Konishi Anomalies and Effective Superpotential for Noncommutative $\mathcal{N} = 1$ Supersymmetric $U(1)$

F. Ardalan & N. Sadooghi

Sharif University of Technology

&

IPM

Tehran-Iran

Part I: Anomalies in NC Field Theory

- hep-th/0002143 by F. Ardalan & N.S.

  $\longrightarrow$ Covariant (Planar) Anomaly

- hep-th/0009233 by F. Ardalan & N.S.

  $\longrightarrow$ Invariant (Nonplanar) Anomaly

- F. Ardalan, H. Arfaei & N.S. (talk by H. Arfaei)

  $\longrightarrow$ Compactification

Part II: Effective Superpotential of NC $\mathcal{N}=1$ Supersymmetric $U(1)$

- 2004 by F. Ardalan & N.S.

I. Anomalies in NC non-SUSY $U(1)$

Talk presented by H. Arfaei

- Two currents associated with one global $U_A(1)$ transformation

$$\psi \to e^{i\alpha\gamma_5}\psi$$

$$J^{\mu,5}_{cov.} = \psi^\beta \star \bar{\psi}^\alpha \left(\gamma^\mu \gamma^5\right)_{\alpha\beta} \qquad j^{\mu,5}_{inv.} = \bar{\psi}^\alpha \star \left(\gamma^\mu \gamma^5\right)_{\alpha\beta} \psi^\beta$$

- Covariant (Planar) Anomaly

$$\langle D_\mu J^\mu_{cov} \rangle = -\frac{g^2}{16\pi^2} F_{\mu\nu} \star \tilde{F}^{\mu\nu}$$

- Invariant (Nonplanar) Anomaly

◇ In an effective theory when we keep the UV-cutoff Λ large but finite

$\Longrightarrow$ UV/IR mixing à la Minwalla *et al.* (1999) occurs

$$\langle \partial_\mu j^\mu_{inv} \rangle = \begin{cases} 0 & (\Theta p)^2 \gg \frac{1}{\Lambda^2} \qquad \text{UV-limit} \\ -\frac{g^2}{16\pi^2} F_{\mu\nu} \star' \tilde{F}^{\mu\nu} + \cdots & (\Theta p)^2 \ll \frac{1}{\Lambda^2} \qquad \text{IR- limit} \end{cases}$$

with generalized $\star$-product

$$f(x) \star' g(x) \equiv f(x+\xi) \frac{\sin\left(\frac{\Theta_{\mu\nu}}{2} \frac{\partial}{\partial \xi_\mu} \frac{\partial}{\partial \zeta_\nu}\right)}{\left(\frac{\Theta_{\mu\nu}}{2} \frac{\partial}{\partial \xi_\mu} \frac{\partial}{\partial \zeta_\nu}\right)} \, g(x+\zeta)\Big|_{\xi=\zeta=0}.$$

$$\langle \partial_\mu j^\mu_{inv} \rangle = \begin{cases} 0 & \text{UV-limit} \\ -\frac{g^2}{16\pi^2} F_{\mu\nu} \star' \tilde{F}^{\mu\nu} + \cdots & \text{IR- limit} \end{cases}$$

◇ Gauge invariant result $\longrightarrow$ attach $F\tilde{F}$ to an open noncommutative Wilson line including an external gauge field (F. Ardalan and N.S., hep-th/0307155)

◇ Here $\cdots$ denote the contributions of higher order terms arising from the expansion of the Wilson line in the order of external gauge field

II. Konishi Anomalies of the NC $\mathcal{N}=1$ SUSY $U(1)$

- The action is invariant under global $\Phi \to e^{i\alpha}\Phi$,

$$I[\Phi, V] = \int d^8z\ \bar{\Phi} \star e^V \star \Phi + \int d^6z\ W_\alpha \star W^\alpha + h.c.$$

- Two Noether currents associated with one global transformation

$$J_{cov} = \Phi \star \bar{\Phi} \star e^V, \qquad J_{inv} = \bar{\Phi} \star e^V \star \Phi$$

- Covariant Konishi (anomaly) equation

$$\bar{D}^2 J_{cov} = \Phi \star \frac{\partial W_{tree}}{\partial \Phi} + S, \qquad \text{with} \qquad S \equiv -\frac{1}{32\pi^2} W_\alpha \star W^\alpha$$

- S contains covariant anomaly $\mathcal{A}_{cov} \sim F_{\mu\nu} \star \tilde{F}^{\mu\nu}$ as component.

