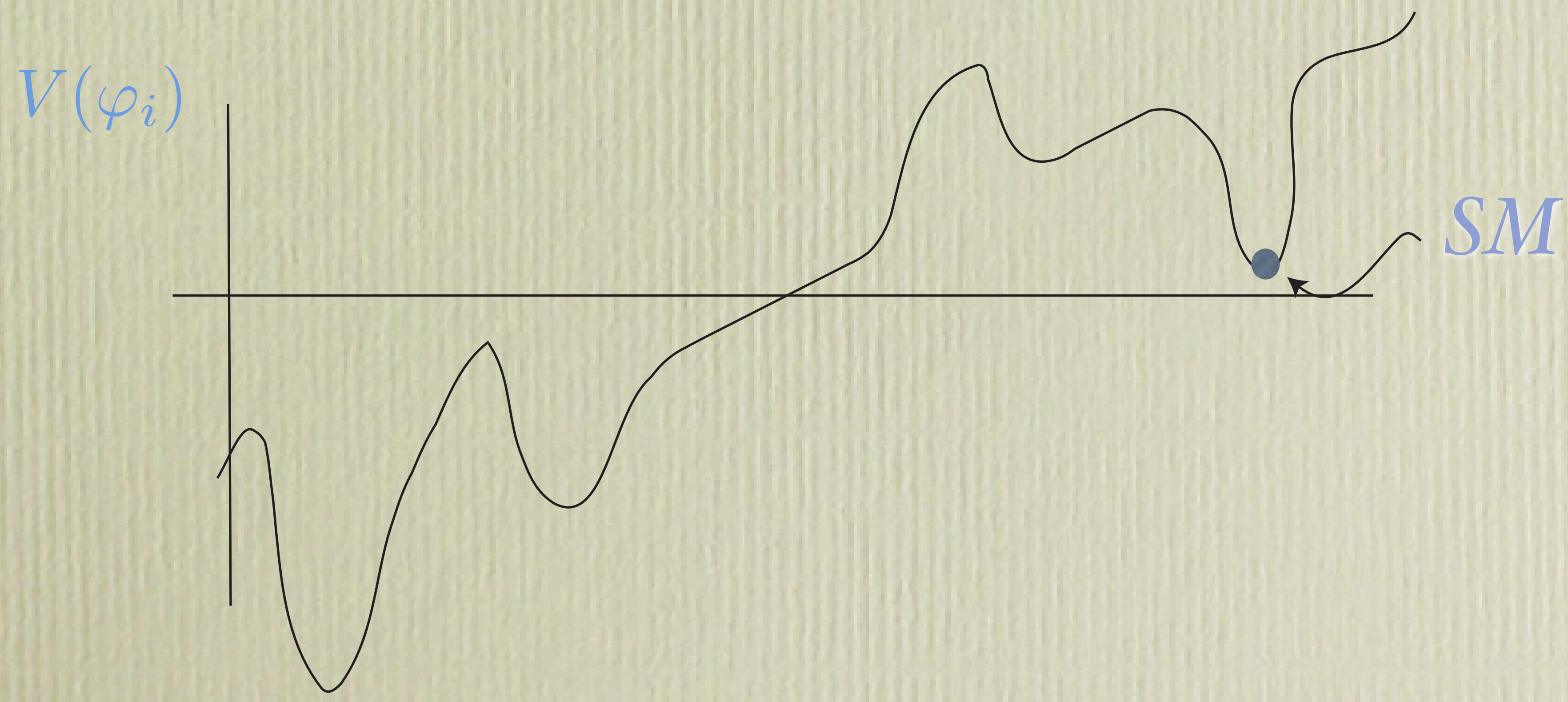

Ising Model in two dimensions

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- *Low temperature: ferromagnetic phase (globally magnetized); order*
- *High temperature: paramagnetic phase (globally disordered); disorder*
- *Low T and high T phases are related by duality*
  H. A. Kramers & G. H. Wannier, Phys.Rev.60:252-262,1941
- *At the fixed point of the duality symmetry there is a second order phase transition, which can be crossed by smoothly changing the temperature*

Non-zero T in field theory

- Temperature $T = \frac{1}{R}$ where R is the radius of a compact, Eucledian dimension
- Bosons periodic and fermions anti-periodic around the compact T-direction
- Non-zero temperature breaks supersymmetry
- Typical four-dimensional field theories do not exhibit an order-disorder duality

Scherk-Schwarz deformations in Field Theory

*useful to **spontaneously break** local symmetries*
~via dimensional reduction~

- J. Scherk & J. H. Schwarz, Nucl.Phys.B153:61-88,1979

$$\mathscr{M}_{1,d} \to \mathscr{M}_{1,d-1} \otimes \mathscr{S}^1$$

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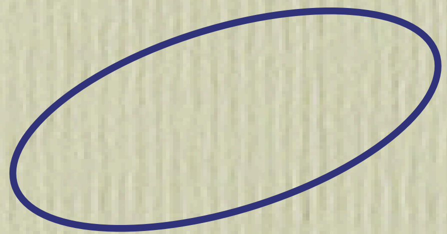

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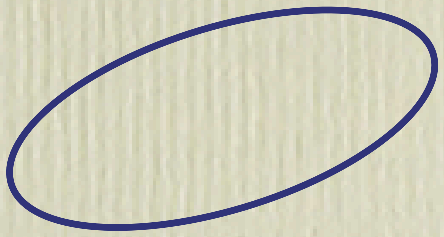

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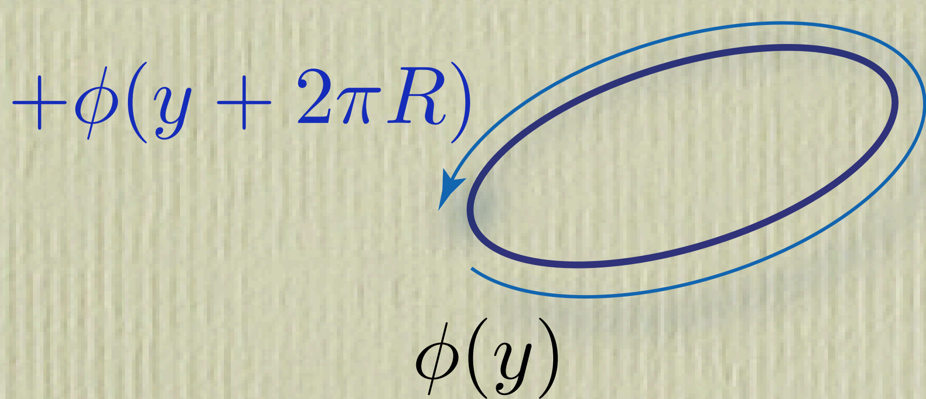

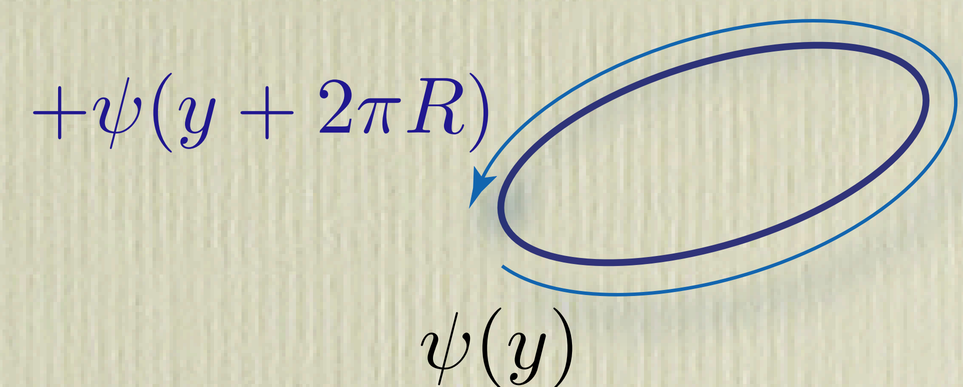

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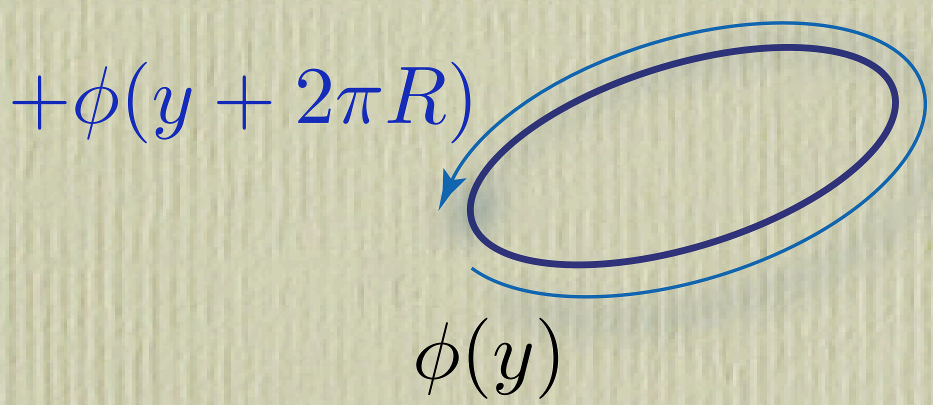

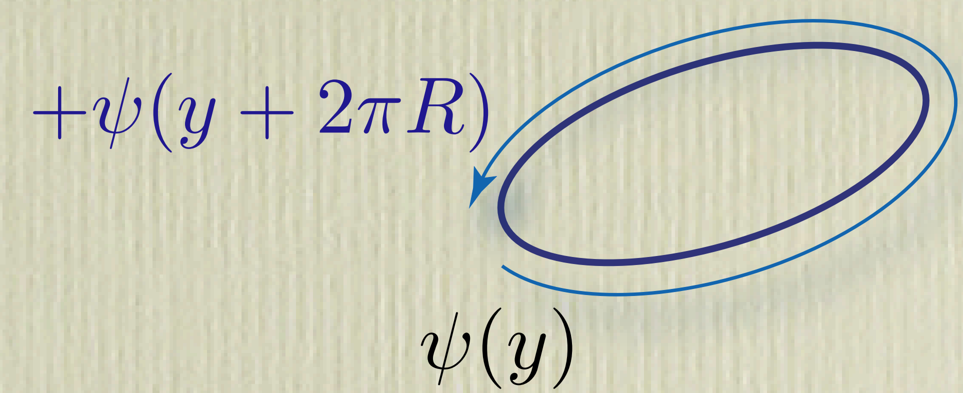

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$$\psi(y) = \sum_n \psi_x e^{iny/R}$$

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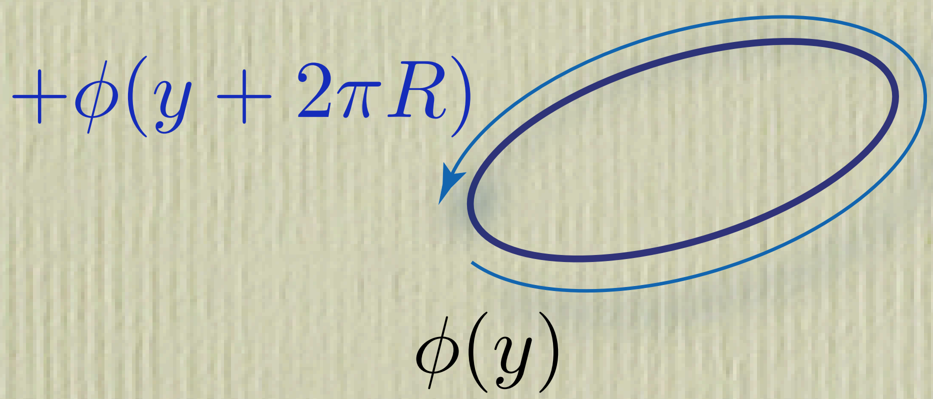

