Non-Critical Covariant Superstrings

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Outline

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Introduction and Motivations

- The critical dimension for superstrings in flat space is $d = 10$. In dimensions $d < 10$, the Liouville mode is dynamical and should be quantized as well (Polyakov). We will call these strings non-critical. The total conformal anomaly vanishes for the non-critical strings due to the Liouville background charge.
- There are various motivations to study non-critical strings. First, non-critical strings can provide and alternative to string compactifications. Second, non-critical strings are expected to provide a dual description to gauge theories.
- Examples of backgrounds one wishes to study are

$$ds^2 = d\varphi^2 + a^2(\varphi)d\vec{x}^2 \tag{1}$$

  where $\vec{x} = (x_1, \ldots, x_{d-1})$, and with other background fields turned on, which may provide a dual description of gauge theories. Depending on the form of the warp factor $a^2(\varphi)$, the gauge theory can be confining, or at a conformal fixed point.
- A complication: The d-dimensional supergravity low-energy effective action contains a cosomlogical constant type term of the form

$$S \sim \int d^d x \sqrt{G} e^{-2\Phi} \left(\frac{d-10}{l_s^2} \right) \tag{2}$$

which vanishes only for $d = 10$. This implies that low energy approximation $E \ll l_s^{-1}$ is not valid when $d \neq 10$, and the higher order curvature terms of the form $\left(l_s^2 \mathcal{R}\right)^n$ cannot be discarded (see Cobi Sonnenschein's lectures in this conference)

- In addition: interesting target space curved geometries include Ramond-Ramond (RR) field fluxes, and we face the need to quantize the strings in such backgrounds.
- The aim: to construct a covariant formulation of the non-critical superstrings, that will allow us to quantize the strings in curved backgrounds with RR fields.
- We start by considering fermionic strings propagating on a linear dilaton background

$$ds^2 = \eta_{ij} dx^i dx^j + dx^2 + d\varphi^2, \qquad \Phi = -\frac{Q}{\sqrt{2}}\varphi \quad (3)$$

where $i, j = 1, \ldots, 2n$, $n = 0, 1, 2, 3$, $x \in S^1$ of radius $R = 2/Q$, $Q = \sqrt{4-n}$, and Φ is the dilaton field.

- Since a flat background with constant dilaton field is not a solution of the non-critical string equations, the linear dilaton background will be used by us to make the dictionary between the NSR non-critical strings and the covariant non-critical strings.
- This dictionary will be used in order to couple the non-critical strings to curved backgrounds with RR fields.

NSR Non-Critical Superstrings

- The $(2n+2)$-dimensional fermionic strings with $n = 0, 1, 2, 3$, are described in the superconformal gauge by $2n+1$ matter superfields $X^i, i = 1, \ldots, 2n+1 \equiv D$, and by a Liouville superfield Φ_l (Kutasov,Seiberg). In components we have (x^i, ψ^i), (φ, ψ_φ). As usual, we have two ghost systems (β, γ) and (b, c).
- The central charges are given by $(2n+1, (2n+1)/2)$, $(1+3Q^2, 1/2)$, 11 and -26. The total central charge is given by $3(2n+1)/2 + 1/2 + (1+3Q^2) - 15 = 3(n+Q^2-4)$. It vanishes for

$$Q(n) = \sqrt{4-n} \tag{4}$$

- The stress energy tensor of the system reads

$$T = T_m + T_{ghost} \tag{5}$$

$$T_m = \sum_{i=1}^{2n+1} \Big(-\frac{1}{2}(\partial x^i)^2 + \frac{1}{2}\psi^i \partial \psi^i \Big) +$$

$$\Big(-\frac{1}{2}(\partial\varphi)^2 + \frac{Q(n)}{2}\partial^2\varphi + \frac{1}{2}\psi_\varphi \partial \psi_\varphi \Big) \tag{6}$$

$$T_{ghost} = -2b\partial c - \partial b c - \frac{3}{2}\beta\partial\gamma - \frac{1}{2}\partial\beta\gamma \tag{7}$$