- Invariant Konishi (anomaly) equation

$$\bar{D}^2 J_{inv} = \frac{\partial W_{tree}}{\partial \Phi} \star \Phi + S',$$

◇ As in Non-SUSY case S' exhibits UV/IR Mixing

$$S' \equiv \begin{cases} 0 & (\Theta p)^2 \gg \frac{1}{\Lambda^2} \quad \text{UV Limit} \\ -\frac{1}{32\pi^2} W_\alpha \star' W^\alpha + \cdots & (\Theta p)^2 \ll \frac{1}{\Lambda^2} \quad \text{IR Limit} \end{cases}$$

◇ S' contains the invariant anomaly $\mathcal{A}_{inv} \sim F_{\mu\nu} \star' \tilde{F}^{\mu\nu} + \cdots$ as component

◇ The $\cdots$ indicate the contribution of higher order diagrams arising from the expansion of susy Wilson line

III. Effective Superpotential

- In SUSY the full effective superpotential

$$W_{eff.} = W_{tree} + W_{dyn.}$$

- Use the method by Veneziano-Yanckielowicz (1982-83) to determine the dynamical part of effective superpotential of our NC model

The main idea behind VY' construction for SQCD

For linear symmetry transformations

the symmetries of the original action $I[Q, V]$ are

automatically

also symmetries of the effective action $\Gamma[T, S]$

- Composite bound states of the low energy effective theory

$$\text{Meson field} \longrightarrow T \sim \text{tr}\left(Q\tilde{Q}\right)$$

$$\text{Gaugino condensate} \longrightarrow S \sim \text{tr}\left(W_\alpha W^\alpha\right)$$

$\longrightarrow$ The Strategy to determine $W_{dyn.}$

i) Looking for the symmetry properties of the original action under Axial and R-symmetry transformations $\longrightarrow$ Anomalies

$$\delta_{U_A(1)}\mathcal{L}[Q,V] = 2N_f i\alpha A(\psi)\left(\overbrace{\frac{1}{32\pi^2}\mathrm{tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)}^{S\sim\mathbf{tr}(W_\alpha W^\alpha)}\right)$$

$$\delta_{U_R(1)}\mathcal{L}[Q,V] = 2(R(\lambda)N_c + R(\psi)N_f)i\alpha\left(\underbrace{\frac{1}{32\pi^2}\mathrm{tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)}_{S\sim\mathbf{tr}(W_\alpha W^\alpha)}\right)$$

ii) Then using the fact that the symmetries of the original action $I[Q, V]$ are automatically also symmetries of the effective actions

$$\delta_{U_A(1)} \int d^2\theta \; W_{dyn}[T, S] \;\sim\; \delta_{U_A(1)} \mathcal{L}$$
$$\delta_{U_R(1)} \int d^2\theta \; W_{dyn}[T, S] \;\sim\; \delta_{U_R(1)} \mathcal{L}$$

- In $\delta\mathcal{L}$ replace $\mathrm{tr}\left(F\tilde{F}\right)$ by the Konishi anomaly $S \sim \mathrm{tr}\left(W_\alpha W^\alpha\right)$

$\Longrightarrow$ two diff. Eqs.

$$T\frac{\partial W_{dyn}}{\partial T} = -N_f S$$

$$-W_{dyn} + S\frac{\partial W_{dyn}}{\partial S} - \frac{N_c - N_f}{N_f}\, T\frac{\partial W_{dyn}}{\partial T} = 0$$

iii) Two diff. Eq. determine W_{dyn} up to some integration constant

$$W_{dyn}(T,S) = +S\left(\log\frac{S^{N_c-N_f}}{\Lambda^{\kappa}\ \det T} - (N_c - N_f)\right)$$

Λ is the RG invariant (dimensionful) scale

$$\Lambda \equiv \mu \exp\left(-\frac{8\pi^2}{b_{N_f} g^2(\mu)}\right)$$

determined by the one-loop β-function of the theory

$$\beta_{SQCD} = -\frac{g^3}{16\pi^2} b_{N_f}, \qquad b_{N_f} = (3N_c - N_f)$$

iv) Using the selection rules $\Longrightarrow$Exponent $\kappa = b_{N_f}$ can be determined (the argument of the $\log$ has no Axial and R charges and has zero mass dimension)