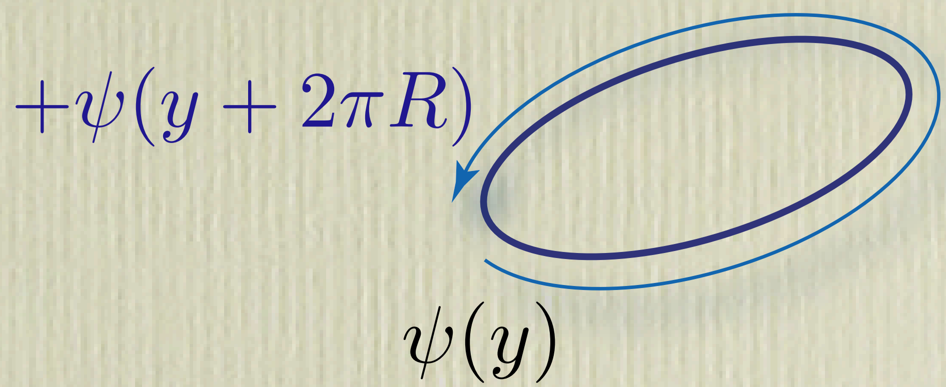

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Standard Kaluza-Klein

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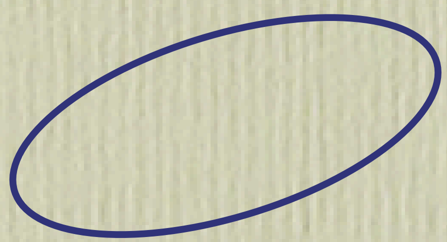

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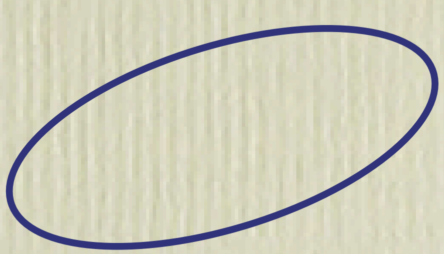

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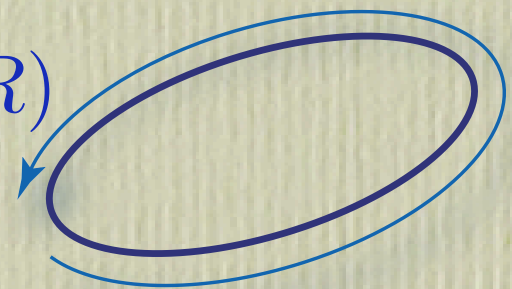

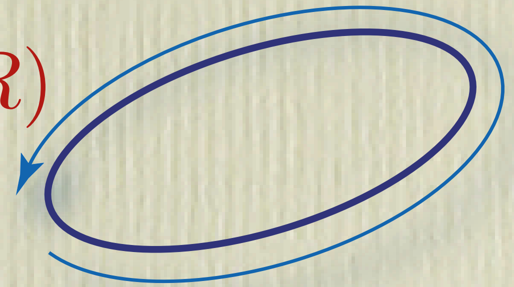

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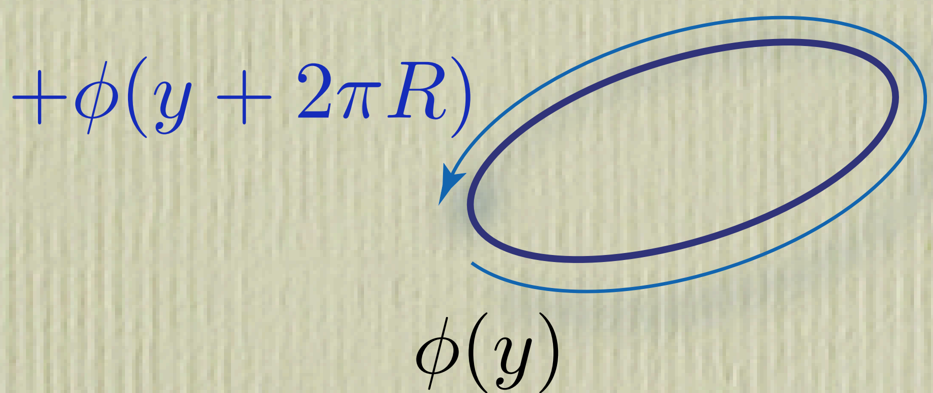

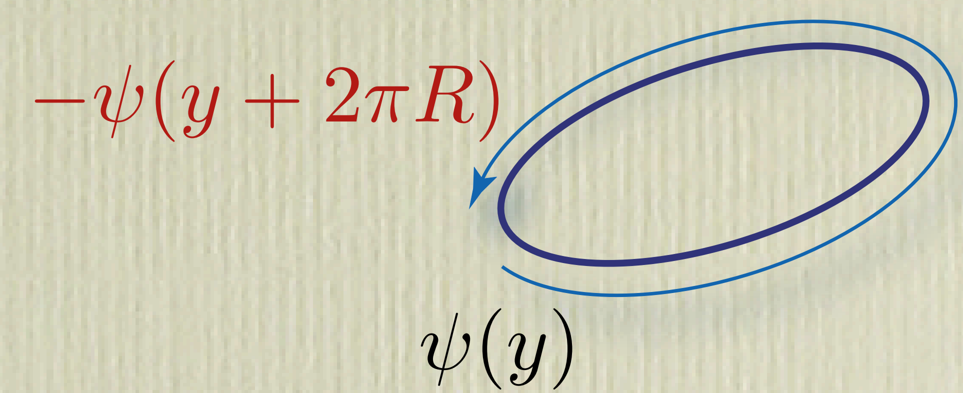

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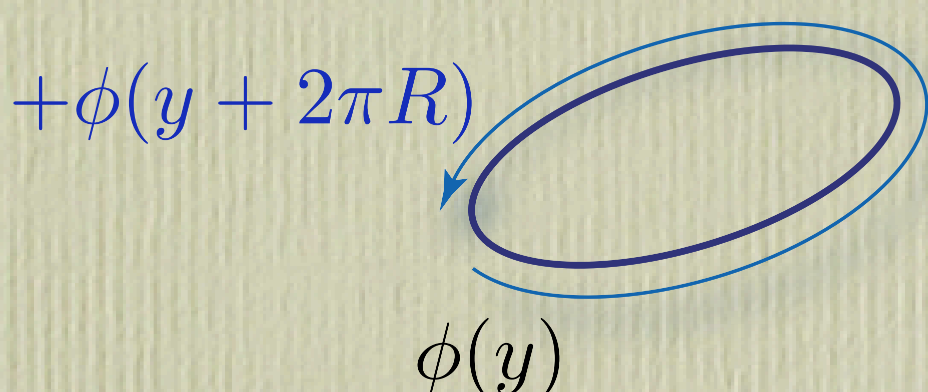

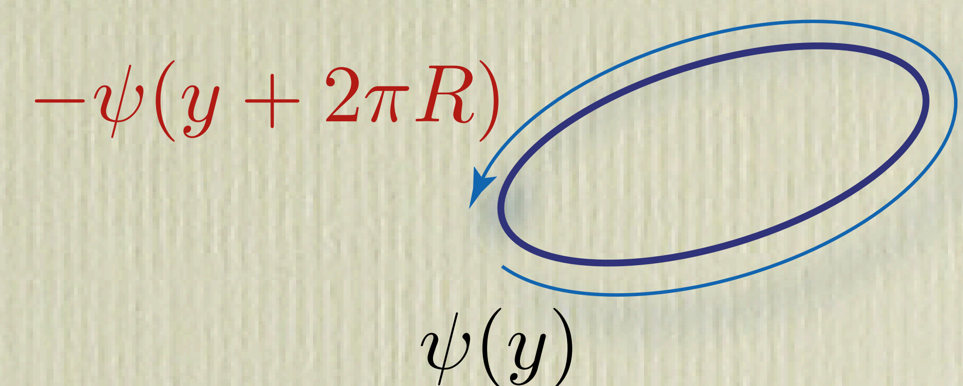

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$$\Lambda \sim 1/R^4 \qquad \Delta M \sim 1/R$$

String Theory at "finite Temperature"

$$\{\text{Het, II}\}/\{(-1)^F \cdot \delta\}$$

F = *Space-time fermion number*

$\delta = (-1)^m$ *"momentum twist"*

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Comapctified on a T^d

Momentum/Winding twists

A lot of information is contained in the
torus partition function

$$\mathcal{T} = \frac{1}{2}\left[|V_8 - S_8|^2 \Lambda_{\vec{m},\vec{n}} + |V_8 + S_8|^2 (-1)^{\vec{m}\cdot\epsilon} \Lambda_{\vec{m},\vec{n}}\right] + \frac{1}{2}\left[|O_8 - C_8|^2 \Lambda_{\vec{m}+\frac{1}{2},\vec{n}} + |O_8 + C_8|^2 (-1)^{\vec{m}\cdot\epsilon} \Lambda_{\vec{m}+\frac{1}{2},\vec{n}}\right]$$

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The potential is simply the integral over the fundamental domain

- *Modular invariant*
- *Non-Supersymmetric*

$$\int_{\mathcal{F}} (1 + S + ST) f(z) = \int_{\tilde{\mathcal{F}}} f(z)$$

$$\tilde{\mathcal{F}} = (1 + S + ST)\mathcal{F}$$

$\mathcal{F}$ *is the fundamental domain*

$$V(\rho) \sim -(-1)^{\vec{m}\cdot\epsilon} \int_{\tilde{\mathcal{F}}} \frac{d\tau_2}{\tau_2^{6-d/2}} e^{-2\pi\tau_2 m^2(\rho)}$$

- *For* $m^2(\rho) > 0$ *the integral is finite as* $\tau_2 \longrightarrow \infty$
- *For* $m^2(\rho) \leq 0$ *there are divergences in the IR*

The (twisted) mass formula for radial moduli

$$m^2(\rho_i) = \frac{1}{2}\sum_{i=1}^{d}\left[(n_i + \frac{1}{2})\rho_i - m_i\frac{1}{\rho_i}\right]^2 + \left[\sum_{i=1}^{d}\left(n_i + \frac{1}{2}\right)m_i - 1\right]$$

$$+N_X + N_\psi + \tilde{N}_X + \tilde{N}_\psi,$$

(for $\rho_i = \rho, n_i = m_i = N_X = \tilde{N}_X = N_\psi = \tilde{N}_\psi = 0$)

