D-branes and SQCD in non-critical superstring theory

Nikolaos Prezas

Institut de Physique

Université de Neuchâtel

based on:

- *D-branes and SQCD in non-critical superstring theory*, with A. Fotopoulos and V. Niarchos; `hep-th/0504010`.
- *D-branes and extended characters in $SL(2,\mathbb{R})/U(1)$*, with A. Fotopoulos and V. Niarchos, Nucl. Phys **B710** (2005); `hep-th/0406017`.

Motivation: Non-critical superstrings and D-branes

Non-critical superstrings

• Strings in $d = 0, 2, 4, 6, 8$ flat dimensions,
$\mathcal{N} = (2,2)$ superconformal worldsheet theories:

$$\mathbb{R}^{d-1,1} \times \mathbb{R}^{Q}_{\phi} \times S^1 \times \mathcal{M}$$

- $\mathbb{R}^{Q}_{\phi}$ → linear dilaton
- S^1 → compact boson
- $\mathcal{M}$ → $\mathcal{N} = (2,2)$ SCFT (e.g. Landau-Ginzburg model)

Kutasov, Seiberg '90

• Dilaton $\Longrightarrow$ *strong coupling problem*. Resolution:

- $\mathbb{R}^{Q}_{\phi} \times S^1 \longrightarrow \mathcal{N} = 2$ Liouville theory **or**

$$\longrightarrow SL(2)_k/U(1) \text{ Kazama-Suzuki model, } k = \frac{2}{Q^2}.$$

• Non-critical superstrings and their D-branes are interesting:

- Backgrounds with linear dilaton: holographic (i.e. dual to theories without gravity in lower dimensions).

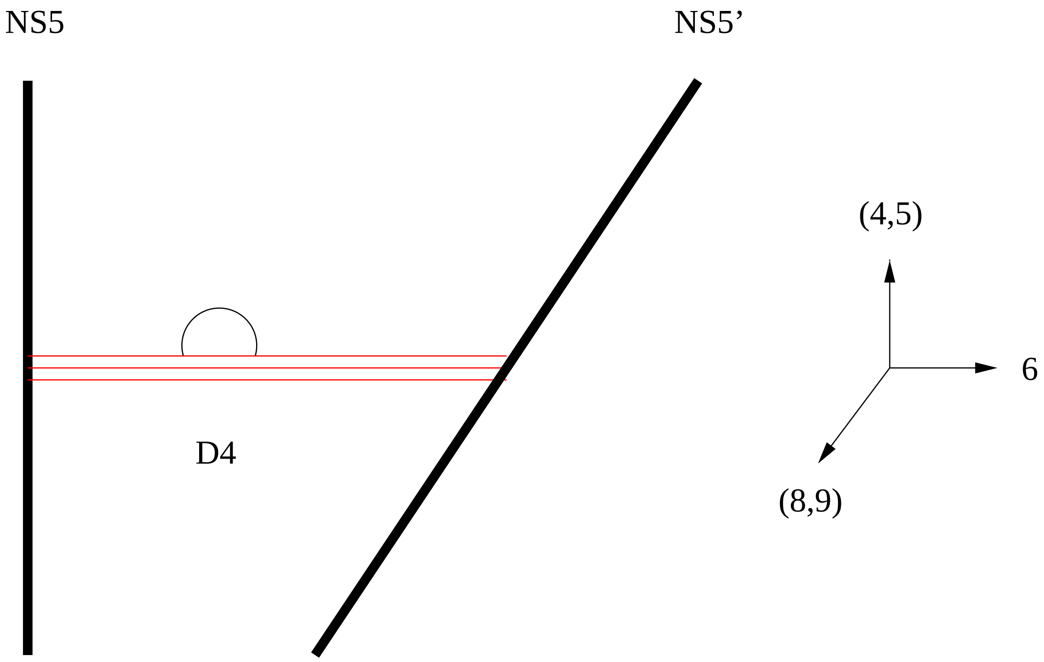
Aharony, Berkooz, Kutasov, Seiberg '98

- The above examples are dual to d-dimensional *Little String Theories* (LSTs).

