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BLACK HOLES AND UNITARITY

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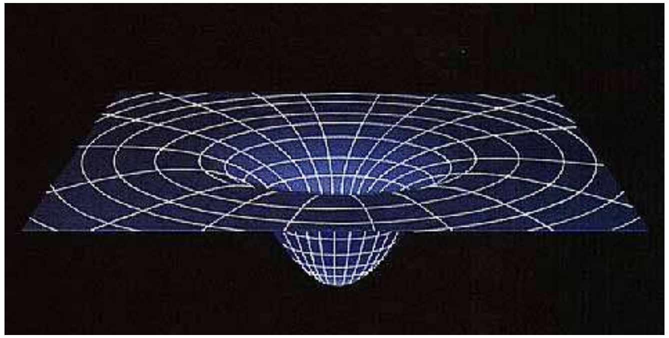
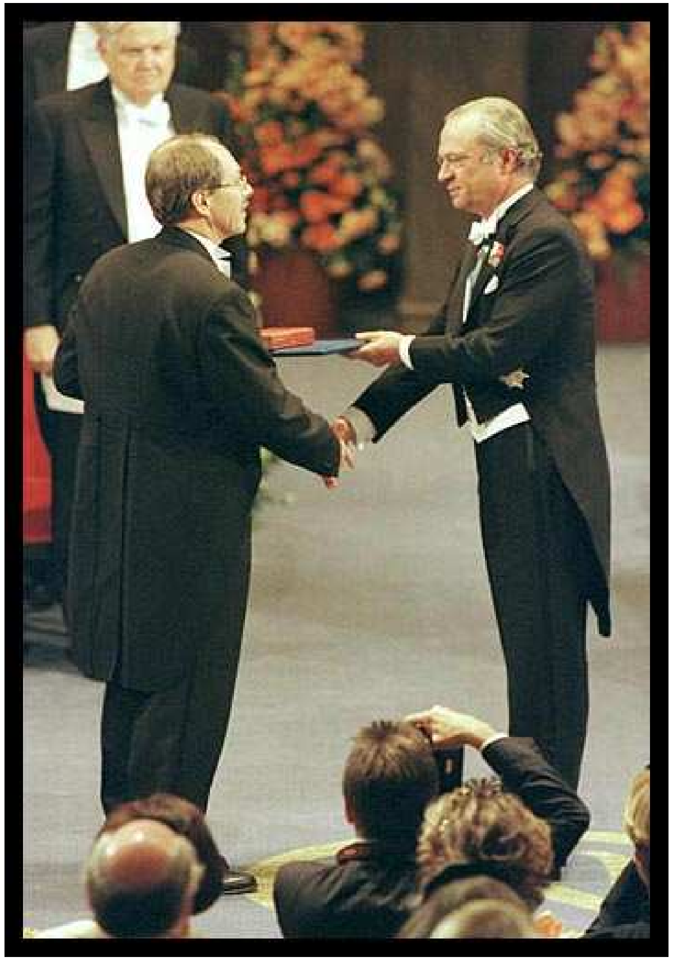
OUTLINE

- Information loss paradox
- Black hole perturbations
- AdS_3/CFT_2 correspondence
- 't Hooft's brick wall
- Solodukhin's wormhole
- Conclusions



G.S., *Class. Quant. Grav.* **22** (2005) 1425; hep-th/0408091



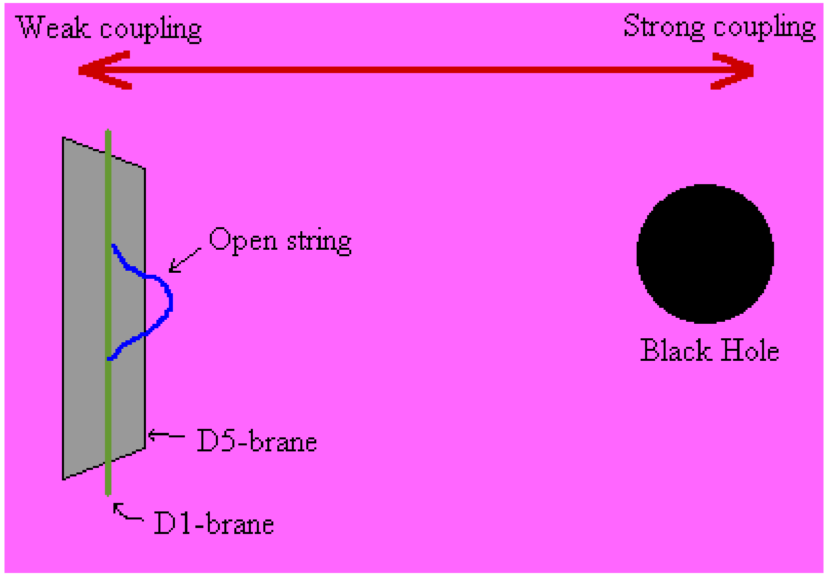
“To many practitioners of quantum gravity the black hole plays the role of a soliton, a non-perturbative field configuration that is added to the spectrum of particle-like objects only after the basic equations of their theory have been put down, much like what is done in gauge theories of elementary particles, where Yang-Mills equations with small coupling constants determine the small-distance structure, and solitons and instantons govern the large-distance behavior.