IV. Results for NC $\mathcal{N}=1$ SUSY $U(1)$ gauge theory

♠ UV / IR Mixing ♠

$$\left.\begin{array}{c}\text{One-loop } \beta \text{ -function} \\ U_A(1) \text{ and } U_R(1) \text{ anomalies}\end{array}\right\} \Longrightarrow \left\{\begin{array}{l}\text{Selection Rules} \\ \text{Relevant d.o.f. for the eff. theory} \\ \text{Appropriate diff. Eqs.}\end{array}\right.$$

leading to exact dynamical superpotential for NC $\mathcal{N}=1$ SUSY $U(1)$

- One-loop β-function

Adjoint matters:

Khoze *et al.* 2001 and Alvarez-Gaumé *et al.* 2003

Fundamental matters: F. Ardalan and N.S. 2003

$$\beta(g) = +\frac{g^3}{16\pi^2} b_{N_f} \qquad \text{with} \qquad b_{N_f} = \begin{cases} -2(3-N_f) & \text{UV Limit} \\ +2(3-N_f) & \text{IR Limit} \end{cases}$$

- $U_A(1)$ and $U_R(1)$ anomalies

$$\delta_{U_A(1)}\mathcal{L} = \begin{cases} 0 & \text{UV Limit} \\ 2N_f \alpha A(\psi)\left(-\frac{1}{32\pi^2}F_{\mu\nu}\star' \tilde{F}^{\mu\nu} + \cdots\right) \sim S' & \text{IR Limit} \end{cases}$$

$$\delta_{U_R(1)}\mathcal{L} = \begin{cases} 2\alpha R(\lambda)\left(-\frac{1}{32\pi^2}F_{\mu\nu}\star \tilde{F}^{\mu\nu}\right) \sim S & \text{UV Limit} \\ 2N_f \alpha R(\psi)\left(-\frac{1}{32\pi^2}F_{\mu\nu}\star' \tilde{F}^{\mu\nu} + \cdots\right) \sim S' & \text{IR Limit} \end{cases}$$

- A NC novelty: Separation of the contribution of the fermions in the fundamental and gauginos in the adjoint representation to the R-anomaly ($A(\psi) = 1$, $R(\psi) = 1/2$ and $R(\lambda) = 3/2$)

- D.o.f. for UV and IR limit

Composite bound states of the effective field theory

- UV Limit

- ◇ Meson field: $T = \tilde{Q} \star Q$
- ◇ Gaugino field: $S \sim W_\alpha \star W^\alpha$

- IR Limit

- ◇ Meson field: $T = \tilde{Q} \star Q$
- ◇ Gaugino field: $S' \sim W_\alpha \star' W^\alpha + \cdots$

Selection Rules UV Limit	$U_R(1)$-charge	$U_A(1)$-charge	m-dim
det$T$	$3N_f$	$2N_f$	$2N_f$
$\left(\Lambda_{N_f}\right)^{b_{N_f}=+2(3-N_f)}$	3	0	b_{N_f}
S	3	0	3
Λ_Θ	0	0	$+1$

Using the results for the axial and R-anomalies $\Longrightarrow$ Differential Eqs.

NC Dynamical Superpotential in UV Limit

$$W_{dyn}(S; \Lambda_{N_f}, \Lambda_\Theta) = -S\left(\log\left(\frac{S{\Lambda_\Theta}^{3-2N_f}}{{\Lambda_{N_f}}^{2(3-N_f)}}\right) - 1\right)$$

$\Lambda_\Theta \equiv 1/\sqrt{\Theta}$ as a new dimensionful constant

Selection Rules IR Limit	$U_R(1)$-charge	$U_A(1)$-charge	m-dim
det$T$	$3N_f$	$2N_f$	$2N_f$
$\left(\Lambda_{N_f}\right)^{b_{N_f}=-2(3-N_f)}$	0	$2N_f$	b_{N_f}
S'	3	0	3
Λ_Θ	0	0	$+1$

Using the results for the axial and R-anomalies $\Longrightarrow$ Differential Eqs.

NC Dynamical Superpotential in IR Limit

$$W_{dyn}(T,S';\Lambda_{N_f},\Lambda_\Theta) = S'\left(\log\left(\frac{S'^{N_f}\Lambda_\Theta{}^{(N_f+6)}}{\Lambda_{N_f}{}^{2(3+N_f)}\ \det T}\right) - N_f\right)$$

To Summarize

- Using the VY' method (anomalies) we have determined the effective superpotential of NC $\mathcal{N} = 1$ susy $U(1)$ gauge theory in two different UV and IR regimes

- To check the results: A matrix model formulation à la Dijkgraaf and Vafa for this NC model

- Then study the underlying physical phenomena as those appearing in the ordinary commutative SQCD