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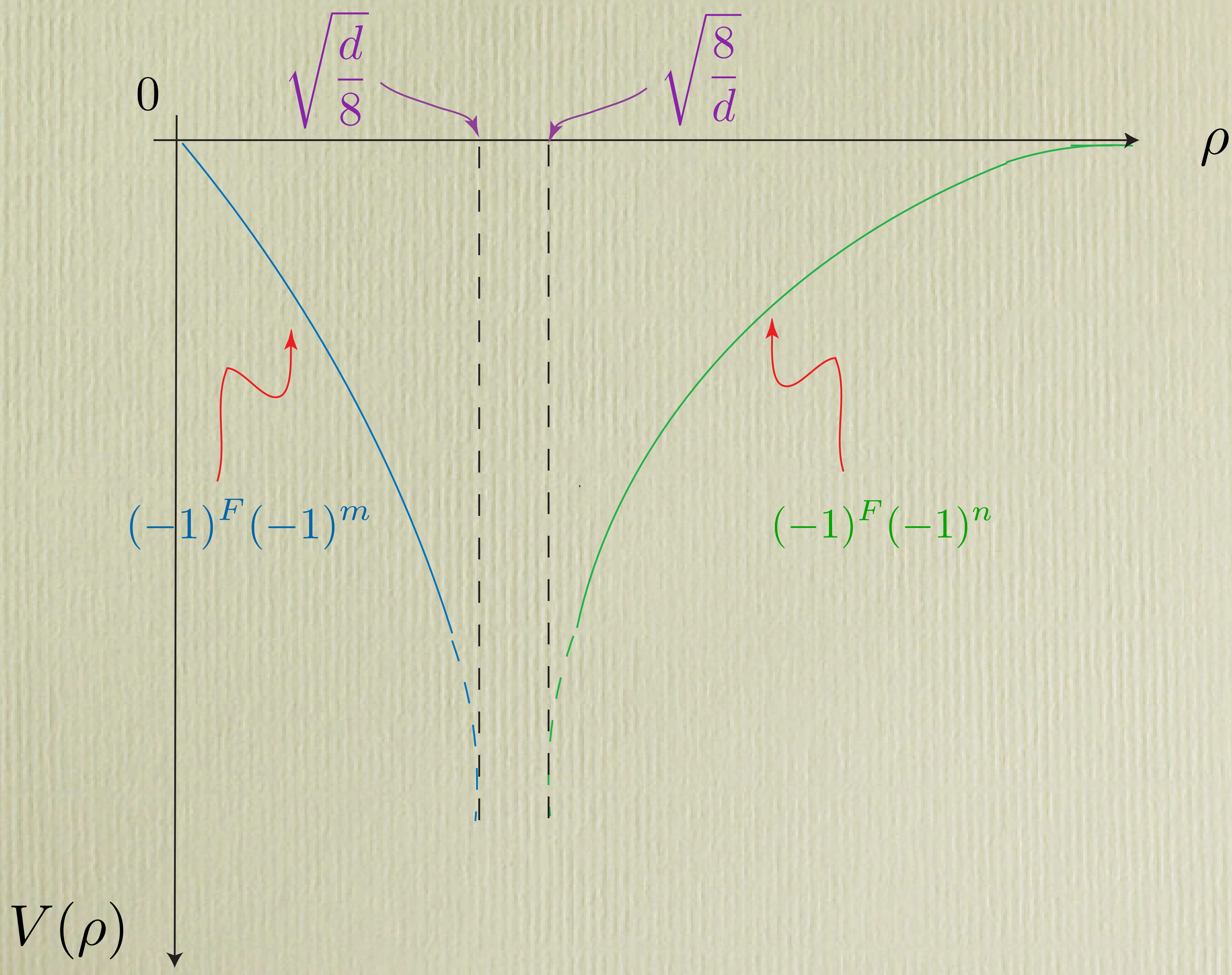
- R. Hagedorn, Nuovo Cim.Suppl.3:147-186,1965
- J. J. Atick & E. Witten, Nucl.Phys.B310:291-334,1988

The Hagedorn phase transition



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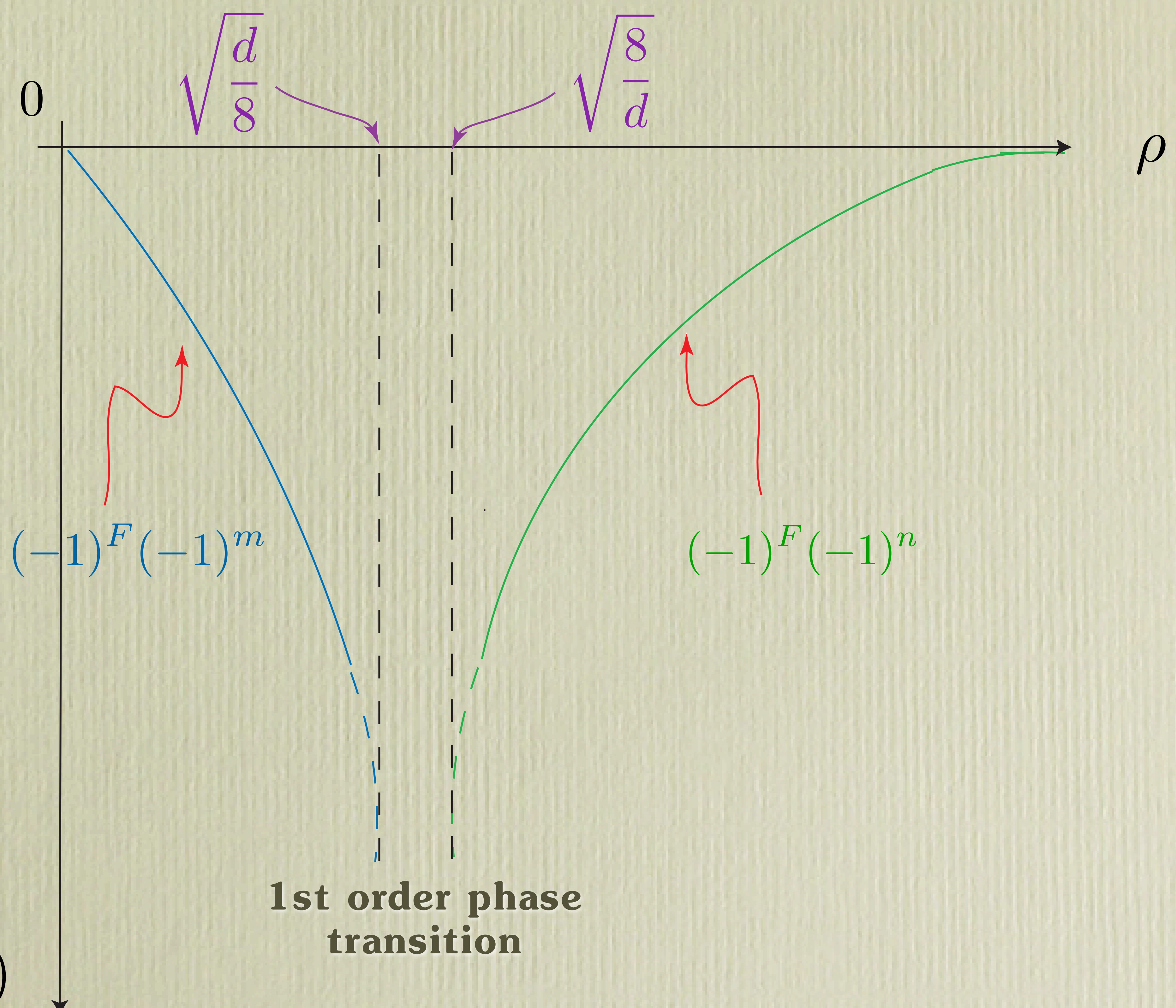

The self T-dual twist

The simplest self-dual twist is $\delta = (-1)^{m+n}$

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The symmetries now are different

- *Modular invariant*
- *Non-Supersymmetric*
- *T-duality invariant*

The mass formula

$$m^2(G) = \frac{1}{2}\mathbf{m}^T G^{-1}\mathbf{m} + \frac{1}{2}\mathbf{n}^T G\mathbf{n} - 2 + \eta + N_X + N_\psi + \tilde{N}_X + \tilde{N}_\psi$$

Unntwisted sector: $\eta = 0,\ m, n \in \text{integers}$

Twisted sector: $\eta = 1,\ m, n \in 1/2\,\text{integers}$

- *Extra massless states at the self-dual radius in the untwisted sector; enhanced gauge symmetry*