Giveon, Kutasov, Pelc '99
Giveon, Kutasov '99

- Strings dynamics near *NS5-branes* or *singularities* (in certain decoupling limits).

- Consider configuration of tilted NS5-branes and N_c *finite* D4-branes:



- Decoupling limit:

$$\boxed{g_s \to 0 \ , \ \frac{L}{l_s} \to 0 \ , \ g_{YM}^2 = \frac{g_s l_s}{L} \ : \ \text{fixed}}$$

- D-brane realization of pure $\mathcal{N} = 1$ 4-dimensional $SU(N_c)$ SYM .
- Adding D4/D6-branes: matter.

Elitzur, Giveon, Kutasov '97
Elitzur, Giveon, Kutasov, Rabinovici, Schwimmer '97

- In this decoupling limit ("double scaling limit") the setup is holographically dual to a non-critical superstring:

Giveon-Kutasov '99

$$\mathbb{R}^{3,1} \times SL(2)_1/U(1)$$

- Adding D3-branes in $\mathbb{R}^{3,1} \times SL(2)_1/U(1)$:
$\mathcal{N} = 1$ SYM in 4-dimensions (in setup with exact CFT description).

- Another application: holography in the non-critical superstring setting!

Kuperstein, Sonnenschein '04
Klebanov, Maldacena '04
Bigazzi, Casero, Cotrone, Kiritsis, Paredes '05

- Example: $AdS_5 \times S_1$ dual to $\mathcal{N} = 1$ SQCD in the conformal window.

Klebanov, Maldacena '04

Conclusion: important to study D-branes in non-critical superstrings.

Strategy:

- Employ recent results on the BCFT of the "cigar" theory $SL(2)/U(1)$ and $\mathcal{N} = 2$ Liouville theory (*conformal and modular bootstrap*),

Ribault, Schomerus '03
Eguchi, Sugawara '03
Ahn, Stashnikov, Yamamoto '03,'04
Israel, Pakman, Troost '04
Fotopoulos, Niarchos, NP '04
Hosomichi '04

(which were built upon previous work on boundary Liouville field theory

Fateev, Zamolodchikov^2 '00
Teschner '00
Zamolodchikov^2 '01

and the BCFT of AdS_3).

Lee, Ooguri, Park '01
Ponsot, Schomerus, Teschner '01

- Obtain boundary states for D-branes in $\mathbb{R}^{3,1} \times SL(2)_1/U(1)$, analyze their properties.

- Engineer $\mathcal{N} = 1$ SQCQ.

- Compare with the "dual" NS5-brane configuration.

- Related work.

Elitzur, Giveon, Kutasov, Rabinovici, Sarkissian '00
Ashok, Murthy, Troost '05

Type II strings on $\mathbb{R}^{3,1} \times SL(2,\mathbb{R})_1/U(1)$

Review of $SL(2)_k/U(1)$ conformal field theory

- $\mathcal{N} = (2,2)$ superconformal field theory with central charge

$$\hat{c} = \frac{c}{3} = 1 + \frac{2}{k}$$

- $\mathbb{R}^{3,1} \times SL(2,\mathbb{R})_1/U(1)$: consistent string background $\Longrightarrow k = 1$.

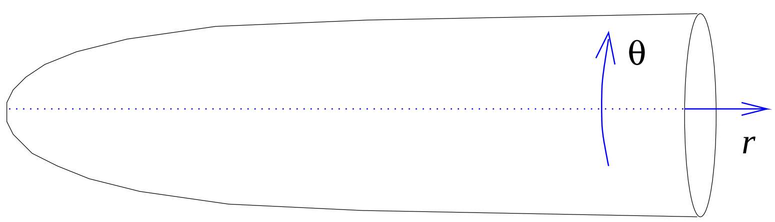
- Describes string propagation on a *cigar-shaped* manifold:

$$ds^2 = k(dr^2 + \tanh^2 r d\theta^2) \ , \ \theta \sim \theta + 2\pi \ ,$$

$$e^{\Phi(r)} = \frac{e^{\Phi_0}}{\cosh r}, \ B_{\mu\nu} = 0$$

- This background is *exact* in α' in the supersymmetric case .

