Non-critical holography and four-dimensional CFT's with fundamentals

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Plan of the talk

- Motivation
- A note on CFT's with fundamentals and their duals
- $D = 6$: $AdS_5 \times S^1$ and $\mathcal{N} = 1$ SCQCD
- D=8: A supersymmetric vacuum and an $AdS_5 \times \tilde{S}^3$ solution
- D=5: AdS_5 and $\mathcal{N} = 0$ conformal QCD
- Outlook

Motivation

GAUGE-STRING DUALITY: D=10 string theory has too many degrees of freedom to incorporate faithful duals of every gauge theory.

Introducing non-critical strings:

A string theory is consistent if the worldsheet SCFT has $c = 15$.

A flat dimension contributes $\frac{3}{2}$.

$$\frac{3}{2}D = 15 \Rightarrow D = 10 \quad \text{(critical superstring theory)}$$

If some dimensions are not flat, like a linear dilaton (Lorentz symmetry is broken):

$$\frac{3}{2}D + \delta c = 15 \;\Rightarrow\; D \neq 10 \quad \text{(non-critical strings)}$$

General feature: Large (order α') curvatures

Fundamental flavors and problems for holography

The conformal anomaly

A conformal field theory coupled to gravity presents a trace anomaly (also called Weyl or conformal anomaly). When $d = 4$:

$$\boxed{< T^{\mu}_{\mu} >= -aE_4 - cI_4}$$

where

$$E_4 : \text{Euler density}$$

$$I_4 : (W_{\mu\nu\rho\sigma})^2$$

Their relation to the fundamentals:

The central charges can be written as:

$$a = \frac{3}{32}[3TrR^3 - TrR], \quad c = \frac{1}{32}[9TrR^3 - 5TrR].$$

This, along with the vanishing of the β-functions yields:

$$a - c = \frac{N_c}{32}\left[\sum_{f=1}^{N_f}(R_f - 1) + \sum_{\tilde{f}=1}^{N_f}(R_{\tilde{f}} - 1)\right]$$

General theorem in AdS/CFT:

Henningson, Skenderis

$a = c$ at leading order in N_c if the action is that of weakly curved supergravity.

Conclusion:

String duals to $d = 4$ SCFT's with fundamental matter ($N_f \sim N_c$) need in general large curvature!

The non-critical dual of $\mathcal{N}=1$ SCQCD

Klebanov, Maldacena

The setup: Consider the $D = 6$ vacuum (8 supercharges):

$$\boxed{\mathrm{IR}^{1,3} \times \frac{SL(2,\mathrm{IR})}{U(1)}}$$

The $U(1)_R$ simmetry is realized as the isometry of the cigar.

$\mathcal{N} = 1$ super QCD has an IR conformal fixed point in the conformal window:

$$\boxed{\frac{1}{3} < \frac{N_c}{N_f} < \frac{2}{3}}$$

After backreaction, we should have $AdS_5 \times S^1$.

	$x_{1,3}$				τ	ψ
(color) D3	—	—	—	—	·	·
(flavor) D5	—	—	—	—	—	○

$\overbrace{\tau\ \psi}^{cigar}$, $\underbrace{x_{1,3}\ \tau}_{AdS_5}$, $\underbrace{\psi}_{S^1}$

Consider the action:

$$S = \int d^6x\sqrt{g}\left(e^{-2\phi}(R + 4(\partial\phi)^2 + \frac{10-D}{\alpha'}) - F_{(5)}^2\right)$$

$$-2N_f \int d^6x e^{-\phi}\sqrt{g}$$

where the last term accounts for the backreaction of the flavor branes.

There is an $AdS_5 \times S^1$ for every $\frac{N_f}{N_c} \neq 0$ with:

$$\boxed{R_{AdS} = \sqrt{6\alpha'}\ , \qquad R_{S^1} = \frac{N_c}{N_f}\sqrt{\frac{2}{3}}\sqrt{\alpha'}\ , \qquad e^{\phi} = \frac{2}{3N_f}\ ,}$$

Conjecture: The conformal window should be selected by keeping only the solutions not destabilized by the open string tachyon living on the D5 branes.