- *In the twisted sector all states are massive for generic radius, and*

- *At the self dual radius, tachyons appear only for* $d < 4$

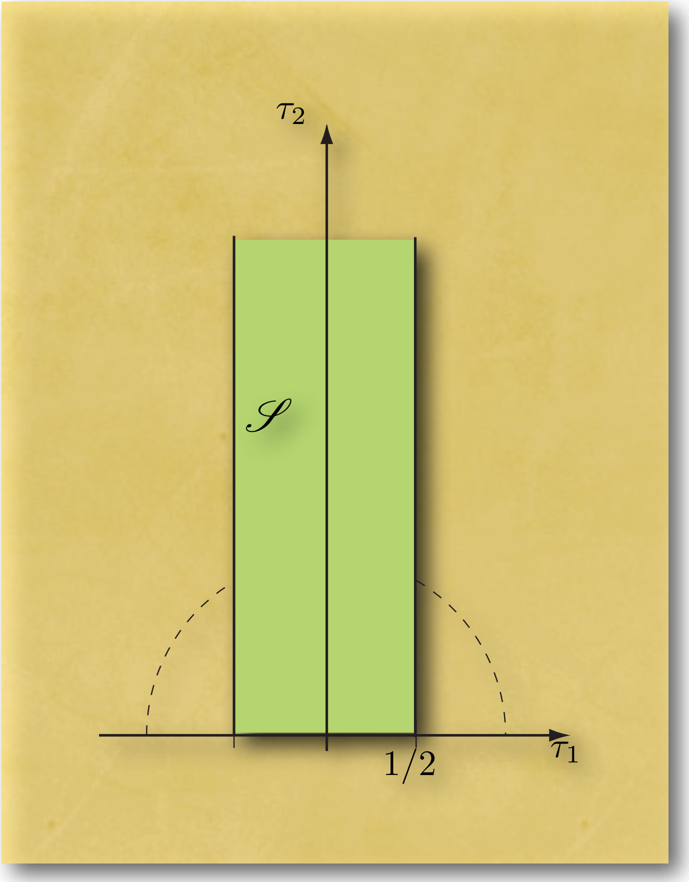
Unfolding the fundamental domain

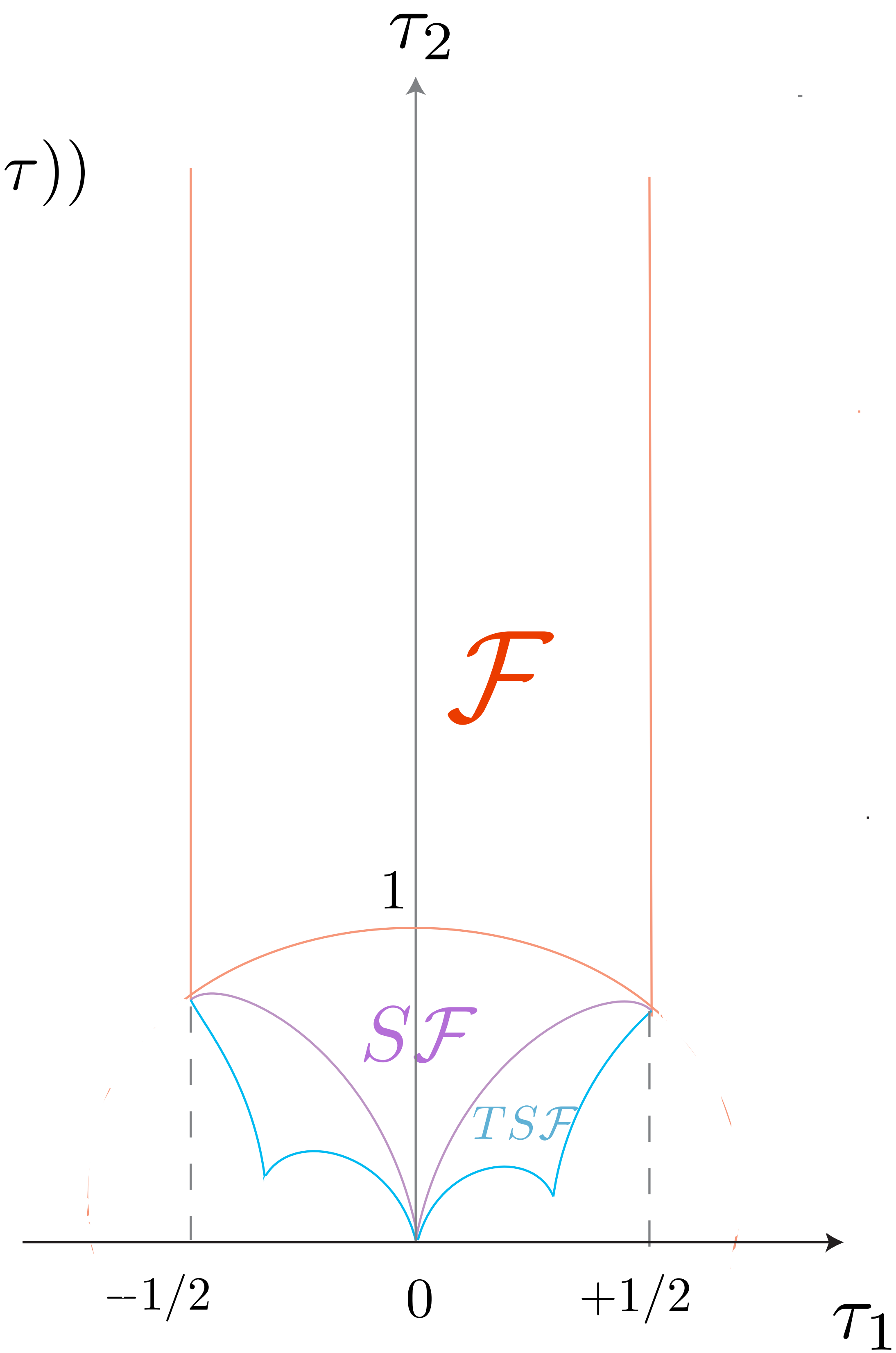
$$\int_{\mathcal{F}}(1+S+ST)f(\frac{\Gamma_0[2]}{T}(\tau)) = \int_{\tilde{\mathcal{F}}} f(\frac{\Gamma_0[2]}{T}(\tau))$$

$$= \int_{\frac{\Gamma_0[2]}{T}\tilde{\mathcal{F}}} f(\tau) = \int_{S^+} f(\tau)$$

$$S: \ \tau \longrightarrow -\frac{1}{\tau}$$

$$T: \ \tau_1 \longrightarrow \tau_1 + 1$$





The one-loop potential

$$V = -\mathcal{N}\int_0^\infty \frac{d\tau_2}{\tau_2^6}\int_{-1/2}^{+1/2} d\tau_1 \sum_{N,N'=0} d_N d_{N'} e^{2i\pi(N-N')\tau_1} e^{2\pi(N+N')\tau_2}$$

$$\times \sum_{\{l_i\}} e^{-\frac{\pi T_2}{4\tau_2 U_2}[(\tau_1-U_1)^2+(\tau_2-U_2)^2]}$$

$$\mathcal{N} = \frac{v_{10-d}\sqrt{detG}}{16(4\pi\alpha')^{(10-d)/2}(\alpha')^{d/2}}$$

$$T = T_1 + iT_2 = -\hat{B}_{12} + i\sqrt{\hat{G}} \qquad \hat{G} = \mathbf{M}G\mathbf{M}^T$$

$$U = U_1 + iU_2 = \frac{1}{\hat{G}_{11}}[-\hat{G}_{12} + i\sqrt{\hat{G}}] \qquad \hat{B} = \mathbf{M}B\mathbf{M}^T$$

Decomposing under $\Gamma_0[2]$

- *The degenerate orbit:* $\mathbf{M} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 2l_1+1 & 2l_2+1 & \cdots & 2l_d+1 \end{pmatrix}$

- *The non-degenerate orbit:* $\mathbf{M} = \begin{pmatrix} 2n_1 & 2n_2 & \cdots & 2n_d \\ 2l_1+1 & 2l_2+1 & \cdots & 2l_d+1 \end{pmatrix}$

For large $\sqrt{detG}$ we can safely interchange sums and integrals. Perform a Poisson resummation over the momenta and then do the τ_1 integral, which enforces level mathcing.

The degenerate orbit

$$I_0^{deg} \sim -\frac{1}{R^{10-d}} \sum_{\{l_i\}} \frac{1}{\left[\sum_i (2l_i + 1)^2\right]^5}$$

$$I_N^{deg} \sim -\frac{1}{R^6} \sum_{N=1}^{\infty} \sum_{\{l_i\}} \frac{N^5 d_N^2}{x^5} K_5\left(\frac{R}{\sqrt{\alpha'}} x\right)$$

$$x = 2\pi \sqrt{N \sum_i^d (2l_i + 1)^2}$$

For large N, $d_N^2 \sim e^{2\pi\sqrt{8N}}$ *and therefore*

$$d_N^2 K_5 \sim e^{-2\pi\sqrt{N}\left(\sqrt{d}\,\frac{R}{\alpha'} - \sqrt{8}\right)}$$

The expression is well defined for $\rho^2 = \dfrac{R^2}{\alpha'} > \dfrac{8}{d}$

i. e. as long as tachyons are absent

By Poisson resumming over the windings and computing, one can conclude that the region

$$\frac{d}{8} < \rho^2 < \frac{8}{d}$$

can not be probed analytically

The non-degenerate orbit

$$I_N^{non-deg} = -\mathcal{N}\sqrt{\frac{U_2}{4T_2}} \sum_{N,N'=0} d_N d_{N'} \sum_{\{n_i,l_i\}} e^{\frac{i\pi\overline{T}}{2}} e^{2\pi i(N-N')U_1} e^{\frac{\pi T_2}{2}} \cdot I_2$$

$$I_2 = \frac{2^{-\frac{7}{2}}}{c_2^{\frac{9}{2}}} y^{\frac{9}{2}} K_{\frac{9}{2}}(y) \qquad y = 2\sqrt{c_1 c_2}$$

$$c_1 = 2\pi(N+N') + \frac{\pi T_2}{4U_2} + \frac{\pi T_2}{U_2}(N-N'), \qquad c_2 = \frac{\pi T_2 U_2}{4}$$

The non-degenerate orbit is exponetially suppressed with respect to the degenertate orbit

Minimizing the potential

$$V(\rho_i) = -(-1)^{(m+n)\cdot\epsilon} \int_{\mathcal{F}} \frac{d\tau_2}{\tau_2^{6-d/2}} e^{-2\pi\tau_2 m^2(\rho_i)}$$

$$\frac{\partial V_2(\rho_i)}{\partial \rho_{j_1}} = (-1)^{(m+n)\cdot\epsilon} \int_{\mathcal{F}} \frac{d\tau_2}{\tau_2^{6-d/2}} (2\pi\tau_2) e^{-2\pi\tau_2 m^2(\rho_i)} \frac{\partial m^2(\rho_i)}{\partial \rho_{j_1}}$$

Extremization condition

$$\frac{\partial m^2(\rho_i)}{\partial \rho_{j_1}} = \left[\sum_i \left(|n_i|\rho_i - |m_i|\frac{1}{\rho_i}\right)\right]\left[|n_{j_1}| + \frac{|m_{j_1}|}{\rho_{j_1}^2}\right] = 0$$

Unique solution: $\rho_i = 1 \longrightarrow$ *Global minimum*

The potential around the self-dual point

$$\mathcal{F} = \Lambda + \lambda$$

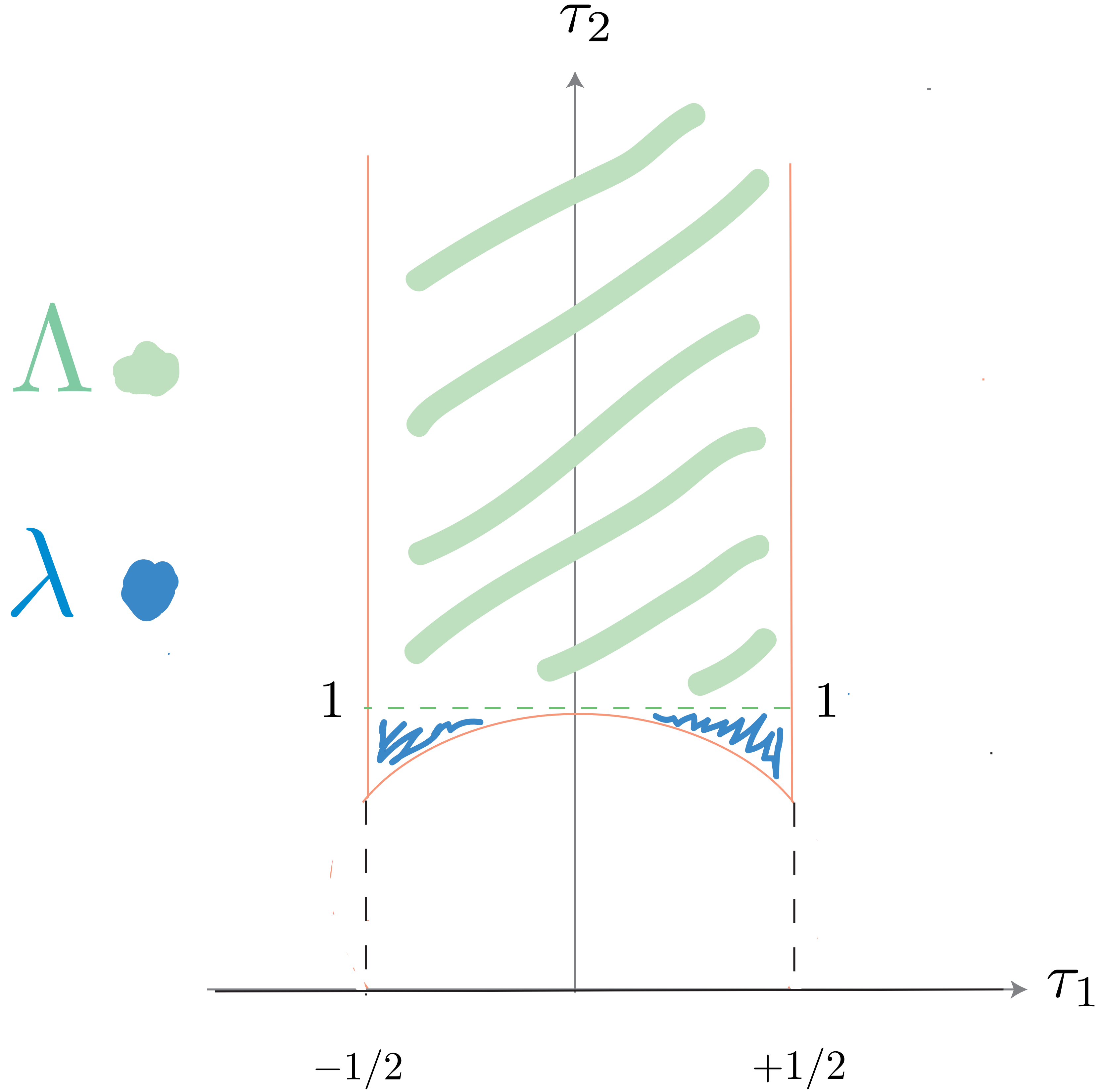
$$V \equiv I_{\mathcal{F}} = I_\Lambda + I_\lambda, \qquad\qquad I_\lambda << I_\Lambda$$

Compute I_Λ *numerically;*
The largest contribution comes from small N, m, n.