Bars, Sfetsos '92
Tseytlin ' 93



- T-dual (mirror) target space: trumpet $\Longrightarrow \mathcal{N} = 2$ Liouville theory.

Giveon, Kutasov '99
Hori, Kapustin '01

• Spectrum: unitary representations of the $SL(2)/U(1)$ non-compact parafermionic algebra (j, m):

Dobrev '87
Kiritsis '87

- *Continuous:* $j = \frac{1}{2} + is$, $s \in \mathbb{R}_{\geq 0}$, $m = r + \alpha$, $r \in \mathbb{Z}$, $\alpha \in [0, 1)$
- *Discrete:* $j \in \mathbb{R}$, $0 < j < \frac{k+2}{2}$, $m = j + r$, $r \in \mathbb{Z}$
- *Identity:* $j = 0$, $m = r \in \mathbb{Z}$

Closed string spectrum of $\mathbb{R}^{3,1} \times SL(2,\mathbb{R})_1/U(1)$

• Type OA/B and Type IIA/B non-critical superstrings on $\mathbb{R}^{3,1} \times SL(2,\mathbb{R})_1/U(1)$ as in critical case: diagonal or chiral GSO projection.

• Toroidal partition function $\Longrightarrow$ closed string physical spectrum.

Mizoguchi '00
Eguchi, Sugawara '00, '04
Hanany, NP, Troost '02
Murthy '03
Israel, Kounnas, Pakman, Troost '04

- General k: both continuous and discrete representations contribute (but not the identity).

- Continuous: wavelike modes along the cigar, contribute with a volume factor.

- Discrete: bound states near the tip, yield a finite contribution. j is truncated to

$$\frac{1}{2} < j < \frac{k+1}{2}, \quad 2j \in \mathbb{Z}$$

- In our case ($k = 1$) we have no discrete states!
- Momenta and winding around cigar:

$$m = \frac{n - kw}{2}, \qquad \bar{m} = -\frac{n + kw}{2}.$$

• Chiral GSO projection: supersymmetric spectrum (8 supercharges), partition function $= 0$ due to generalized Jacobi identities.

Bilal, Gervais '87

Theory	Sector	Fields
IIA and IIB	$NS+NS+$	$G_{\mu\nu},\ B_{\mu\nu},\ \phi$
	$NS-NS-$	$T,\ T'$
IIA	$R+R-$	A_1
	$R-R+$	A_1'
IIB	$R+R+$	$C_0,\ C_2^+$
	$R-R-$	$C_0',\ C_2^-$

- Massive graviton (NS+NS+).

- Two complex massless "tachyons" (NS-NS-).

- Continuous spectrum of RR fields, type IIB: massless C_0, C_2^+ for $s = 0$.

D-branes and BCFT on $\mathbb{R}^{3,1} \times SL(2,\mathbb{R})_1/U(1)$

• D-branes in conformal field theory $\Longrightarrow$ *boundary states*:
encode couplings of D-branes to closed string modes.

Boundary conditions

• $\mathbb{R}^{3,1}$ part:

- We want to engineer 3+1-dimensional gauge theories: *Neumann* in $\mathbb{R}^{3,1}$.
- Such boundary states are well-known.