(This setup has been further studied using worldsheet CFT by Fotopoulos, Niarchos, Prezas and Ashok, Murthy, Troost)

D=8: more non-critical vacua

One may consider the vacuum (16 supercharges):

$$\boxed{\mathrm{I\!R}^{1,3} \times \mathrm{I\!R}^2 \times \frac{SL(2,\mathrm{I\!R})}{U(1)}}$$

so placing D3 and D5 branes in it would engineer $\mathcal{N} = 2$ SQCD.

I will consider a different vacuum (8 supercharges):

$$\boxed{\mathrm{I\!R}^{1,3} \times KKL_4}$$

The metric is: Kiritsis, Kounnas, Lüst

$$ds_4^2 = \frac{d\zeta^2}{4f_2(\zeta)} + \frac{\zeta}{4}(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{f_2(\zeta)}{4}(d\psi + \cos\theta d\varphi)^2$$

with:

$$f_2(\zeta) = 32\,\frac{e^{-\frac{\zeta}{4}} - 1 + \frac{\zeta}{4}}{\zeta}$$

$$\phi = \phi_0 - \frac{\zeta}{8}$$

The isometry is $SU(2) \times U(1)$

We can construct the dual of an $\mathcal{N}=1$ SCFT with $SU(2) \times U(1)_R$ global symmetry. The backreacted solution should be $AdS_5 \times \tilde{S}^3$ where:

$$ds^2_{\tilde{S}^3} = e^{2\nu}(d\theta^2 + \sin^2\theta d\varphi^2) + e^{2f}(d\psi + \cos\theta d\varphi)^2$$

is a homogeneous, anisotropic space.

Brane configuration:

					KKL_4			
	$x_{1,3}$				τ	ψ	θ	φ
(color) D3	—	—	—	—	·	·	·	·
(flavor) D5	—	—	—	—	—	◯	$\sim$	$\sim$
	AdS_5					$\tilde{S}^3$		

In order to preserve the $SU(2)$ symmetry, the D5-branes are smeared along their transverse space.

Consider the action is:

$$S = \frac{1}{2\kappa^2_{(8)}} \int d^8x \sqrt{-g_{(8)}} \Big(e^{-2\phi} \Big(R + 4(\partial_\mu \phi)^2 + \frac{10-D}{\alpha'} \Big) - F^2_{(5)} \Big) + S_{flavor}$$

The action for the flavor branes comes from their DBI action:

$$S_{flavor} = -T_5 \sum_{i=1}^{N_f} \int_{\mathcal{M}_6} d^6\xi e^{-\phi} \sqrt{-\hat{g}_{(6)}} \rightarrow$$

$$\rightarrow -\frac{T_5 N_f}{4\pi} \int d^8x \sin\theta \, e^{-\phi} \sqrt{-\hat{g}_{(6)}}$$

where $\mathcal{M}_6$ is the 6-dimensional worldvolume of the flavor branes along $x_0, \ldots, x_3, \tau, \psi$.

There is a one-parameter family of $AdS_5 \times \tilde{S}^3$ solutions, depending on N_f/N_c

The special solution:

For just one value of $\frac{N_f}{N_c}$ we can argue that the solution is supersymmetric. Naively, it corresponds to:

$$\boxed{\frac{N_f}{N_c} \approx 2.56}$$

and the metric is:

$$ds_8^2 = e^{\frac{2}{\sqrt{6}}\tau}\, dx_{1,3}^2 + d\tau^2 +$$

$$(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{2}{3}(d\psi + \cos\theta d\varphi)^2$$

There are two arguments that indicate this special solution is supersymmetric:

- It comes from a set of first order equations obtained from a simple superpotential

- It can be interpreted as the backreaction of branes in $\mathbb{R}^{1,3} \times KKL_4$

$\boxed{\text{1. Existence of a superpotential}}$

For an action of the type:

$$S = \int du\, e^{4\alpha} \left(3(\alpha')^2 - \frac{1}{2} G_{ab}(f) f'^a f'^b - V(f) \right)$$

if one can find W such that:

$$V = \frac{1}{8} G^{ab} \frac{\partial W}{\partial f^a} \frac{\partial W}{\partial f^b} - \frac{1}{3} W^2$$

then the second order equations of motion can be solved by a first order system. In supersymmetric cases, one can always find a W and it turns out to be of a simple form.