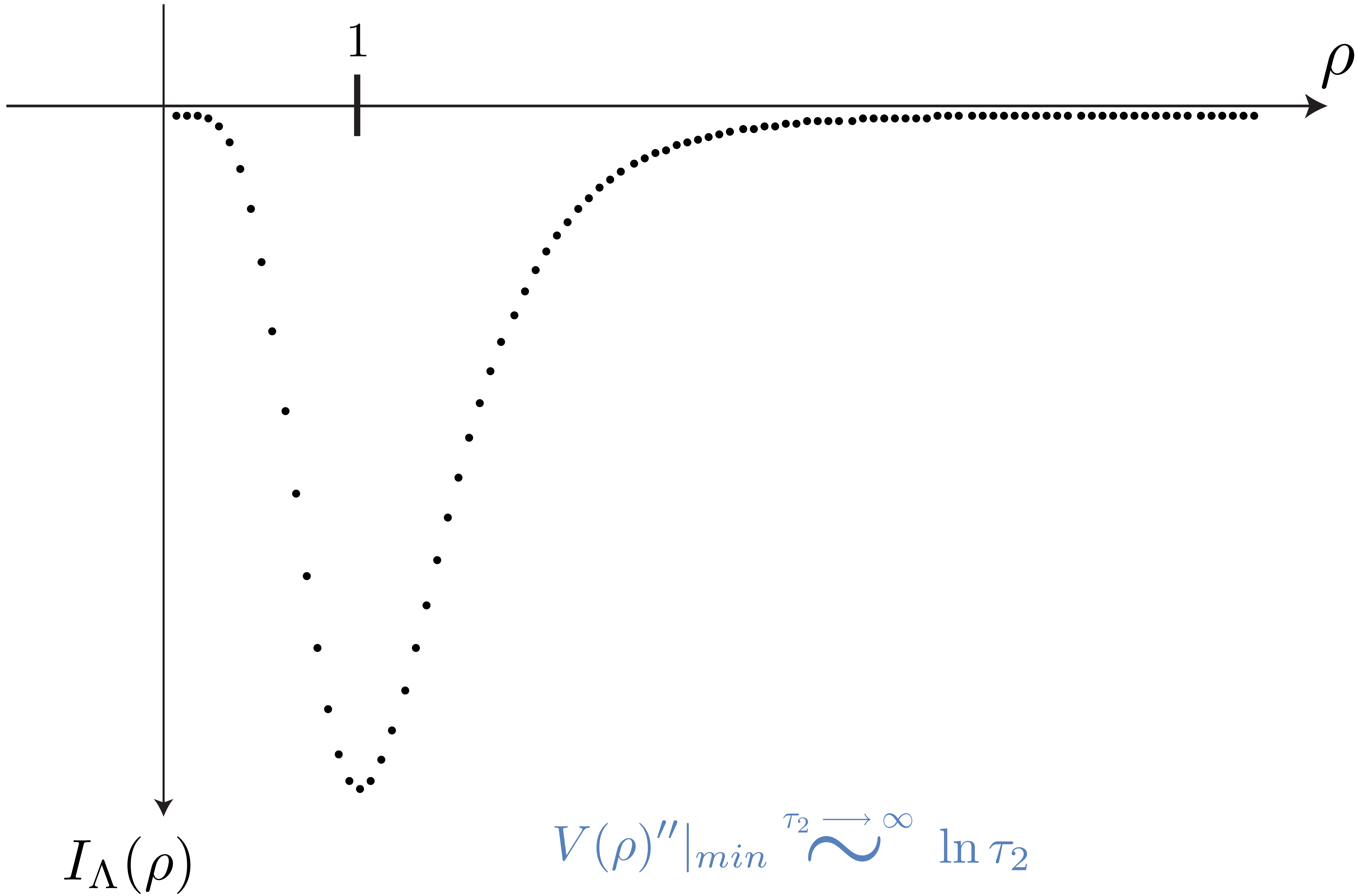
$$I_\Lambda = \sum_{N=0}^{\infty} \sum_{\vec{m},\vec{n}} (-1)^{(\vec{m}+\vec{n})\cdot\epsilon} d_N d_{N+\vec{m}\cdot\vec{n}} \frac{1}{6} \left[\left(2 + A(A-1)\right) e^{-A} + A^3 Ei(-A) \right]$$

$$A = 2\pi \left[2N + \vec{m}\cdot\vec{n} + \frac{1}{2} \left((\frac{\vec{m}}{\rho})^2 + (\vec{n}\rho)^2 \right) \right]$$