Di Vecchia, Liccardo '99
Gaberdiel '00

• $SL(2)/U(1)$ part:

-Two types of $\mathcal{N} = 2$ boundary conditions:

$$\begin{array}{lll} \texttt{A-type:} & (J_n - \bar{J}_{-n})|B\rangle = 0 \ , & (G_r^{\pm} - i\eta\bar{G}_{-r}^{\mp})|B\rangle = 0 \ , \\ \texttt{B-type:} & (J_n + \bar{J}_{-n})|B\rangle = 0 \ , & (G_r^{\pm} - i\eta\bar{G}_{-r}^{\pm})|B\rangle = 0 \ . \end{array}$$

- A-type: Dirichlet bc's on θ, B-type: Neumann bc's on θ.
- Corresponding solutions: A- and B-Ishibashi states.
- Boundary states have been proposed (*conformal and modular bootstrap*).

Eguchi, Sugawara '03
Ahn, Stashnikov, Yamamoto '04
Israel, Pakman, Troost '04
Fotopoulos, Niarchos, NP '04
Hosomichi '04

• Construct boundary states in full theory:

- tensor flat and coset Ishibashi states.
- same spin structure on flat and coset parts.
- GSO projection.

- Type IIB theory: only B-type boundary states are BPS.

- Generic B-type boundary state: $|B;\xi\rangle_{\mathrm{NS}} \pm |B;\xi\rangle_{\mathrm{R}}$,

$\xi : SL(2)/U(1)$ representation.

B-type boundary states:

- Class 1 (analogs of ZZ branes)

- ξ: identity representation.

- Bulk one-point functions on the disk:

$$\langle \mathcal{V}^{NS+NS+}_{\frac{1}{2}+is,m,\bar{m}}(p^\mu)\rangle_{D3} = \langle \mathcal{V}^{R-R-}_{\frac{1}{2}+is,m+\frac{1}{2},\bar{m}+\frac{1}{2}}(p^\mu)\rangle_{D3} =$$

$$= \delta^{(4)}(p^\mu)\ \delta_{m,\bar{m}}\ \frac{1}{2}\frac{\Gamma(\frac{1}{2}+is+m)\Gamma(\frac{1}{2}+is-m)}{\Gamma(1+2is)\Gamma(2is)} \sim$$

$$\sim \delta^{(4)}(p^\mu)\ \delta_{m,\bar{m}}\ \sinh(\pi s)\ ,\ m \in \mathbb{Z}$$

$$\langle \mathcal{V}^{NS-NS-}_{\frac{1}{2}+is,m,\bar{m}}(p^\mu)\rangle_{D3} = \langle \mathcal{V}^{R+R+}_{\frac{1}{2}+is,m+\frac{1}{2},\bar{m}+\frac{1}{2}}(p^\mu)\rangle_{D3} =$$

$$= \delta^{(4)}(p^\mu)\ \delta_{m,\bar{m}}\ \frac{1}{2}\frac{\Gamma(\frac{1}{2}+is+m)\Gamma(\frac{1}{2}+is-m)}{\Gamma(1+2is)\Gamma(2is)} \sim$$

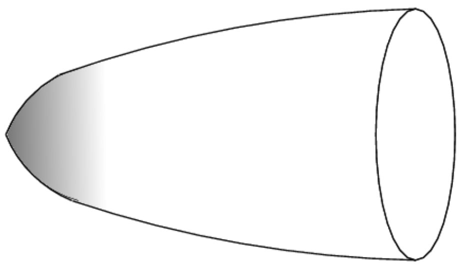
$$\sim \delta^{(4)}(p^\mu)\ \delta_{m,\bar{m}}\ \cosh(\pi s)\ ,\ m \in \mathbb{Z}+\frac{1}{2}$$

- Features:

- Open string spectrum: *identity* representation.

- Geometry: branes localized near the tip of the cigar.

- Full theory: D3-branes.