For $\boxed{\frac{N_f}{N_c} \approx 2.56}$ one finds

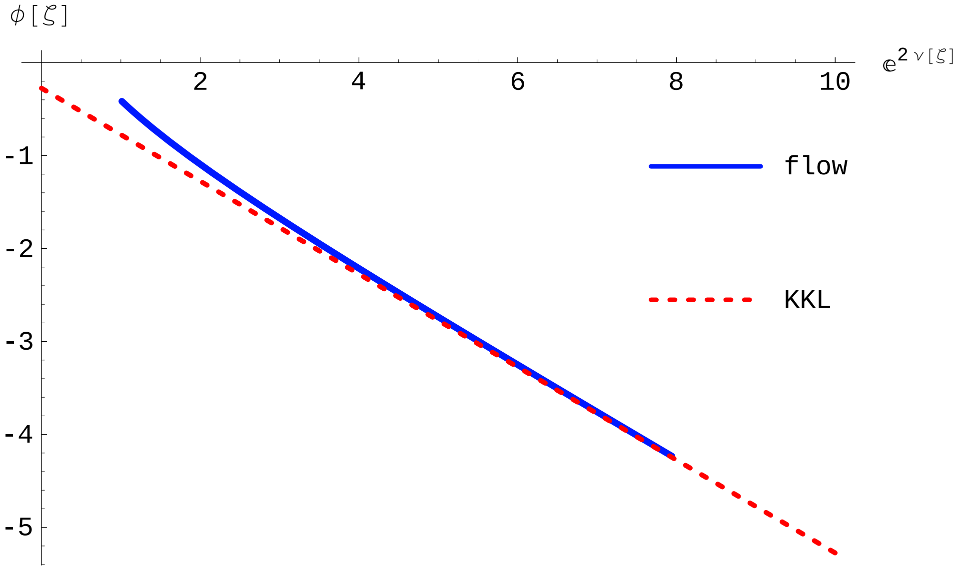
$$W = -\frac{1}{2} e^{\frac{2}{3}\phi - 2\gamma} - e^{-2\beta+2\gamma} - \frac{1}{2} e^{-2\beta-4\gamma} + Q_f e^{\frac{1}{3}\phi - 4\beta}$$

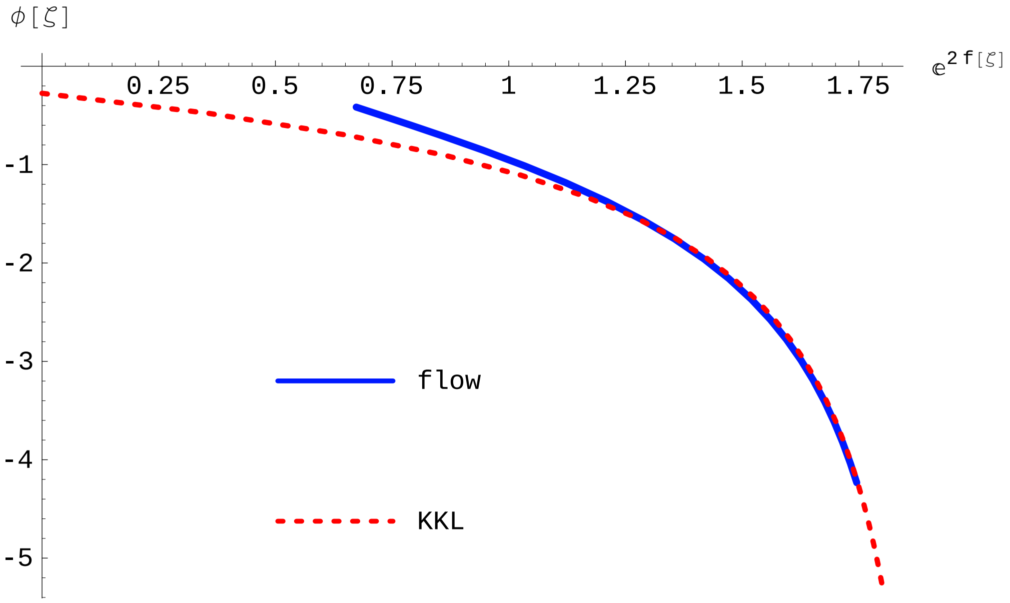
- For $N_f = 0$, the solution is KKL_4

- For constant ϕ, one gets the special $AdS_5 \times \tilde{S}^3$ solution.