Such an attitude however is probably not correct in quantum gravity. The coupling constant increases with decreasing distance scale which implies that the smaller the distance scale, the stronger the influences of “solitons”. At the Planck scale it may well be impossible to disentangle black holes from elementary particles.”

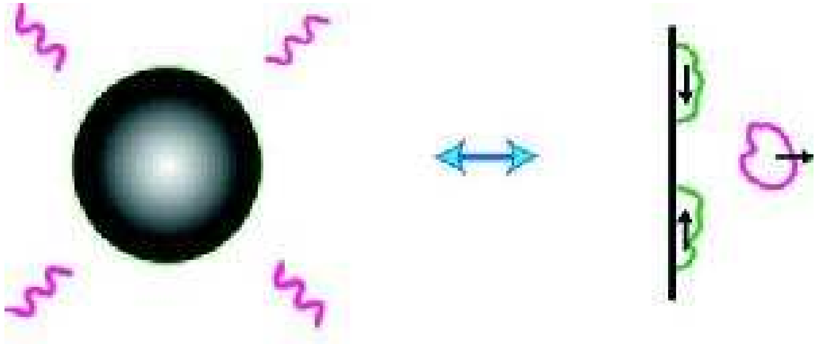
– G. ’t Hooft

Strings explain black hole entropy quantitatively in terms of D-branes

[Strominger, Vafa]



black-hole entropy



- Hawking radiation
  $\Rightarrow$ *Information loss paradox*
- Greybody factors

Quasi-normal modes (QNMs)

describe small perturbations of a black hole.

- A black hole is a thermodynamical system whose (Hawking) temperature and entropy are given in terms of its global characteristics (total mass, charge and angular momentum).

QNMs obtained by solving a wave equation for small fluctuations subject to the conditions that the flux be

- ingoing at the horizon and
- outgoing at asymptotic infinity.

$\Rightarrow$ discrete spectrum of complex frequencies.

- imaginary part determines the decay time of the small fluctuations

$$\Im\omega = \frac{1}{\tau}$$

imaginary part of QNM is negative

⇒ black hole eventually relaxes back to its original (thermal) equilibrium at (Hawking) temperature T_H

$$|e^{-i\omega t}| = e^{-t/\tau}$$

⇒ leakage of information into the horizon

⇒ breakdown of unitarity

⇒ closely related to Hawking's information loss paradox

resolution will require understanding of quantum gravity beyond semi-classical approximation.

asymptotically AdS space-times

additional tool due to AdS/CFT correspondence:

- complex QNM frequencies are poles of the retarded propagator in CFT
- puzzling: CFT is unitary $\therefore$ propagator should possess *real* poles only.

Poincaré recurrence theorem

- two-point function quasi-periodic with a period

$$t_P \sim e^S$$

  S: entropy.

- For times $t \ll t_P$, system may look like it is decaying back to thermal equilibrium, but for $t \gtrsim t_P$, it should return to its original state (or close) an infinite number of times.

- system will *never* relax back to its original state.

AdS Black Holes

establishment of AdS/CFT correspondence hindered by difficulties in solving the wave equation.

- In 3d: Hypergeometric equation $\therefore$ solvable

  [Cardoso, Lemos; Birmingham, Sachs, Solodukhin]
  quasi-normal frequencies are the poles of the retarded Green function of the corresponding perturbations in the dual CFT (on the cylinder $\mathbb{R} \times S^1$).

