Towards Donaldson–Thomas Theory on Orbifolds

Michele Cirafici

University of Patras
Πανεπιστήμιο Πατρών

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work in progress with A. Sinkovics and R.J. Szabo

Topological Strings on Calabi–Yau Manifolds

- Topological Strings play a very important role in modern mathematical physics
- They compute F–terms in supersymmetric theories
  Antoniadis Gava Narain Taylor
  Bershadsky Cecotti Ooguri Vafa
- Black Holes: counting of microstates and statistical interpretation of the entropy
  Beckenridge Myers Peet Vafa
  Ooguri Strominger Vafa
- Geometric engineering of gauge theories
  Katz Klemm Vafa
- Interplay with enumerative geometry and characterization of the Calabi–Yau moduli space
- Test our understanding of the full String Theory in a controllable setup

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The Enumerative Geometry of Threefolds

- The Topological String computes invariants that characterize the geometry
- Gromov–Witten : count worldsheet instantons
- Gopakumar–Vafa: count massive BPS states
- Donaldson–Thomas: count D0–D2–D6 bound states
- All these invariants are equivalent since they are different expansions of the same topological amplitude: remarkable prediction!
- Problem: usually these invariants are known only in the large radius limit where classical geometry is a good concept.
- To learn more about quantum geometry we can try to move away from the large radius limit

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Gromov–Witten theory for orbifolds

- Very hard problem but with a recent solution
- The Topological B model has an interpretation as a wave function over the Calabi–Yau moduli space

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- Recently the B–model was solved on a threefold by using the properties of modularity and holomorphicity of the free energy

  Aganagic Bouchard Klemm ; Huang Klemm Quackenbush
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The Local Threefold

- We will focus on the local threefold $\mathcal{O}(-3) \longrightarrow \mathbb{P}^2$ and its orbifold limit $\mathbb{C}^3/\mathbb{Z}_3$
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Counting Ideal Sheaves

- The Donaldson–Thomas invariants count the number of bound states formed by a single D6 brane wrapping the Calabi–Yau X with an arbitrary number of D2 branes wrapping a curve $C \subset X$ and D0 branes
- The curve C and the set of points where the D0 branes are supported can be described by an ideal sheaf: the holomorphic functions that vanish on the prescribed locus
- Counting D6–D2–D0 bound states leads us to consider the moduli spaces of ideal sheaves $I_m(X, \beta)$ such that

$$\begin{aligned} \chi(\mathcal{O}_Y) &= m && \text{number of D0} \\ \beta &= [C] \in H_2(X, \mathbb{Z}) && \text{curve the D2 are wrapping} \end{aligned}$$

- The DT invariant $D^m_\beta(X)$ is the "volume" of this moduli space

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The Calabi–Yau Crystal Picture

- When X is toric DT theory is more easily understood in terms of the Calabi–Yau crystal

Okounkov Reshetikhin Vafa

- A toric manifold has the structure of a fibration where the fibers are tori. The information of the locus where the cycles of the torus degenerate can be encoded in a (trivalent) toric diagram that characterize completely the manifold.
- The topological partition function is

$$\mathcal{Z}_X(q,t) = \sum_{\substack{\{\pi_f\} \\ f \in \text{vertices}}} (-q)^{|\pi_f|} \prod_{e \in \text{edge}} (-1)^{m_e |\lambda_e|} \, \mathrm{e}^{-t_e |\lambda_e|}$$

- Partition function of a classical crystal whose edges are given by the toric diagram. The "atoms" of the crystal correspond to boxes in a 3D Young tableaux π_f: combinatorial interpretation of DT theory

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The Topological Gauge Theory Picture

- The states of the crystal can be described as "instanton" solutions of 6–dimensional $\mathcal{N}_T = 2$ abelian super Yang–Mills topologically twisted

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- The bosonic matter content is $A_\mu, \varphi^{3,0}, \Phi$ and the gauge theory localizes on solutions of the Donaldson-Uhlenbeck-Yau equations

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$$F^{(0,2)} = 0\,, \qquad F^{(1,1)} \wedge \omega \wedge \omega = 0\,, \qquad \mathrm{d}_A \Phi = 0$$

  that characterize holomorphic bundles ($\sim$ ideal sheaves)