Breaking up the fundamental domain

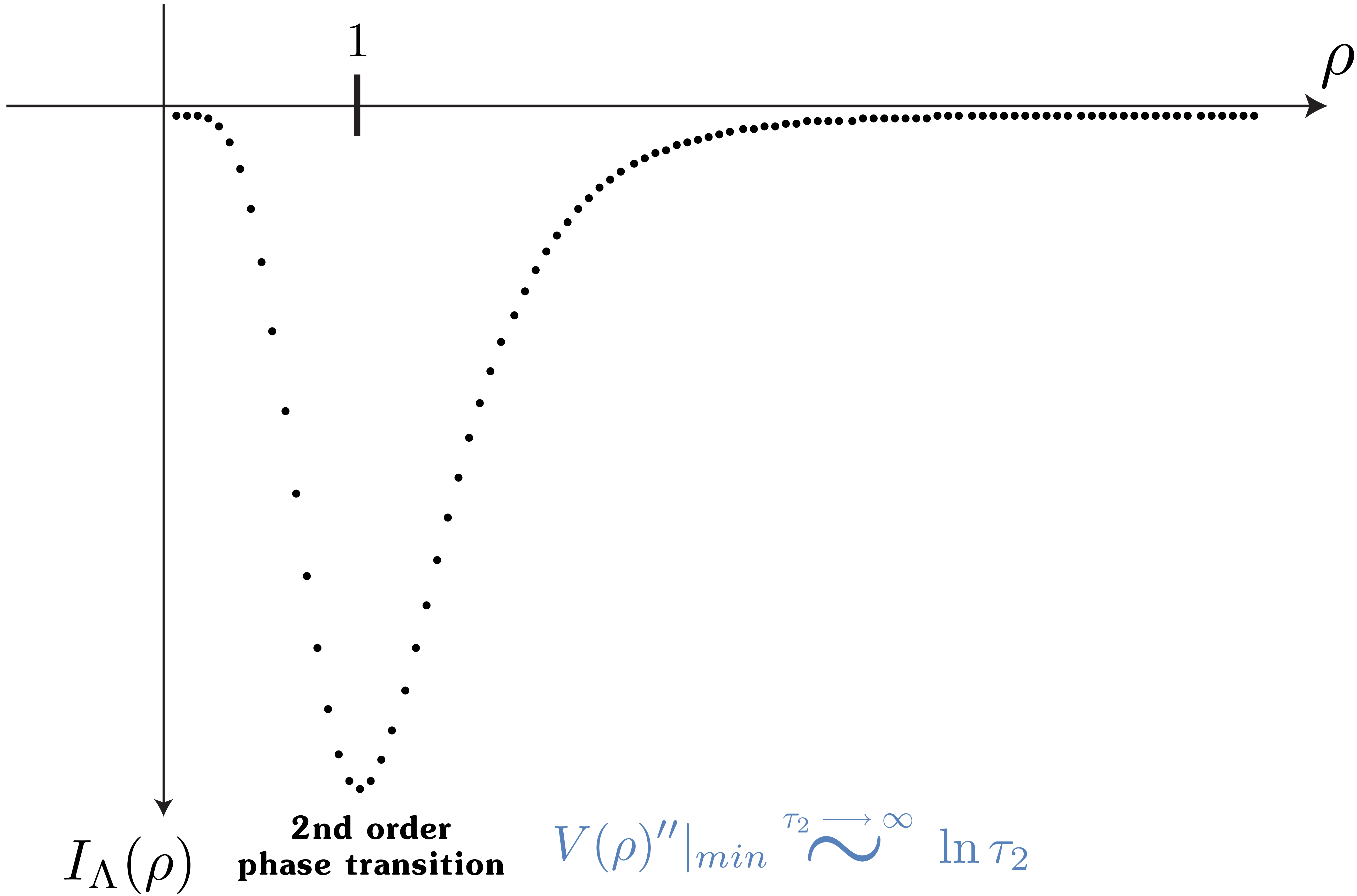


The numerical computation

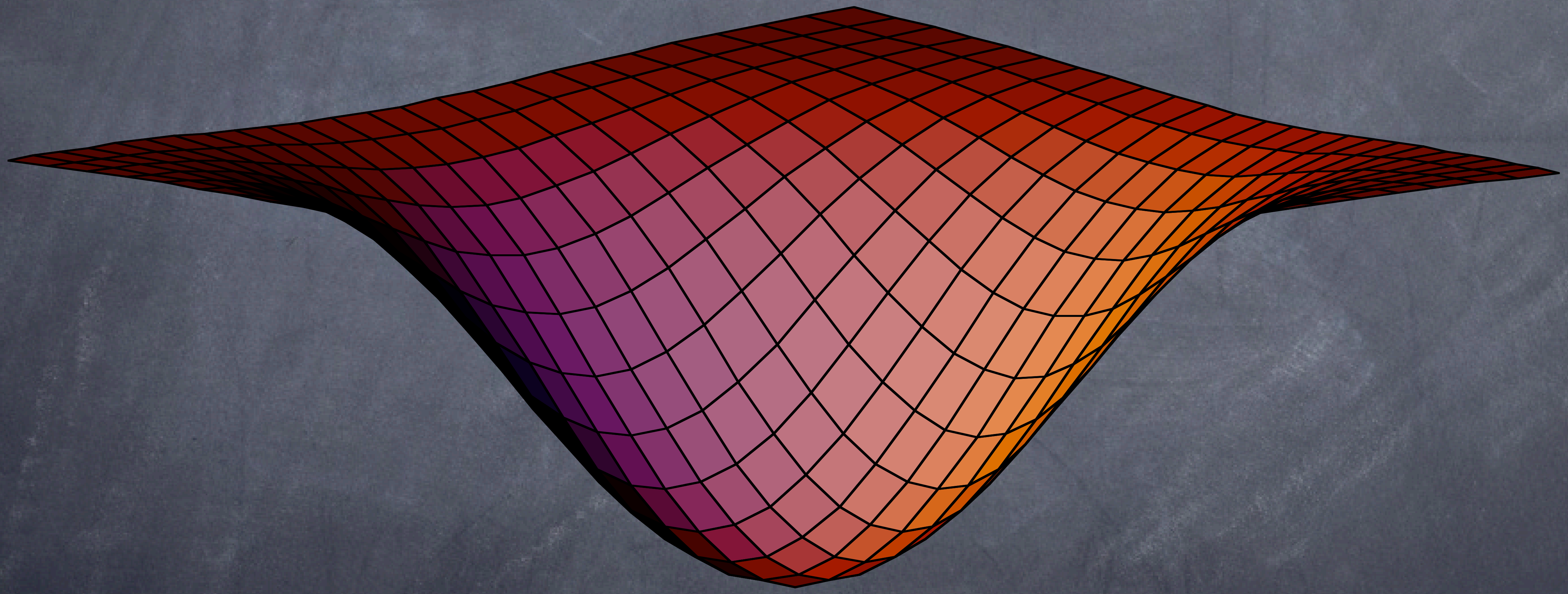


$$V(\rho)''|_{min} \overset{\tau_2 \longrightarrow \infty}{\sim} \ln \tau_2$$

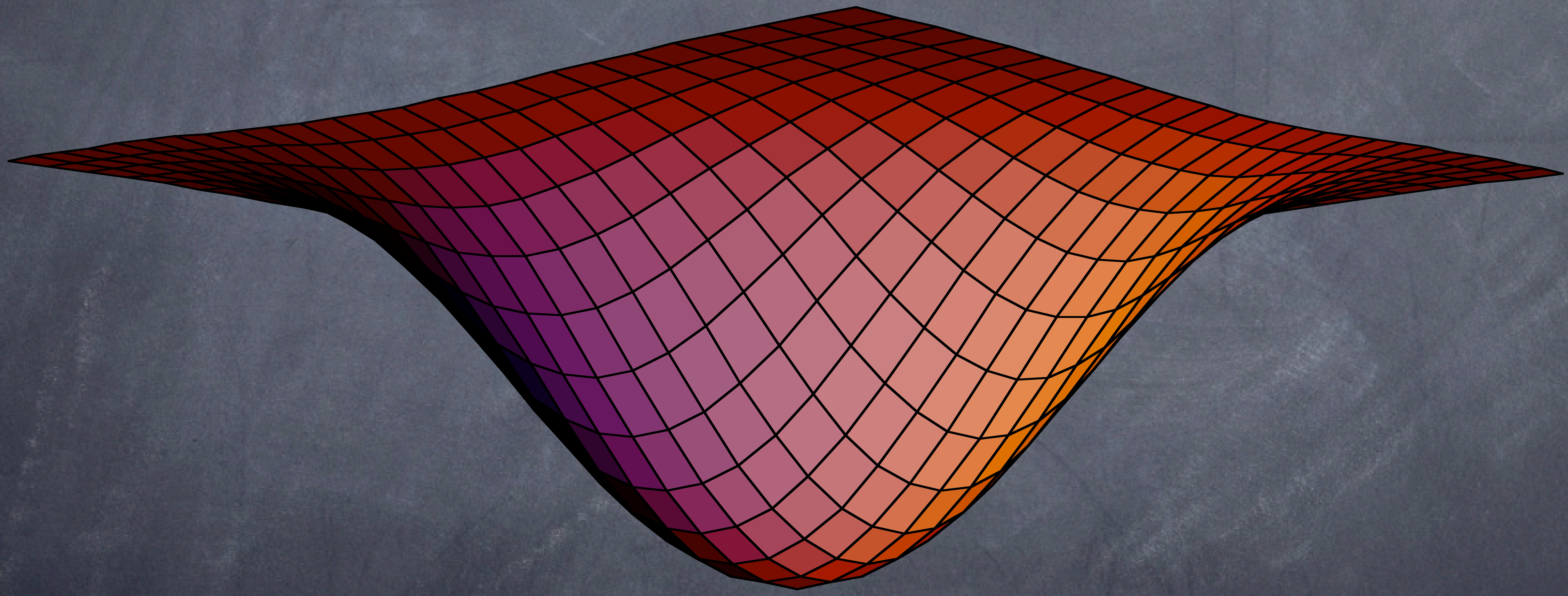
The numerical computation



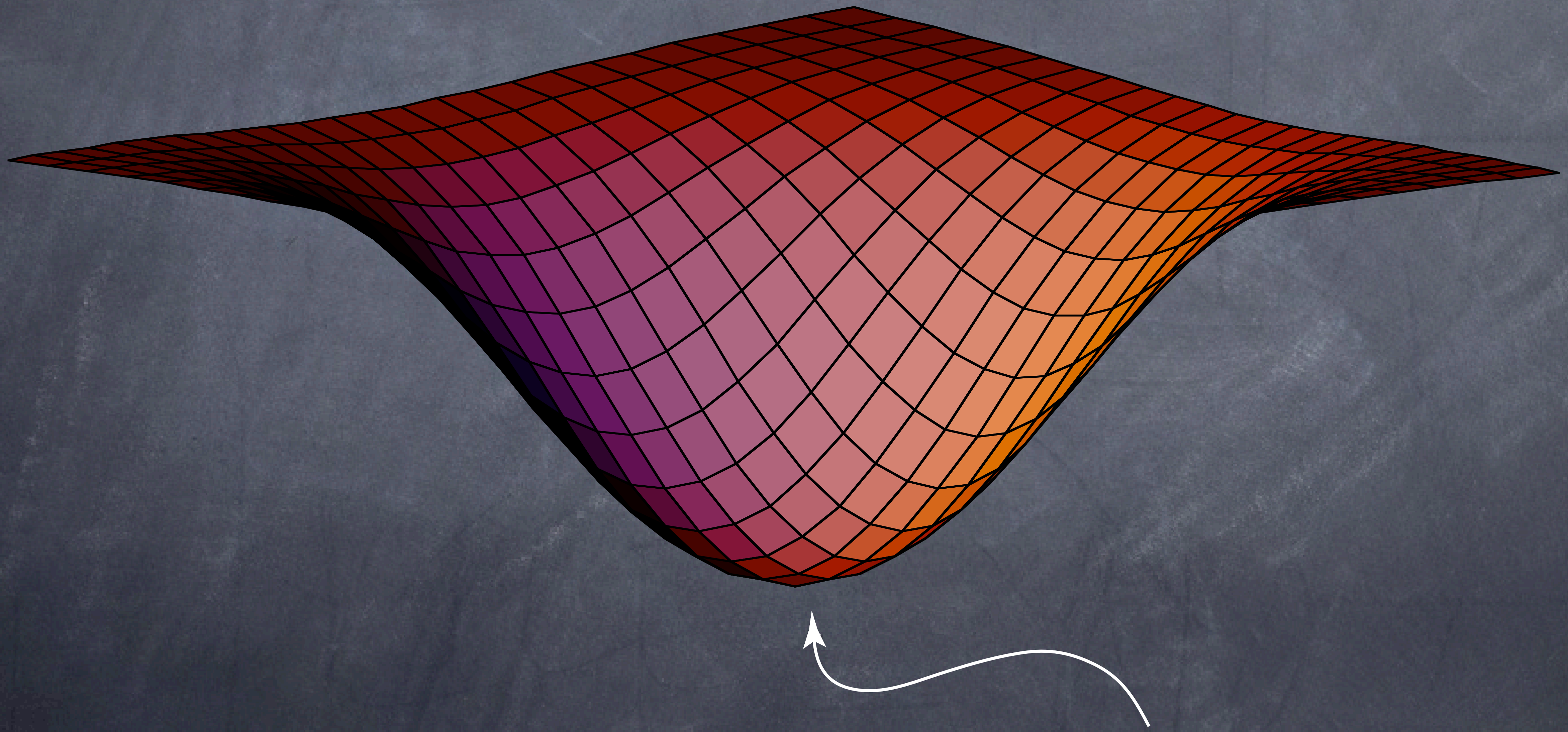
$$V(\rho)''|_{min} \overset{\tau_2 \longrightarrow \infty}{\sim} \ln \tau_2$$