- Numerical results in 4d, 5d and 7d

  [Horowitz, Hubeny; Starinets; Konoplya]

- In 5d: Heun equation $\therefore$ unsolvable.
  in high frequency regime Heun equation $\rightarrow$ Hypergeometric equation.

  [GS]
  - analytical expression for asymptotic form of QNM frequencies
  - in agreement with numerical results.

Monodromies

Asymptotic expressions for QNMs may also be easily obtained by considering the **monodromies** around the singularities of the wave equation.

- singularities lie in the unphysical region.
  - In three dimensions, they are located at the horizon $r = r_h$, where r_h is the radius of the horizon, and at the black hole singularity, $r = 0$.
  - In higher dimensions, it is necessary to analytically continue r into the complex plane. The singularities lie on the circle $|r| = r_h$.
  - similar to asymptotically flat space where analytic continuation of r yields asymptotic form of QNMs *[Motl and Neitzke]*.
- It is curious that **unphysical** singularities determine the behavior of QNMs.

AdS$_5$/CFT$_4$ correspondence

Despite a considerable amount of work on quasi-normal modes of black holes in asymptotically AdS space-times their relation to the AdS/CFT correspondence is not well understood.

In 5d, the large real part of the QN frequencies challenges our understanding of the AdS/CFT correspondence in five dimensions.

QN frequencies determine the poles of the retarded correlation functions of dual operators in finite-temperature $\mathcal{N} = 4$ $SU(N)$ SYM theory in the large-N, large 't Hooft coupling limit.
[Núñez and Starinets]

Also arise in complexified geodesics.
[Fidkowski, Hubeny, Kleban and Shenker; Balasubramanian and Levi]

► QN frequencies are obtained as the poles of a CFT on $\mathbb{R} \times S^3$ in a certain scaling limit where Euclidean time is identified with one of the periodic coordinates of S^3.
[GS]

Linear response theory

[Birmingham, Sachs and Solodukhin]

system in thermal equilibrium described by density matrix ρ.

perturbation

$$H' = \int dx J(t,x)\mathcal{O}(t,x)$$

J: external source

change in ensemble average

$$\delta\langle\mathcal{O}(t,x)\rangle = \int_{-\infty}^{\infty} dt' \int dx' J(t',x') G_R(t,x;t',x')$$

in terms of retarded propagator

$$G_R(t,x;t',x') = -i\theta(t-t')\mathrm{Tr}\left(\rho[\mathcal{O}(t,x),\mathcal{O}(t',x')]\right)$$

Fourier transform $\tilde{G}_R(\omega, p)$:

- analytic in upper-half ω-plane.
- discrete energy levels
  $\Rightarrow$ simple poles on real axis
  $\Rightarrow$ meromorphic in lower-half ω-plane
  $\Rightarrow$ oscillatory behavior
- continuous energy levels
  $\Rightarrow$ poles (stable states) or cuts (multi-particle states) on real axis
  $\Rightarrow$ poles (resonances) in lower-half ω-plane

BTZ black hole

The wave equation (with $m = 0$) is

$$\frac{1}{R^2\, r}\partial_r\left(r^3\left(1-\frac{r_h^2}{r^2}\right)\partial_r\Phi\right)-\frac{R^2}{r^2-r_h^2}\partial_t^2\Phi+\frac{1}{r^2}\partial_x^2\Phi=0$$

One normally solves this in physical interval:

$$r\in[r_h,\infty)$$

Instead, we shall solve it inside the horizon

$$0\le r\le r_h$$

Solution:

$$\Phi=e^{i(\omega t-px)}\Psi(y),\qquad y=\frac{r^2}{r_h^2}$$

where Ψ satisfies

$$\Big(y(1-y)\Psi'\Big)'+\left(\frac{\widehat{\omega}^2}{1-y}+\frac{\widehat{p}^2}{y}\right)\Psi=0$$

in terms of dimensionless variables

$$\hat{\omega} = \frac{\omega}{2r_h} = \frac{\omega}{4\pi T_H}, \quad \hat{p}^2 = \frac{p}{2r_h} = \frac{p}{4\pi T_H}$$

$T_H = r_h/(2\pi)$: Hawking temperature.