- The critical points ("instantons") correspond to 3D partitions
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- Key Idea: work equivariantly with respect to the toric action (i.e. $Q \longrightarrow Q + Q_\mu \Omega_{\mu\nu} x_\nu$) and consider a noncommutative deformation of the gauge theory
- After this deformation the instanton moduli space is regularized and the critical points isolated: the localization problem is well posed
- Taking this into account the gauge theory partition function localizes as a sum over instanton solutions

$$Z \sim \sum_{x \in \{\text{critical}\}} \left(\int_{\mathcal{M}_{\text{inst}}(\text{ch}_2, \text{ch}_3)} 1 \right) e^{\,S_{\text{inst}}(x)}$$

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Donaldson–Thomas on $\mathbb{C}^3/\mathbb{Z}_3$

- In the following we will adopt this gauge theoretical point of view
- But how do we define the theory on the orbifold?
- We propose to work on $\mathbb{C}^3$ and restrict attention to $\mathbb{Z}_3$–invariant sheaves
- Motivation: the mathematical theory of Gromov–Witten and its formulation as " orbifold cohomology on a quotient stack "

  Chen Ruan
- The localization procedure is well defined since *the orbifold action and the toric action commute* on $\mathbb{C}^3$:

$$\begin{aligned} (\mathbb{C}^\times)^3 &: \quad (z_1, z_2, z_3) \mapsto \left(e^{i\epsilon_1} z_1, e^{i\epsilon_2} z_2, e^{i\epsilon_3} z_3 \right) \\ \mathbb{Z}_3 &: \quad (z_1, z_2, z_3) \mapsto \left(e^{\frac{2\pi i}{3}} z_1, e^{\frac{2\pi i}{3}} z_2, e^{\frac{2\pi i}{3}} z_3 \right) \end{aligned}$$

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- We propose to work on $\mathbb{C}^3$ and restrict attention to $\mathbb{Z}_3$–invariant sheaves
- Motivation: the mathematical theory of Gromov–Witten and its formulation as " orbifold cohomology on a quotient stack " `Chen Ruan`
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- We introduce a noncommutative deformation of the gauge theory:

$$[x^i, x^j] = i\theta^{ij} \quad i = 1 \dots 6$$

  and work equivariantly with respect to the toric action on $\mathbb{C}^3$

- More precisely we work with the fields

$$Z^1 = \frac{1}{\sqrt{2\theta_1}}(X^1 + iX^2) \dots \qquad X^i = x^i + i\theta^{ij} A_j$$

- The fixed point equations now read

$$\begin{aligned} [Z_i, Z_j] &= 0 \qquad [Z_i, \Phi] = \epsilon_i Z_i \\ \sum_i [Z_i, Z_i^\dagger] &= \begin{pmatrix} P^{(0)} & 0 & 0 \\ 0 & P^{(1)} & 0 \\ 0 & 0 & P^{(2)} \end{pmatrix} \end{aligned}$$

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Equivariant localization

- The fixed points can be classified in terms of *colored* 3D partitions: each box of a 3D partition has a different color labeled by $0, 1, 2$ corresponding to the orbifold action
- These equations can be solved as

$$Z_i = U_I a_i F U_I^\dagger \qquad a_i = \begin{pmatrix} 0 & \alpha_i^{(1)} & 0 \\ 0 & 0 & \alpha_i^{(2)} \\ \alpha_i^{(0)} & 0 & 0 \end{pmatrix}$$

- Here U_I are the von Neumann partial isometry split into the orbifold sectors and (for example)

$$\alpha_1^{(r)} = \sum_{k=0}^{\infty} \sum_{\substack{\{n\} \\ n_1+n_2+n_3=r+3k}} \sqrt{n_1} |n_1 - 1, n_2, n_3\rangle\langle n_1, n_2, n_3|$$

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Conclusions and Work in Progress

- It is possible to define Donaldson–Thomas theory on an orbifold through equivariant localization of a topological gauge theory
- The fixed points are classified by colored 3D partitions
- Can we express the result in a closed form? Maybe through modular forms?
- It would be nice to make contact with the Gromov–Witten theory. However we are missing something: how to compute the descendants? Quiver representation theory?

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