- Class 2 (analogs of FZZT branes)

- ξ: continuous representation, $|B;s,m\rangle_{NS} \pm |B;s,m\rangle_R$, $s \in \mathbb{R}^+, m = 0, 1/2$.
- Bulk one-point functions on the disc:

$$\langle \mathcal{V}^{NS+NS+}_{\frac{1}{2}+is',m',\bar{m}'}(p^\mu)\rangle_{D5(s,m)} = (-)^{2m}\langle \mathcal{V}^{R-R-}_{\frac{1}{2}+is',m'+\frac{1}{2},\bar{m}'+\frac{1}{2}}(p^\mu)\rangle_{D5(s,m)}$$

$$= \delta^{(4)}(p^\mu)\ \delta_{m',\bar{m}'}\ \cos(4\pi s s')\frac{\Gamma(1+2is')\Gamma(2is')}{\Gamma(\frac{1}{2}+is'+m')\Gamma(\frac{1}{2}+is'-m')}$$

$$\sim \delta^{(4)}(p^\mu)\ \delta_{m',\bar{m}'}\ \frac{\cos(4\pi s s')}{2\sinh(\pi s')}\ ,\ m' \in \mathbb{Z}$$

$$\langle \mathcal{V}^{NS-NS-}_{\frac{1}{2}+is',m',\bar{m}'}(p^\mu)\rangle_{D5(s,m)} = (-)^{2m}\langle \mathcal{V}^{R+R+}_{\frac{1}{2}+is',m'+\frac{1}{2},\bar{m}'+\frac{1}{2}}(p^\mu)\rangle_{D5(s,m)}$$

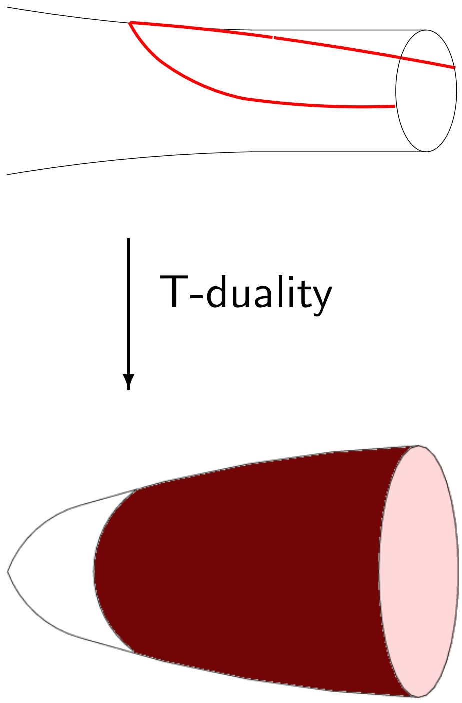
$$= \delta^{(4)}(p^\mu)\delta_{m',\bar{m}'}\cos(4\pi s s')\frac{(-)^{2m}\Gamma(1+2is')\Gamma(2is')}{\Gamma(\frac{1}{2}+is'+m')\Gamma(\frac{1}{2}+is'-m')}$$

$$\sim \delta^{(4)}(p^\mu)\ \delta_{m',\bar{m}'}\ (-)^{2m}\frac{\cos(4\pi s s')}{2\cosh(\pi s')}\ ,\ m' \in \mathbb{Z}+\frac{1}{2}$$

- Comments:

- Semiclassics: Fotopoulos '03, boundary states: Fotopoulos, Niarchos, NP '04.
- Derivation: T-duality from A-type D4-branes in $\mathcal{N}=2$ Liouville theory.
- Consistency: *generalized Cardy check* and *factorization constraints*.

Fotopoulos, Niarchos, NP '04
Hosomichi '04



- Features:

- Wrap the cigar from $\rho = s \longrightarrow \rho = \infty$.
- $m \in \mathbb{Z}_2$: Wilson line.
- Full theory $\Longrightarrow$ D5-branes.
- Open string spectrum: continuous representations.
- Modulus: changing s.
- Integer and half-integer momentum modes: <u>two</u> "sheets" (compare to T-dual theory).