Two solutions obtained by examining behavior near the horizon ($y \to 1$),

$$\Psi_\pm \sim (1-y)^{\pm i\hat{\omega}}$$

A different set obtained by studying behavior at **black hole singularity** ($y \to 0$)

$$\Psi \sim y^{\pm i\hat{p}}$$

For QNMs, Ψ ingoing at the horizon

$$\Psi \sim \Psi_- \text{ as } y \to 1$$

By writing

$$\Psi(y) = y^{\pm i\hat{p}}(1-y)^{-i\hat{\omega}}F(y)$$

we deduce

$$y(1-y)F'' + \{1 \pm 2i\hat{p} - (2 - 2i(\hat{\omega} \mp \hat{p})y\} F' + (\hat{\omega} \mp \hat{p})(\hat{\omega} \mp \hat{p} + i) F = 0$$

Solution:

$$F(y) = F(1 - i(\hat{\omega} \mp \hat{p}), -i(\hat{\omega} \mp \hat{p}); 1 \pm 2i\hat{p}; y)$$

- near the horizon ($y \to 1$): mixture of ingoing and outgoing waves.
- blows up at infinity ($y \to \infty$).

For a QNM, we demand that $F(y)$ be a **Polynomial**

$\Rightarrow$ takes care of both limits $y \to 1, \infty$

$\Rightarrow$

$$\hat{\omega} = \pm\hat{p} - in \quad , \quad n = 1, 2, \dots$$

$\Rightarrow$

$$F(y) = {}_2F_1(1 - n, -n; 1 \pm 2i\hat{p}; y)$$

is a Polynomial of order $n - 1$.

$\therefore$ constant at $y = 1$, as desired and

$\therefore$

$$F(y) \sim y^{n-1} \sim y^{i(\hat{\omega}\mp\hat{p})-1} \text{ as } y \to \infty$$

so $\Psi \sim y^{-1}$ as $y \to \infty$, as expected.

A monodromy argument

Let $\mathcal{M}(y_0)$ be the monodromy around the singular point $y = y_0$ computed along a small circle centered at $y = y_0$ running counterclockwise.

For $y = 1$,

$$\mathcal{M}(1) = e^{2\pi\widehat{\omega}}$$

For $y = 0$,

$$\mathcal{M}(0) = e^{\mp 2\pi\widehat{p}}$$

Since the function vanishes at infinity, the two contours around the two singular points $y = 0, 1$ may be deformed into each other without encountering any singularities,

$\therefore$

$$\mathcal{M}(1)\,\mathcal{M}(0) = 1$$

$$\therefore \quad e^{2\pi(\widehat{\omega}\mp\widehat{p})} = e^{2\pi i n} \quad (n \in \mathbb{Z})$$

same QNM frequencies as before, if we demand $\Im\widehat{\omega} < 0$.

Massive scalar

wave equation for massive scalar of mass m

$$\frac{1}{r}\partial_r\left(r^3\left(1-\frac{r_h^2}{r^2}\right)\partial_r\Phi\right)-\frac{1}{r^2-r_h^2}\partial_t^2\Phi+\frac{1}{r^2}\partial_x^2\Phi=m^2\Phi$$

r_h: radius of horizon (set AdS radius $R=1$)

Let

$$\Phi=e^{i(\omega t-px)}\Psi(y),\qquad \Psi(y)=y^{i\widehat{p}}(1-y)^{-i\widehat{\omega}}F(y)$$

Solution

$$F(y)=F(a_+,a_-;c;1-y)$$

where

$$a_\pm=\tfrac{1}{2}\Delta_\pm-i(\widehat{\omega}-\widehat{p})\ ,\ c=1-2i\widehat{\omega}\ \ ,\ \ \Delta_\pm=1\pm\sqrt{1+m^2}$$

As $y \to \infty$, this function behaves as

$$F(y) \sim \mathcal{A} y^{-a_+} + \mathcal{B} y^{-a_-}$$

where

$$\mathcal{A} = \frac{\Gamma(c)\Gamma(a_- - a_+)}{\Gamma(a_-)\Gamma(c - a_+)} \ , \ \mathcal{B} = \frac{\Gamma(c)\Gamma(a_+ - a_-)}{\Gamma(a_+)\Gamma(c - a_-)}$$

For desired behavior ($\Psi \sim y^{-\Delta_+/2}$ as $y \to \infty$), set

$$\mathcal{B} = 0$$

This condition implies

$$\hat{\omega} = \pm \hat{p} - i(n + \tfrac{1}{2}\Delta_+ - 1) \quad , \quad n = 1, 2, \ldots$$

AdS/CFT correspondence:

- flux at the boundary ($y \to \infty$) is related to the retarded propagator of the corresponding CFT living on the boundary.