For the rest, we have only been able to integrate numerically.

2. Flow from $AdS_5 \times \tilde{S}^3$ to $\mathrm{IR}^{1,3} \times KKL$





On the dual gauge theory

Unfortunately, because of the intrinsic limitations of the gravity approximation and a lack of knowledge of brane dynamics on the KKL_4 vacuum, we have not found a final answer.

From work of Hori, Kapustin, we know that KKL_4 is related to NS5-branes wrapping the vanishing CP_1 inside Eguchi-Hanson and it should have a marginal deformation corresponding to the blowing up of the CP_1.

Some hints:

- Contains fundamental matter
- $\mathcal{N} = 1$ supersymmetry
- $SU(2) \times U(1)_R$ global symmetry
- It is conformal only for a fixed N_f/N_c ratio.
- There is a marginal operator
- $a, c \sim N_c^2$ and $\Delta_{baryons} \sim N_c$

D=5 non-critical strings and $\mathcal{N}=0$ CQCD

As $\mathcal{N}=1$ SQCD, non-supersymmetric QCD is supposed to have also a conformal window, although its bounds in N_f/N_c are not so clear.

On general grounds, we expect the dual string theory to live in $D=5$, and the geometry should be just AdS_5.

We consider:

	$x_{1,3}$				τ
(color) D3	—	—	—	—	·
(flavor) D4-anti D4	—	—	—	—	—

$SU(N_f) \times SU(N_f)$ $\leftrightarrow$ gauge group D4-anti D4

θ_{YM} $\leftrightarrow$ constant axion

$U(1)_A$ $\leftrightarrow$ symmetry of the complex tachyon

Gravity solution

With the action:

$$S = \frac{1}{2\kappa_{(5)}^2} \int d^5x \sqrt{-g_{(5)}} \Big(e^{-2\phi} \left(R + 4(\partial_\mu \phi)^2 + 5 \right) - F_{(5)}^2 - 2Q_f e^{-\phi} \Big)$$

there is an AdS_5 solution with constant dilaton $e^{\phi} \sim N^{-1}$. Calling $\boxed{x \equiv Q_f/Q_c}$:

$$R_{AdS}^2 = \frac{200}{50 + 7x^2 - x\sqrt{200 + 49x^2}}$$

Fluctuations of the flavor branes

Gauge field $\rightarrow$ vector mesons $(\Delta = 3)$

Complex tachyon $\rightarrow$ scalar mesons $\left(\Delta = 2 \pm \sqrt{4 - \frac{R_{AdS}^2}{2}}\ \right)$

Conjecture: In the complete (string corrected) solution, the condition $6 \leq R_{AdS}^2 \leq 8$ should select the values of N_f/N_c in the conformal window.

Summary

- SCFT's with $N_f \sim N_c$ cannot have, in general, weakly curved gravitational duals.

- D3 and D5-branes on the cigar engineer $\mathcal{N} = 1 SQCD$.

- An $AdS_5 \times \tilde{S}^3$ is the solution corresponding to the backreaction of (color) D3-branes and (flavor) D5-branes in a non-critical supersymmetric vacuum $\mathbb{R}^{1,3} \times KKL_4$.

- The dual theory seems to be superconformal only for a fixed ratio of $\frac{N_f}{N_c}$.

- A five dimensional string dual for QCD in the conformal window was proposed. Some qualitative features can be matched. We have conjectured how the limits of the conformal window should show up in this setup.

Outlook

- Non-critical strings are needed to study the holographic duals of realistic theories.

- The geometries obtained have curvatures of the order of the string scale.

- Two complementary approaches can be followed:

  – Use worldsheet CFT techniques to study the physic of strings in the relevant vacua

  – Use the gravity approximation in order to get some qualitative insight (at least for symmetric enough spaces)

- There is a long way to go...