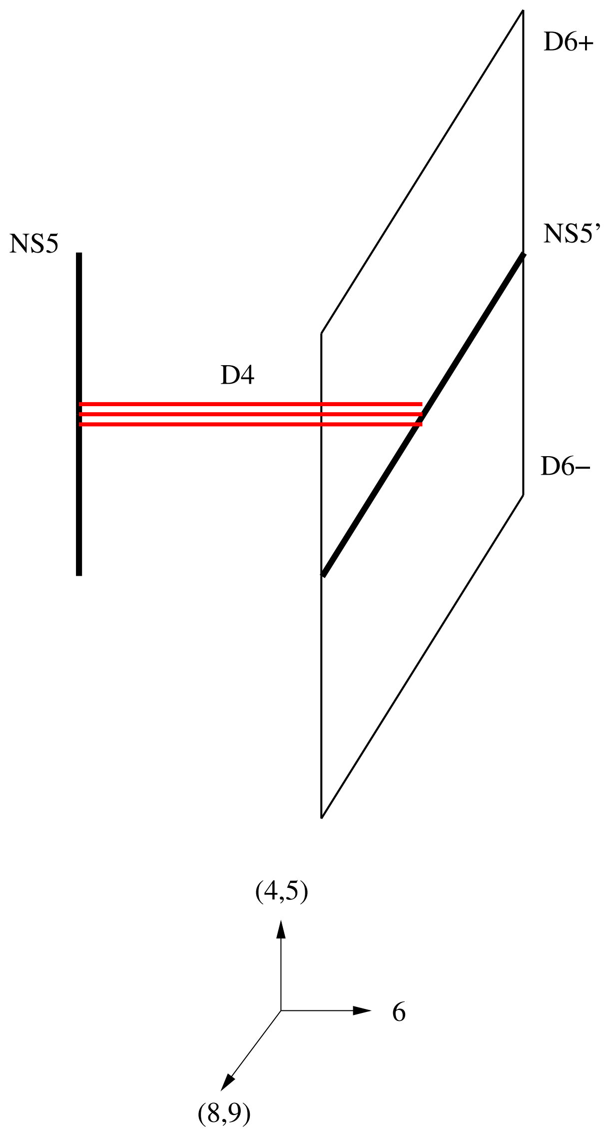
- Annulus amplitudes: self-overlaps are zero (SUSY), only $|B\rangle, |B, s, 0\rangle, \overline{|B, s, \frac{1}{2}\rangle}$ are mutually SUSY

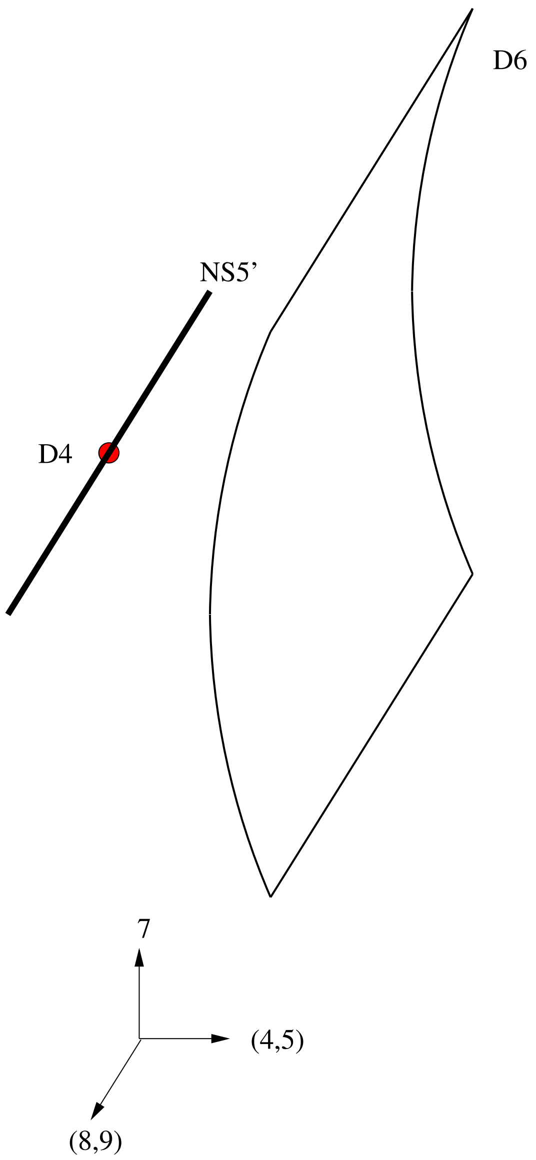
Supersymmetric QCD in non-critical D3-D5 systems