A standard calculation yields

$$\tilde{G}_R(\omega, p) \sim \lim_{y\to\infty} \frac{F'(y)}{F(y)}$$

Explicitly,

$$\begin{aligned}\tilde{G}_R(\omega, p) &\sim \frac{\mathcal{A}}{\mathcal{B}} \\ &\sim |\Gamma(\tfrac{1}{2}\Delta_+ - i(\hat{\omega} - \hat{p}))\Gamma(\tfrac{1}{2}\Delta_+ - i(\hat{\omega} + \hat{p}))|^2 \\ &\quad \times \sin\pi(\tfrac{1}{2}\Delta_+ - i(\hat{\omega} - \hat{p}))\sin\pi(\tfrac{1}{2}\Delta_+ - i(\hat{\omega} + \hat{p}))\end{aligned}$$

Plainly, QNMs (zeroes of $\mathcal{B}$) are poles of the retarded propagator $\because$

$$\tilde{G}_R \sim 1/\mathcal{B}$$

2-point correlator

$$\langle \mathcal{O}(t,x)\mathcal{O}(0,0)\rangle = \frac{(\pi T_H)^{2\Delta_+}}{(\sinh \pi T_H(t-x)\sinh \pi T_H(t+x))^{\Delta_+}}$$

decays exponentially as $t \to \infty$,

$$\langle \mathcal{O}(t,x)\mathcal{O}(0,0)\rangle \sim e^{-2\pi T_H \Delta_+ t}$$

AdS$_3$

associated with zero temperature

Metric

$$ds^2 = -\cosh^2\rho d\tau^2 + d\rho^2 + \sinh^2\rho d\phi^2$$

The boundary on which the corresponding CFT lives is the cylinder $\mathbb{R}\times S^1$.

Upon a change of coordinates,

$$y = \cosh^2\rho \ , \quad x = \frac{\tau}{2\pi T}$$

metric identical to the BTZ black hole metric with $y = r^2/r_h^2$, $r_h = 2\pi T$.

cf. with corresponding CFT:

- write propagator in terms of invariant distance in the embedding

$$\mathcal{P}(X,X') = (X - X')^2$$

where

$$X^0 = \cosh\rho\cos\tau \ , \quad X^3 = \cosh\rho\sin\tau$$
$$X^1 = \sinh\rho\cos\phi \ , \quad X^2 = \sinh\rho\sin\phi$$

and similarly for X'.

propagator on the boundary

$$G(\tau,\phi;\tau',\phi') \sim \lim_{\rho,\rho'\to\infty} \mathcal{P}^{-\Delta_+/2}$$

In this limit,

$$\mathcal{P} \sim \cosh(\tau-\tau') - \cos(\phi-\phi')$$

therefore,

$$G(\tau,\phi;\tau',\phi') \sim \frac{1}{(\cos(\tau-\tau')-\cos(\phi-\phi'))^{\Delta_+/2}}$$

real poles

$$\omega = p + 2(n + \tfrac{1}{2}\Delta_+ - 1)\ ,\ \ p\in\mathbb{Z}\ ,\ \ n = 1,2,\ldots$$

$\Rightarrow$ oscillatory behavior

CFT calculation

(without reference to the corresponding AdS)

two-point function of a massless scalar on the cylinder $\mathbb{R} \times S^1$ is

$$G_0(\tau, \phi; \tau', \phi') \sim T \sum_{j=-\infty}^{\infty} \int \frac{dk}{2\pi} \, e^{-ik\cdot x} \, \frac{i}{k^2}\Big|_{k^0=2\pi jT}$$

After integrating over k, summing over j and subtracting an irrelevant (infinite) constant, we obtain

$$G_0(\tau, \phi; \tau', \phi') \sim \ln \mathcal{P}$$

For a scalar operator $\mathcal{O}$ of dimension Δ, the two point function then reads

$$G(\tau, \phi; \tau', \phi') \equiv \langle T(\mathcal{O}(\tau, \phi)\mathcal{O}(\tau', \phi'))\rangle \sim \frac{1}{\mathcal{P}^{\Delta/2}}$$

as before.

SUMMARY

- high temperature limit: BTZ black hole
- zero temperature limit: AdS space (locally equivalent to BTZ black hole)
- intermediate temperature? Hard
  Correlator on torus of periods $1/T$ and 1.

