BACKREACTION
OF THE
HAWKING RADIATION

Three recent papers* by
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Presented by

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A semi-classical derivation in 1 + 3 dimensions for spherically symmetric, uncharged, collapsing matter of mass $M >> \mu$ (Planck mass) having compact spatial support.

As a black hole forms and evolves, its Hawking process decreases its initial mass by about 10%, and creates a vacuum-induced "charge" equal to the remaining mass, corresponding to a stress-energy tensor of the type associated with a long-range field.

* hep-th/0511182, hep-th/0511183, hep-th/0511184

INTRODUCTION

Toy models (1 + 1 dimension) miss essential physics

VILKOVISKY PAPERS

Kinematics of evaporating black holes	hep-th/0511182
Radiation equations for black holes	hep-th/0511183
Backreaction of the Hawking radiation	hep-th/0511184

Definitions:

★ *Classical*, *Semiclassical*, and *Ultraviolet Regions* of a macroscopic collapsing mass — based on the correspondence principle and the concept of causality.

The semiclassical region covers the black hole evolution from macroscopic to microscopic scale.

★ *Charge*: the strength of a long-range field having its source in a compact domain.

Bondi charges $\mathcal{M}$ and $\mathcal{Q}^2$ (not necessarily an electric charge): the coefficients of a weak field metric at $\mathcal{I}^+$. In the (u,r) coordinates:

$$(\nabla r)^2\Big|_{\mathcal{I}^+} = 1 - \frac{2\mathcal{M}(u)}{r} + \frac{\mathcal{Q}^2(u)}{r^2} + \dots$$

$$u\Big|_{\mathcal{I}^+} = ct - r_* , \quad r_* := r + r_g \ln\left|\frac{r}{r_g} - 1\right| , \quad r_g := \frac{2GM}{c^2}$$

Conclusion: The collapse metric in the semiclassical region can be written in terms of the 2 arbitrary Bondi charges.

RADIATION EQUATIONS for black holes

Bondi charges equations:

$$\frac{d\mathcal{M}}{du} = -\frac{\mu^2}{48\pi}\mathcal{K}^2(1+\Gamma)$$

$$\frac{d\mathcal{Q}^2}{du} = \frac{\mu^2}{24\pi}\mathcal{K}$$

μ Planck mass, M initial mass, $\frac{\mu}{M} << 1$

$$\mathcal{K} := \frac{(\mathcal{M}^2 - \mathcal{Q}^2)^{1/2}}{r_{AH}^2} \qquad \text{for } u > u_0$$

Apparent horizon r_{AH} calculated to satisfy $1 - \frac{2\mathcal{M}(u)}{r_{AH}} + \frac{\mathcal{Q}^2}{r_{AH}^2} = 0$

Γ function of $\mathcal{M}$ and $\mathcal{Q}$; uniformly bounded, small: $\Gamma \leq 27/160$

Derivation:

The collapse metric satisfies the WKB approximation of the solution of expectation-value equation

$$R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}R = 8\pi T^{\mu\nu}_{source} + 8\pi\langle T^{\mu\nu}_{vac}\rangle$$

The vacuum is modeled as a massless scalar field Φ in the collapse metric. Its stress-energy tensor is computed at $\mathcal{I}^+$:

$$\langle T^{\mu\nu}_{vac}\rangle\Big|_{\mathcal{I}^+} = \langle invac|T^{\mu\nu}|invac\rangle\Big|_{\mathcal{I}^+} - \langle outvac|T^{\mu\nu}|outvac\rangle\Big|_{\mathcal{I}^+}$$

Φ is expanded in spherical harmonics.

Φ in 4 dimensions is a sequence of φ_ℓ in 2 dimensions.

2-d models are not 4-d models truncated at $\ell = 0$.

BACKREACTION of the HAWKING RADIATION

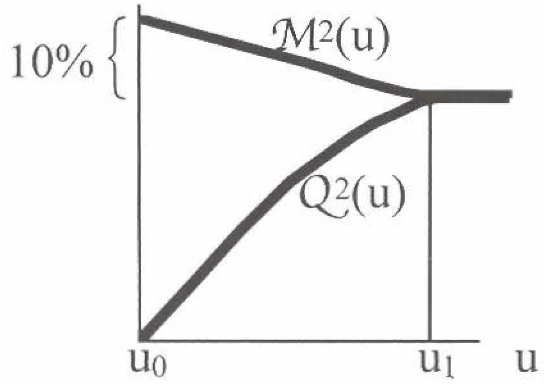
The Bondi charges equations are solved with initial conditions at u_0 when the apparent horizon is tangent to an outgoing ray.

$$\mathcal{M}(u_0) = M\,, \quad \mathcal{Q}(u_0) = \mathcal{O} \quad (\text{vanishes as } \mu/M \to 0)$$

Let u_1 be the value of u such that

$$\mathcal{M}^2(u_1) = \mathcal{Q}^2(u_1)$$

At u_1 the apparent horizon is tangent to an incoming light ray.



Beyond u_1 the calculation is not valid; the WKB approximation remains valid.

Approximate numerical values.

$$u_1 - u_0 = 96\pi \frac{M^3}{\mu^2}\,, \qquad 0.098 < \frac{M - \mathcal{M}(u_1)}{M} < 0.112$$

The mass decreases by 10%: The Hawking process liberates more than half of the energy of the black hole.

[redacted]