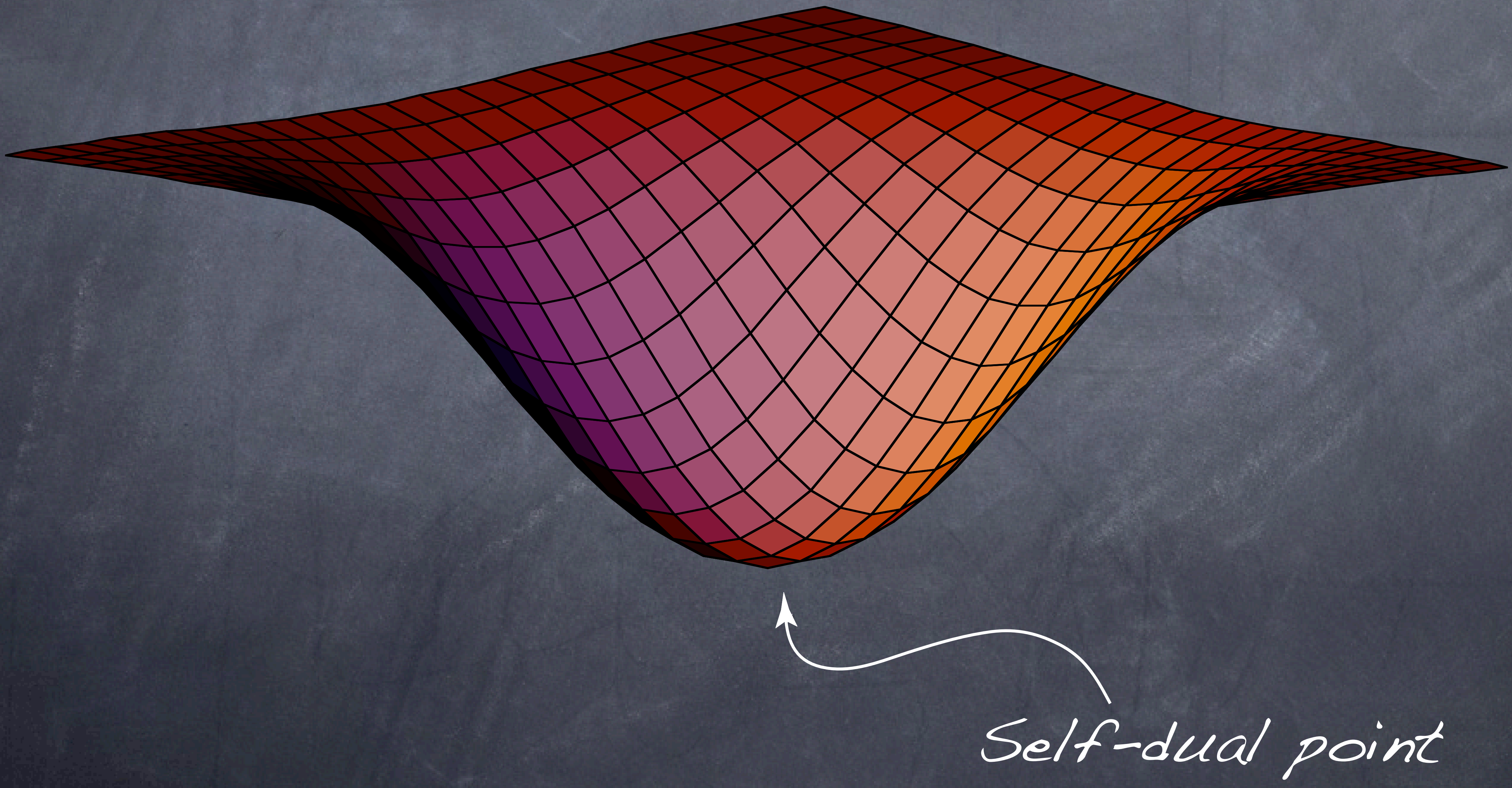
Potential for only radial moduli



Potential for only radial moduli



Potential for only radial moduli

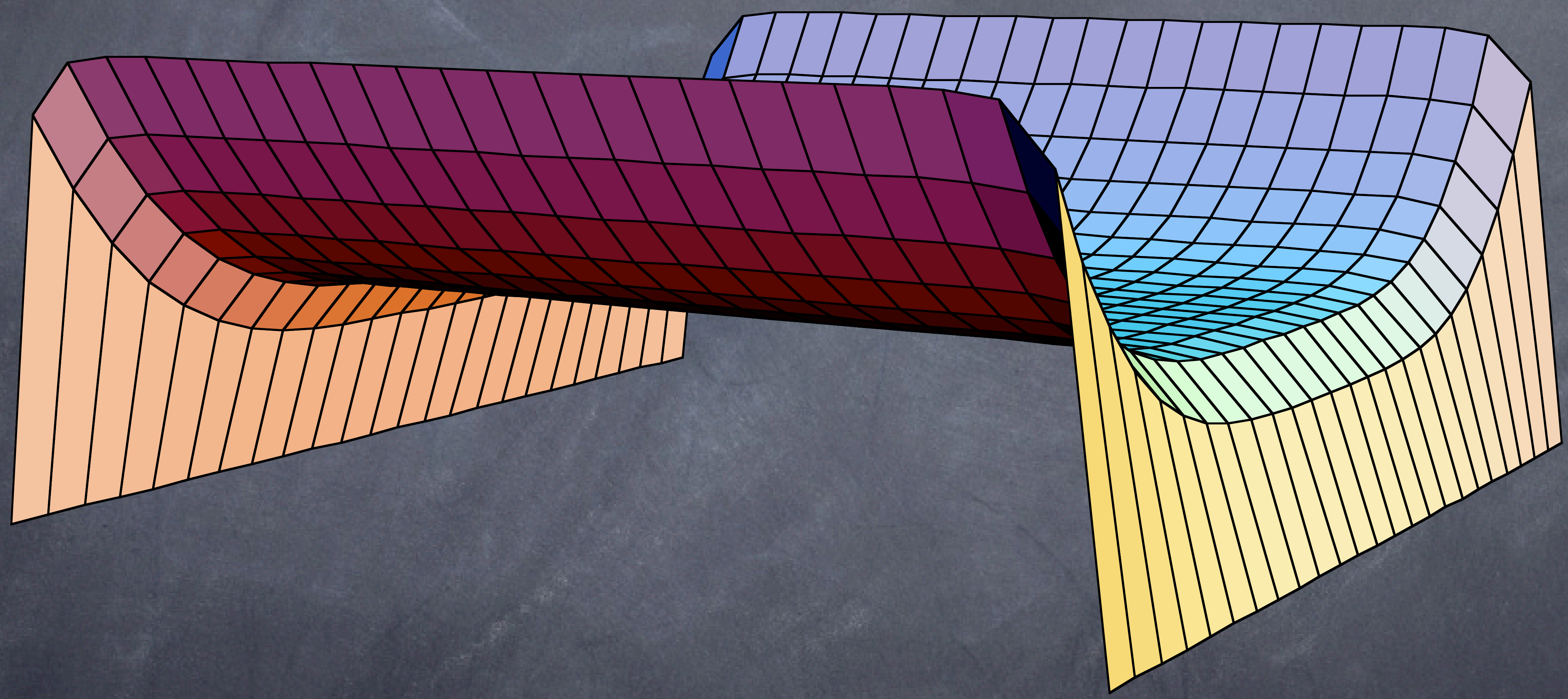


A caveat and the way out

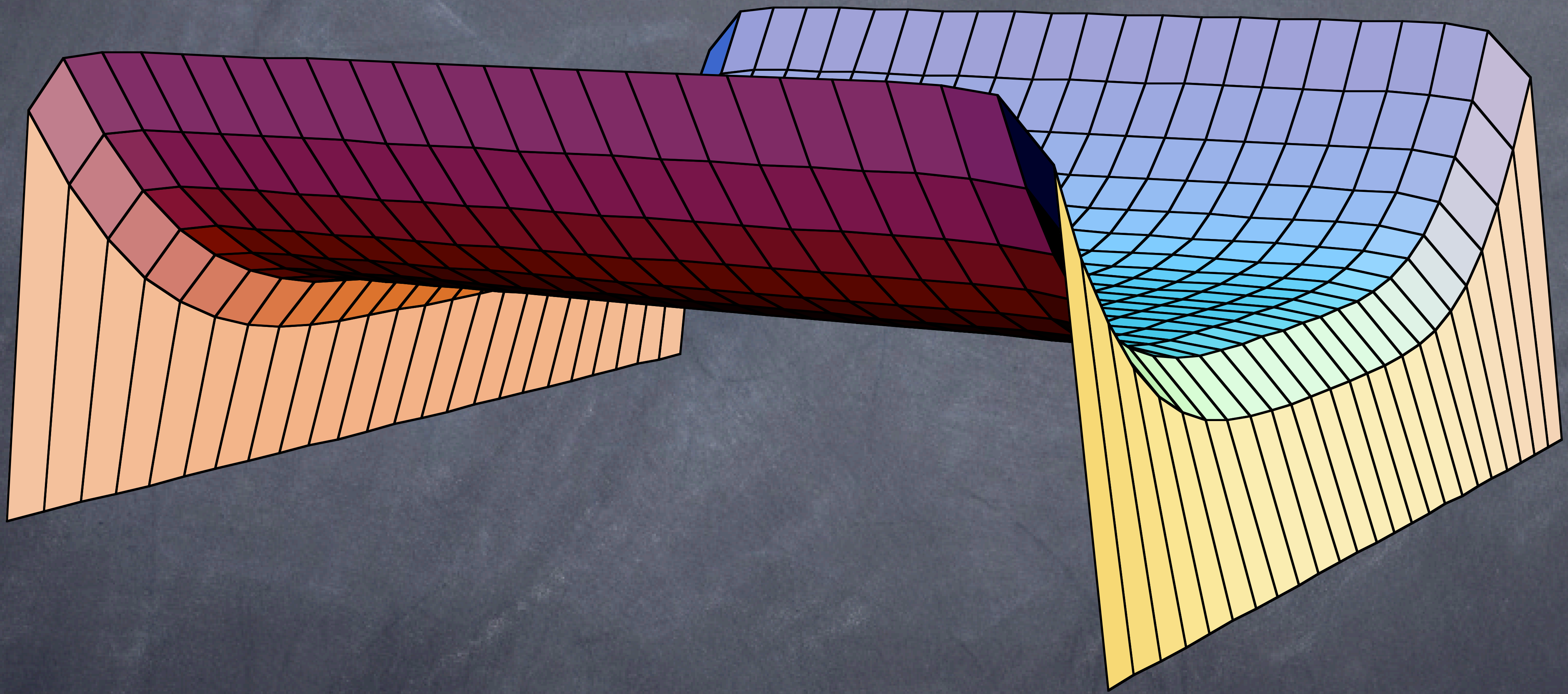
Off-diagonal G/B moduli introduce tachyons, just as a tiny magnetic field destabilizes the system in the Ising model.

The only way to be safe is to project them out

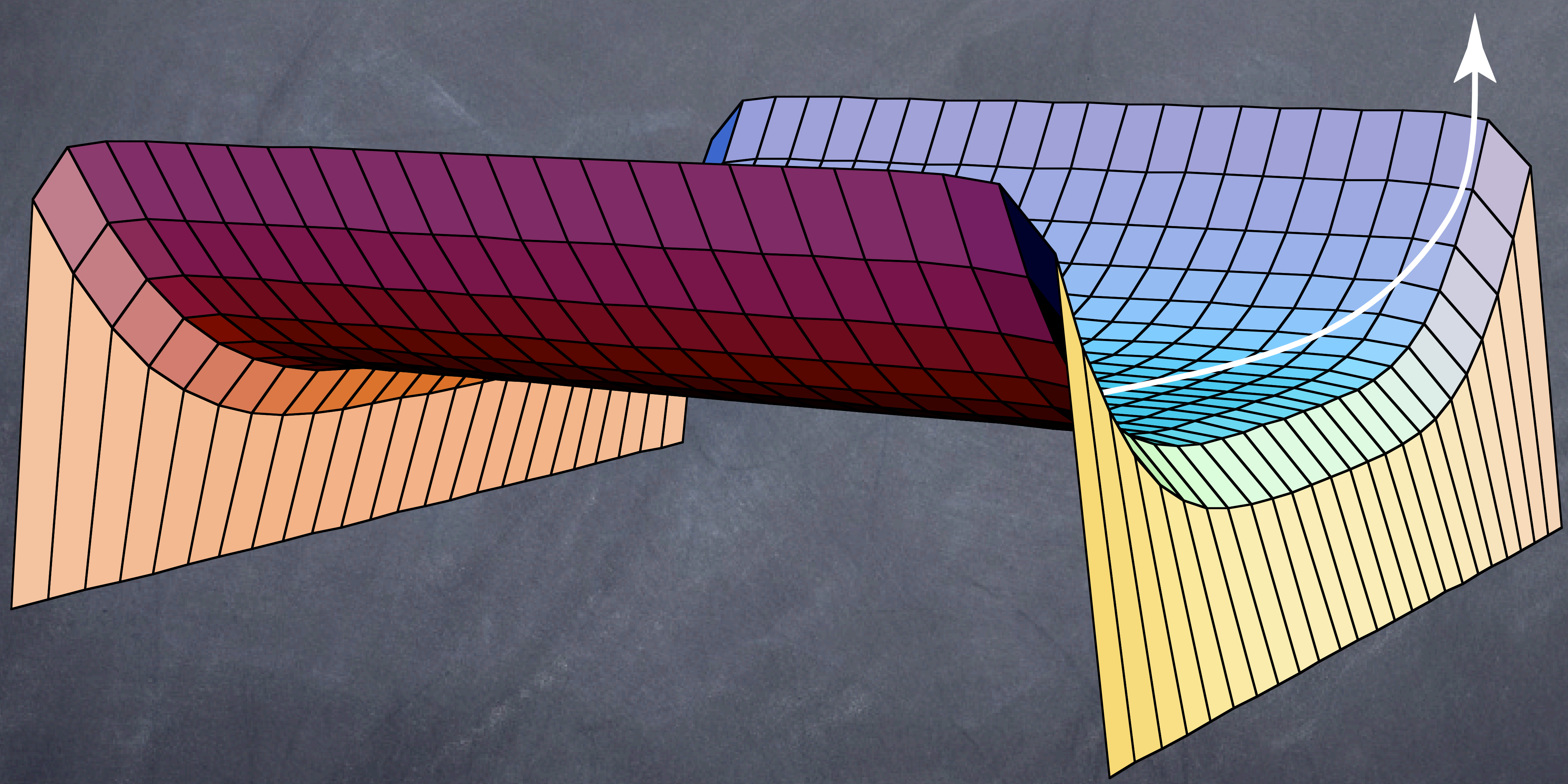
- *A* $T^6/\{Z_4 \times Z_4\}$ *orbifold projects out all off-diagonal G-moduli*
- *Orientifolding projects out the B-moduli*
- *These operations are not expected to introduce new contributions to the potential*