EXAMPLE: free fermion ($\Delta = 1$)

Correlator:

$$\langle \psi(w)\psi(0)\rangle = \frac{\partial_w \vartheta_1(0|T)}{\vartheta_\nu(0|iT)} \frac{\vartheta_\nu(wT|iT)}{\vartheta_1(wT|iT)} \ , \quad \nu = 3, 4$$

$w = i(t+\phi)$, invariant under $w \to w + 1/T$, $w \to w + i$,

$\Rightarrow$ periodic in t, period 1.

poles:

$$w = \frac{m}{T} + in \ , \quad m, n \in \mathbb{Z}$$

As $T \to 0$, oscillating behavior:

$$\langle \psi(w)\psi(0)\rangle \sim \frac{1}{\sin \pi(t+\phi)}$$

As $T \to \infty$, exponential decay

$$\langle \psi(w)\psi(0)\rangle = \frac{\pi T}{4 \sinh \pi T(t+\phi)} \left\{ 1 \pm 2e^{-\pi T} \cosh 2\pi T(t+\phi) + \dots \right\}$$

violation of periodicity ($t \to t+1$) and loss of unitarity? NO!

Two time scales: 1 and $1/T \ll 1$.

- When $t \lesssim 1/T$, system decays
- When $t \sim 1/T$, corrections important
- When $t \gtrsim 1$, periodicity is restored

Strong coupling

AdS_3 arises in type IIB superstring theory in the near horizon limit of a large number of D1 and D5 branes.

- Low energy excitations form a gas of strings wound around a circle with winding number $\mathtt{k}$ and target space T^4.
- They are described by a strongly coupled CFT_2 whose central charge is

$$c = 6\mathtt{k} \sim \frac{1}{G} \gg 1$$

At finite temperature, the thermal CFT_2 has entropy

$$S \sim \mathtt{k} \sim \frac{1}{G}$$

BTZ black hole:

If radius of horizon is $o(1)$, then so is area of horizon

$$A \sim 1$$

Bekenstein-Hawking entropy:

$$S = \frac{A}{4G} \sim \frac{1}{G}$$

in agreement with CFT.

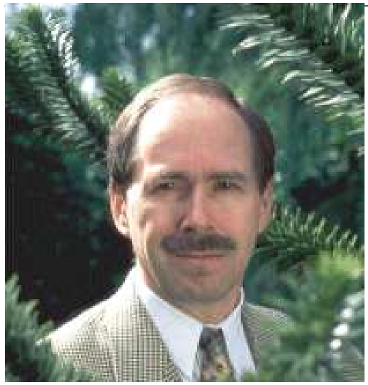
Poincaré recurrence time:

$$t_P \sim e^S \sim o(e^{1/G})$$

To understand this, one ought to include contributions to gravity correlators

- beyond the semi-classical approximation

which will modify the black-hole background.



’t Hooft’s brick wall

near horizon infinite energy levels

$\therefore$ information loss

$\therefore$ Hawking radiation

$\hookrightarrow$ continuous spectrum due to horizon. Set $r_h = 1$.

place brick wall at distance ϵ from horizon

$$\phi(r) = 0 \ , \quad r \leq 1 + \epsilon$$

discrete energy levels

$$\omega_n \sim \frac{n\pi}{-\ln \epsilon}$$

Free energy $F \sim T_H^3 \frac{A}{\epsilon}$.

Entropy: $S = -\frac{\partial F}{\partial T} \sim \frac{A}{\epsilon}$

- ▶ contributes to renormalization of G.

PROBLEM: Unnatural cutoff (coordinate invariance broken) - also need two time scales (ϵ as well as $\ln \epsilon$), more complicated spectrum (fractal wall?)

Solodukhin's wormhole

replace black hole by a wormhole

$\Rightarrow$ eliminate horizon and attendant leakage of information.

size of narrow throat $\lambda \sim o(e^{-1/G})$
leading to a Poincaré recurrence time

$$t_P \sim \frac{1}{\lambda} \sim o(e^{1/G})$$

in agreement with expectations.

GOAL

- calculate two-point functions explicitly
- obtain the *real* poles of the propagator,
  thus demonstrating unitarity.