Dual NS5-brane setup in critical string theory

- Orthogonal NS5-branes, D4- and D6-branes $\Longrightarrow$ SQCD.

Elitzur, Giveon, Kutasov, Rabinovici, Schwimmer '97
Brodie, Hanany '97
Hanany, Zaffaroni '97





- Finite D4-branes $\longrightarrow$ D3-branes in $\mathbb{R}^{3,1} \times SL(2)_1/U(1)$: $|B\rangle$.

• D6-branes $\longrightarrow$ D5-branes $|B,s,0\rangle, \overline{|B,s,\frac{1}{2}\rangle}$.

- D6-brane is intersected by NS'-brane to D6+ and D6- $\Longleftrightarrow$ D5-brane reaches the tip (two sheets): $s=0$.
- D6-brane is moved in x^4, x^5 direction $\Longleftrightarrow$ D5-brane at distance s from the tip (sheets are continuously connected): $s>0$.

• Can we keep only one D6-brane sheet ? No, due to tadpole! $\Longleftrightarrow$ Cannot break consistently $|B;0,0\rangle, \overline{|B;0,\frac{1}{2}\rangle}$.

Engineering of supersymmetric QCD

• Consider N_c D3-branes and N_f D5-branes in $\mathbb{R}^{3,1}\times SL(2)_1/U(1)$.

-Spectrum of D3-brane:

$$\begin{aligned}\langle B|e^{-\pi T H^c}|B\rangle &= \frac{1}{2}\Big(\chi_I(it)\big[{}^0_0\big]\frac{\theta[{}^0_0](it)}{\eta(it)^3} - \chi_I(it)\big[{}^0_1\big]\frac{\theta[{}^0_1](it)}{\eta(it)^3} \\ &- \chi_I(it)\big[{}^1_0\big]\frac{\theta[{}^1_0](it)}{\eta(it)^3} + \chi_I(it)\big[{}^1_1\big]\frac{\theta[{}^1_0](it)}{\eta(it)^3}\Big) \\ &= \big(2+\mathcal{O}(q)\big) - \big(2+\mathcal{O}(q)\big)\end{aligned}$$

- NS+, R-: yields $\mathcal{N}=1$ vector multiplet (gauge field + gauginos).
- Spectrum of D3-D5 strings:

$$\begin{aligned}\langle B|e^{-\pi T H^c}|B;s,0\rangle &= \frac{1}{2}\Lambda_1(s;it)=0 \quad NS+, R- \ \text{(all massive)} \\ \langle B|e^{-\pi T H^c}\overline{|B;s,\frac{1}{2}\rangle} &= \frac{1}{2}\Lambda_{-1}(s;it)=0 \quad NS-, R+ \ (s=0:\text{massless})\end{aligned}$$

- In particular:

$$\begin{aligned}\frac{1}{2}\Lambda_{-1}(s;it) &= \left(\chi_c(s,\frac{1}{2};it)\left[{0 \atop 0}\right]\frac{\theta[{0 \atop 0}]}{\eta(it)^3} + \chi_c(s,\frac{1}{2};it)\left[{0 \atop 1}\right]\frac{\theta[{0 \atop 1}]}{\eta(it)^3}\right) \\ &- \left(\chi_c(s,0;it)\left[{1 \atop 0}\right]\frac{\theta[{1 \atop 0}]}{\eta(it)^3} + \chi_c(s,0;it)\left[{1 \atop 1}\right]\frac{\theta[{1 \atop 1}]}{\eta(it)^3}\right) \\ &= \left(2q^{s^2} + \mathcal{O}(q^{s^2+\frac{1}{2}})\right)_{NS-} - \left(2q^{s^2} + \mathcal{O}(q^{s^2+\frac{1}{2}})\right)_{R+}\end{aligned}$$