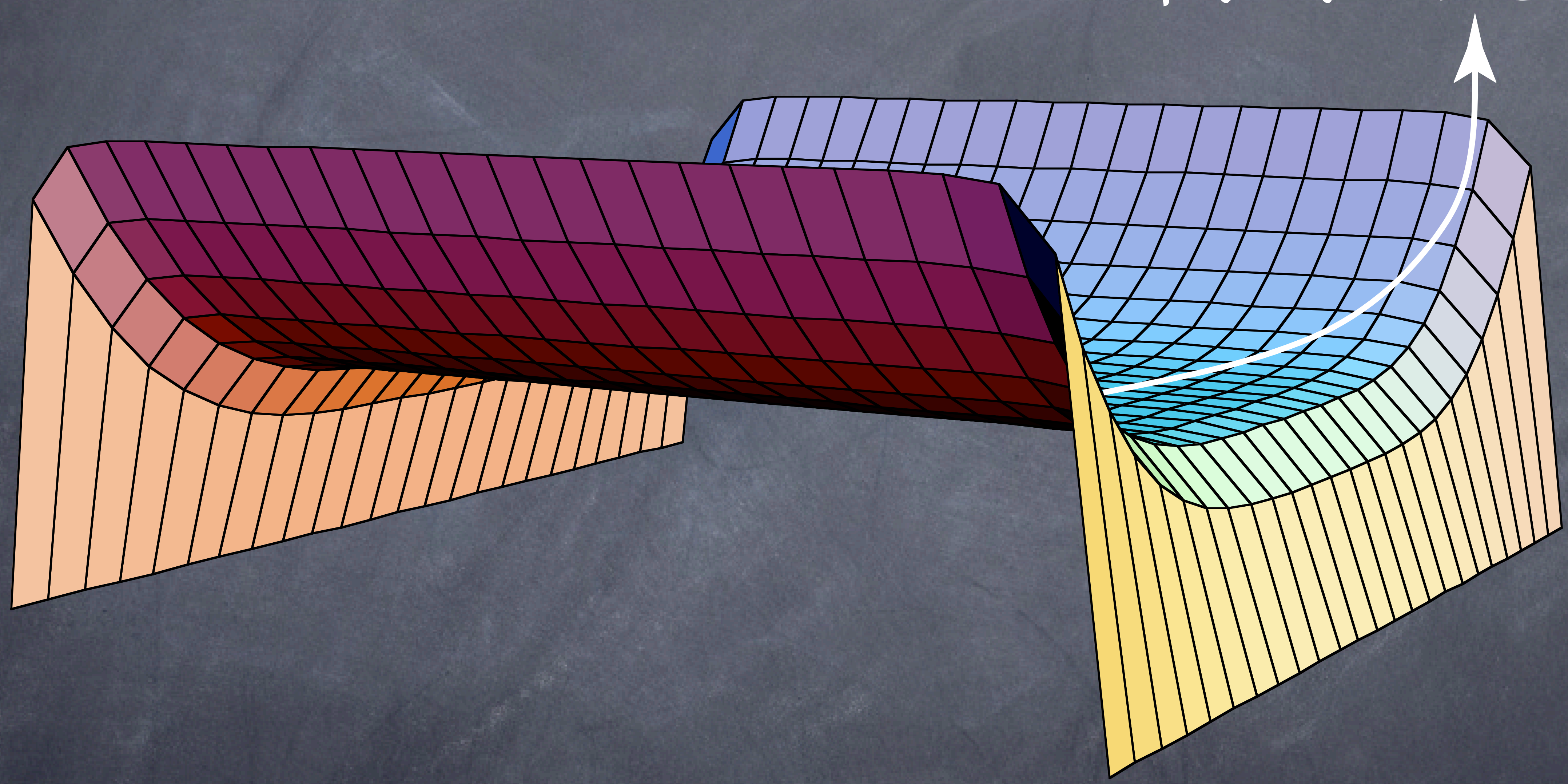
Potential with non-radial moduli



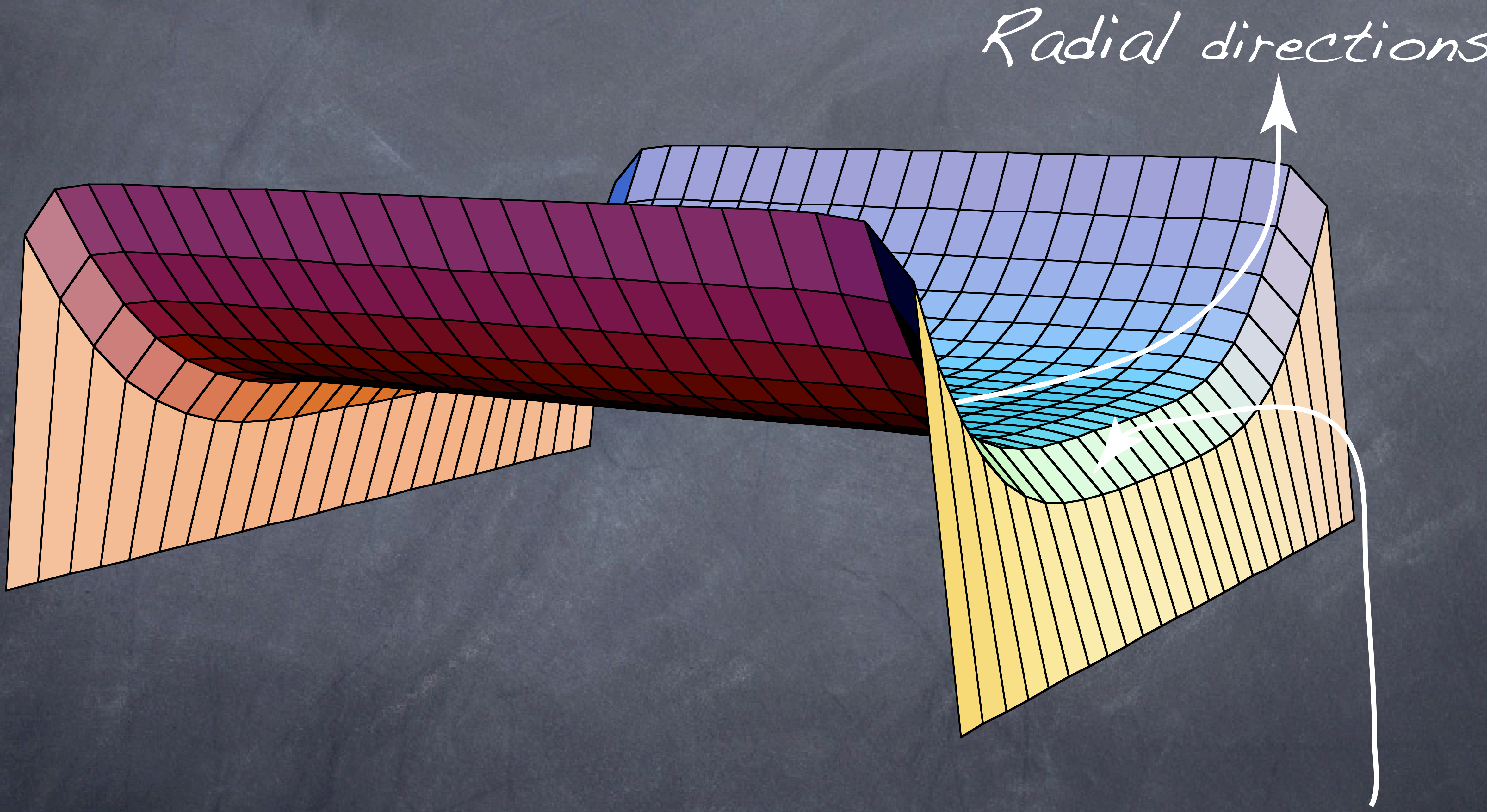
Potential with non-radial moduli



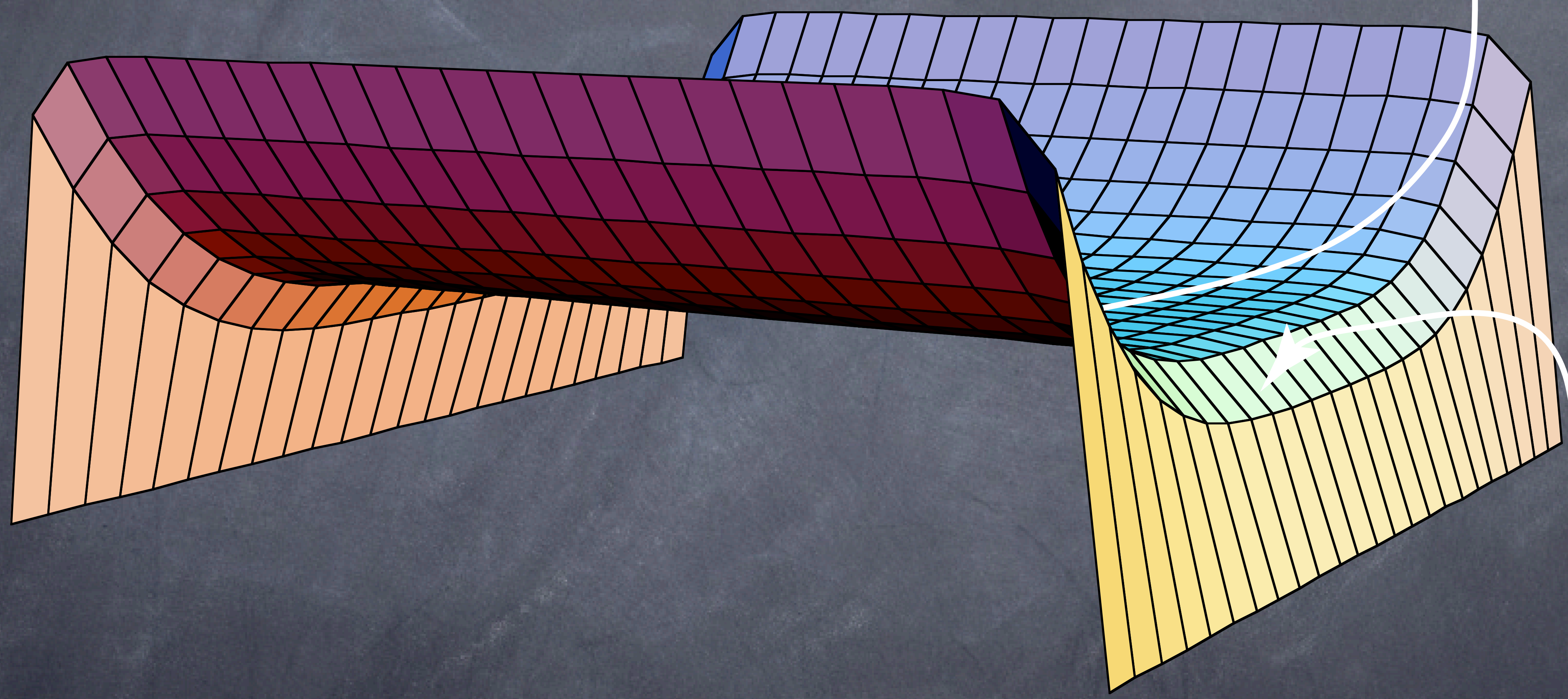
Potential with non-radial moduli



Potential with non-radial moduli



Potential with non-radial moduli



Conclusions

- Non-supersymmetric vacua in string theory can be stable if a self T-dual projection is applied and all tachyonic modes are projected out
- A class of such vacua have a very interesting thermal interpretation
- It is well worth trying to find more general duality invariant projections or look for metastable vacua
- Combine this mechanism with SM model building with intersecting branes