Wormhole metric

$$ds^2 = -(\sinh^2 y + \lambda^2)\, dt^2 + dy^2 + \cosh^2 y\, d\phi^2$$

In the limit $\lambda \to 0$, reduces to BTZ black hole.

no horizon at $y = 0$:

- wormhole has a very narrow throat ($o(\lambda)$) joining two regions of space-time with two distinct boundaries (at $y \to \pm\infty$, respectively).
- Information may flow in both directions through the throat.
- modification significant near the "horizon" point $y = 0$.
- As $y \to 0$, time-like distance is $ds^2 \approx -\lambda^2 dt^2$,

  $\Rightarrow$ time scale of system is $\sim 1/\lambda$.

  $\Rightarrow$ Poincaré recurrence time

$$t_P \sim o(1/\lambda)$$

  as advertised.

λ will be fixed upon comparison with CFT.

wave equation

$$\frac{1}{\cosh y\,(\sinh^2 y+\lambda^2)^{1/2}}\left(\cosh y\,(\sinh^2 y+\lambda^2)^{1/2}\,\Psi'\right)'+\left(\frac{\omega^2}{\sinh^2 y+\lambda^2}+\frac{k^2}{\cosh^2 y}\right)\Psi=m^2\Psi$$

to be solved along the entire real axis ($y \in \mathbb{R}$)

cf. black hole: $y \geq 0$, horizon at $y = 0$.

solve wave equation in the small-λ limit ($\lambda \ll 1$).

$\hookrightarrow$ consider three regions,

(I) $y \gg \lambda$, which includes one of the boundaries,
(II) $y \ll -\lambda$, which includes the other boundary, and
(III) $|y| \ll 1$.

solve in each region and then match solutions in overlapping regions

$$\lambda \ll y \ll 1 \;\;,\;\; -1 \ll y \ll -\lambda$$

Region (II)

wave equation may be approximated by BTZ black hole.

- no physical requirement dictating a choice based on the small-y behavior (no horizon at $y = 0$).

choose a linear combination which behaves nicely at the boundary $(y \to -\infty)$,

$$\Psi_{II} = \cosh^{-2h_+} y \, \tanh^{-i\omega} y \, F(h_+ - \tfrac{i}{2}(\omega + k), h_+ - \tfrac{i}{2}(\omega - k); 2h_+; 1/\cosh^2 y)$$

vanishes at the boundary

$$\Psi_{II} \sim e^{2h_+ y} \quad \text{as} \quad y \to -\infty$$

At small y

$$\Psi_{II} \sim \mathcal{B}_+ y^{-i\omega} + \mathcal{B}_- y^{+i\omega} \quad , \quad \mathcal{B}_\pm = \frac{\Gamma(2h_+)\Gamma(\pm i\omega)}{\Gamma(h_+ \pm \frac{i}{2}(\omega + k))\Gamma(h_+ \pm \frac{i}{2}(\omega - k))}$$

Region (III)

$|y| \ll 1$; wave equation

$$\frac{1}{(y^2+\lambda^2)^{1/2}}\left((y^2+\lambda^2)^{1/2}\,\Psi'_{III}\right)' + \frac{\omega^2}{y^2+\lambda^2}\,\Psi_{III} = 0$$

linearly independent solutions

$$\Psi^{(1)}_{III} = F(\tfrac{i}{2}\omega, -\tfrac{i}{2}\omega; \tfrac{1}{2}; -y^2/\lambda^2) \;,\; \Psi^{(2)}_{III} = \frac{y}{\lambda} F(\tfrac{1}{2}+\tfrac{i}{2}\omega, \tfrac{1}{2}-\tfrac{i}{2}\omega; \tfrac{1}{3}; -y^2/\lambda^2)$$

At large $y > 0$,

$$\Psi^{(1)}_{III} \sim \frac{1}{2}\left(\frac{2y}{\lambda}\right)^{+i\omega} + \frac{1}{2}\left(\frac{2y}{\lambda}\right)^{-i\omega} \;,\; \Psi^{(2)}_{III} \sim \frac{i}{2\omega}\left(\frac{2y}{\lambda}\right)^{+i\omega} - \frac{i}{2\omega}\left(\frac{2y}{\lambda}\right)^{-i\omega}$$

asymptotic behavior as $y \to -\infty$,

$$\Psi^{(1)}_{III} \sim \frac{1}{2}\left(\frac{2y}{\lambda}\right)^{+i\omega} + \frac{1}{2}\left(\frac{2y}{\lambda}\right)^{-i\omega} \;,\; \Psi^{(2)}_{III} \sim -\frac{i}{2\omega}\left(\frac{2y}{\lambda}\right)^{+i\omega} + \frac{i}{2\omega}\left(\frac{2y}{\lambda}\right)^{-i\omega}$$