- Two lowest level contributions: quark multiplets Q^i and $\tilde{Q}_j^\dagger$ (D5-D3 strings give the conjugates).
- $Q, \tilde{Q}$ due to two-D5-sheets.
- $s > 0$: quarks acquire mass $m \sim s$ (as in dual brane setup).
- Q transforms under (N_c, N_f), $\tilde{Q}$ transforms under $(\bar{N}_c, \bar{N}_f)$.

- **Conclusion: appropriate D5-branes $\longrightarrow \overline{|B; s, \frac{1}{2}\rangle}$**
- Spectrum ($e.g\ s = 0$):

$$\begin{aligned}\langle\overline{B;0,\frac{1}{2}}|e^{-\pi T H^c}|\overline{B;0,\frac{1}{2}}\rangle &= \int_0^\infty ds' \left(\rho_+(s';0|0)\Lambda_1(s';it)\right. \\ &+ \left.\rho_-(s';0|0)\Lambda_{-1}(s';it)\right)\end{aligned}$$

- No massless gauge fields (dual NS5-brane setup: wavefunctions not localized near NS5-branes).

Elitzur, Giveon, Kutasov, Rabinovici, Sarkissian '00

- Massless chiral multiplet M_i^j in $(N_f, \bar{N}_f)$: moduli corresponding to s_i.
- M_i^j: parameters in D3-brane theory (live in higher-dimensional brane).
- Three-string interaction:

$$W_M = \mathrm{Tr}\ M_i^{\tilde{j}} Q^i \tilde{Q}_{\tilde{j}}$$

- Turning on $M_i^j \Longrightarrow s_i > 0 \Longrightarrow$ mass terms for $Q, \tilde{Q}$.

• Symmetries:

- Classical symmetry of $\mathcal{N} = 1$ SQCD:

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_a \times U(1)_x$$

- Vector $SU(N_f)$: present when all D5-branes have the same s_i.
- Axial $SU(N_f)$: appears when $s_i = 0$ (two-sheets become disconnected $\Longrightarrow$ global flavor symmetry: $U(N_f) \times U(N_f)$ – compare to D6+/D6-).

Brodie, Hanany '97
Hanany, Zaffaroni '97

- Open string theory on D3- and D5-branes ($s = 0$) has 3 U(1)s:

flavor $U(1) \times U(1)$, angular momentum $U(1)_m \longrightarrow U(1)_B \times U(1)_a \times U(1)_x$.

• Moduli space:

- Moduli space of SQCD: VEVs to mesons/baryons $\Longrightarrow$ Higgsing.
- NS5-brane picture: splitting D4-branes on D6-branes according to *s-rule*.

Hanany, Witten '96

- Non-critical superstrings: unclear how to realize it with available boundary states.

Summary and open problems

Summary

• We have studied D-branes in a specific non-critical superstring theory using BCFT methods.

• We have engineered SQCD with non-critical D-branes and verified several statements concerning the dual NS5-brane setup.

Open problems

• $|B; s, 0\rangle$ role in NS5-brane setup ?

• Brane realization of Higgsing ? Microscopic derivation of s-rule ?

• What is the magnetic description of SQCD in this setting ?

• Can we realize Seiberg duality ?

Elitzur, Giveon, Kutasov, Rabinovici, Schwimmer '97
Oogurі, Vafa '97
Klebanov, Maldacena '04

• Precise form of WZ couplings.

• Quantum aspects:

- Understand further non-critical holography.

- No region of validity of supergravity!

- Backreaction of branes.

Di Vecchia, Frau, Pesando, Sciuto, Lerda, Russo '97
Ashok, Murthy, Troost '05