Match this to the asymptotic behavior of Ψ_{II},

$$\Psi_{III} = \mathcal{B}_+\Psi^{(-)}_{III} + \mathcal{B}_-\Psi^{(+)}_{III} \;,\; \Psi^{(\pm)}_{III} = \left(\frac{2}{\lambda}\right)^{\mp i\omega}\left(\Psi^{(1)}_{III} \pm i\omega\Psi^{(2)}_{III}\right)$$

at large $y > 0$, it behaves as

$$\Psi_{III} \sim \mathcal{B}_+ \left(\frac{4y}{\lambda^2}\right)^{+i\omega} + B_- \left(\frac{4y}{\lambda^2}\right)^{-i\omega}$$

Region (I)

linearly independent solutions

$$\Psi_I^{(\pm)} = \cosh^{-2h_\pm} y \, \tanh^{-i\omega} y \, F(h_\pm - \tfrac{i}{2}(\omega + k), h_\pm - \tfrac{i}{2}(\omega - k); 2h_\pm; 1/\cosh^2 y)$$

Matching asymptotic behavior,

$$\Psi_I = \alpha_+ \Psi_I^{(-)} + \alpha_- \Psi_I^{(+)}$$

where

$$\alpha_+ = \frac{\mathcal{B}_+^2 \left(\frac{2}{\lambda}\right)^{2i\omega} - \mathcal{B}_-^2 \left(\frac{2}{\lambda}\right)^{-2i\omega}}{\mathcal{B}_+\mathcal{C}_- - \mathcal{B}_-\mathcal{C}_+} \;,\quad \alpha_- = \frac{\mathcal{B}_-\mathcal{C}_- \left(\frac{2}{\lambda}\right)^{-2i\omega} - \mathcal{B}_+\mathcal{C}_+ \left(\frac{2}{\lambda}\right)^{2i\omega}}{\mathcal{B}_+\mathcal{C}_- - \mathcal{B}_-\mathcal{C}_+}$$

$\mathcal{B}_\pm$ are given above and

$$\mathcal{C}_\pm = \frac{\Gamma(2h_-)\Gamma(\pm i\omega)}{\Gamma(h_- \pm \frac{i}{2}(\omega + k))\Gamma(h_- \pm \frac{i}{2}(\omega - k))}$$

For a normalizable solution, set

$$\alpha_+ = 0$$

which leads to the quantization condition

$$\left(\frac{2}{\lambda}\right)^{2i\omega} = \frac{\mathcal{B}_-}{\mathcal{B}_+} = \frac{\Gamma(-i\omega)\Gamma(h_+ + \frac{i}{2}(\omega+k))\Gamma(h_+ + \frac{i}{2}(\omega-k))}{\Gamma(+i\omega)\Gamma(h_+ - \frac{i}{2}(\omega+k))\Gamma(h_+ - \frac{i}{2}(\omega-k))}$$

$\Rightarrow$ discrete spectrum of *real* frequencies

For small ω,

$$\omega_n \approx \left(n+\tfrac{1}{2}\right)\frac{\pi}{\ln\frac{2}{\lambda}} \quad , \quad n \in \mathbb{Z}$$

$\Rightarrow$ periodicity with period $L_{eff} \sim \ln(1/\lambda)$.

cf. with CFT (string winding $\mathtt{k}$ times around circle of length $o(1)$)
$L_{eff} \sim \mathtt{k} \sim 1/G$

$$\lambda \sim o(e^{-1/G})$$

as promised.

Notice: $L_{eff} \ll t_P$, $\therefore$ two time scales.

In the limit $\lambda \to 0$ (or, equivalently, $\mathtt{k} \to \infty$),

⇒ spectrum of real frequencies becomes continuous,

⇒ emergence of a horizon.

⇒ QNMs emerge

It should be emphasized that for no other value of λ, no matter how small, do complex poles arise.

Conclusions

- Hawking radiation and information loss paradox semiclassical effects
- String theory should provide a unitary description of evolution of a black hole
- AdS/CFT correspondence indispensible tool
  - need understand AdS/CFT correspondence at *finite* temperature
  - non-perturbative effects
  - explicit results in 3d

▶ generalize to higher dimensions