- We choose an euclidean metric for the bosonic space x^i. However, one of the boson, which we take to be $x^{2n+1} \equiv x$ is compactified on S^1 of radius $\frac{2}{Q}$.
- We define $\Psi = \psi + i\psi_l$ and $\Psi^I = \psi^i + i\psi^{i+n}$ (with $I = 1, \ldots, n$). These are bosonized in the usual way by introducing the bosonic fields H for $\partial H = \frac{i}{2}\Psi\Psi^\dagger$ and $\partial H^I = \frac{i}{2}\Psi^I\Psi^{\dagger I}$ where $\Psi^\dagger = \psi - i\psi_l$, $\Psi^{\dagger I} = \psi^i - i\psi^{i+n}$. We have

$$
\begin{aligned}
H^i(z)H^i(w) &\sim -\log(z-w) \\
H(z)H(w) &\sim -\log(z-w)
\end{aligned}
\tag{8}
$$

- We define the spin fields $\Sigma^\pm = e^{\pm\frac{i}{2}H}$, and $\Sigma^a = e^{\pm H^1 \pm \ldots \pm H^n}$, where the index a runs over the independent $SO(2n)$ spinor representation.
- Supersymmetry:

  For the $(2n+2)$-dimensional strings we can construct in the $-\frac{1}{2}$ picture 2^{n+2} candidates for supercurrents

$$
e^{-\frac{\phi}{2}+\frac{i}{2}\left(\pm H \pm H^1 \pm \ldots \pm H^n \pm Qx\right)}
\tag{9}
$$

  However, only 2^n of them are mutually local w.r.t each other and close a supersymmetry algebra.
- Combining the left and right sectors, one gets an $N = 2$ supersymmetry algebra in $2n$-dimensional space. Type IIA and type IIB strings are distinguished in the way we choose the supersymmetry currents from the left and right

sectors. When the target space allows chiral supersymmetry ($n = 1, 3$), type IIA and type IIB have $(1, 1)$ and $(2, 0)$ target space supersymmetry, respectively.

- In order to work in a covariant formalism we will see that it is convenient to use a bigger superspace with double the amount of supersymmetric coordinates. Such supespace arises naturally when considering the critical superstrings compactified on CY_{4-n} manifolds (Berkovits).
- Consider the simplest model with $D = 1$ ($n = 0$). We have the bosonic fields (x, φ). In this case there is only one nilpotent supercharge. We can choose the corresponding supercurrent $q_+(z)$ in the form

$$q_+(z) = e^{-\frac{\phi}{2}-\frac{iH}{2}-ix} , \tag{10}$$

which gives the supercharge Q_+

$$Q_+ = \oint e^{-\frac{1}{2}\phi-\frac{i}{2}H-ix} , \tag{11}$$

with $Q_+^2 = 0$.

- One can write another supercurrent in the form

$$q_-(z) = e^{-\frac{\phi}{2}+\frac{iH}{2}+ix} . \tag{12}$$

However, it is not local w.r.t. $q_+(z)$.

- There is also a supercurrent from the right sector, which we will denote by $\bar{q}$. If we choose, the same supercurrents $(q_+, \bar{q}_+)$ or $(q_-, \bar{q}_-)$ in the left and right sector, we get type IIB with $0+0$-dimensional $N=2$ supersymmetry (the two supercharges are nilpotent). If, on the other hand, we choose different supercurrents in the left and right sectors $(q_+, \bar{q}_-)$ or $(q_-, \bar{q}_+)$ we get type IIA.

- The affine current

$$R = \frac{2i}{Q(n)} \partial x \tag{13}$$

corresponds to the $U(1)_R$ symmetry. q_+ and q_- have R-charges ± 1.

- Note that while the target space is two-dimensional, the supersymmetry structure is that of two dimensions less. This structure will persist in higher dimensions, namely in $2n+2$ dimensions we will have $2n$-dimensional supersymmetry algebra for the non-critical superstrings.
- In order to construct the covariant hybrid formalism we need to work in a bigger superspace. This can be seen, for instance, by noticing that we have in the NSR formalism four fermionic variables (ψ, ψ_φ) and (b, c). Thus, we need two target superspace fermionic coordinates and their conjugate momenta, for each of the left and right sectors.
- In order to double the superspace, we add one more supercharge

$$Q_{\dot{+}} = \oint e^{-\frac{1}{2}\phi - \frac{i}{2}H + ix} \tag{14}$$

which has the property that

$$\{Q_+, Q_{\dot{+}}\} = \oint e^{-\phi - iH} \tag{15}$$

Similarly, we introduce

$$Q_{\dot{-}} = \oint e^{-\frac{1}{2}\phi + \frac{i}{2}H - ix} \tag{16}$$

which has the same property

$$\{Q_{-}, Q_{\dot{-}}\} = \oint e^{-\phi + iH} \tag{17}$$

Using the two charges $Q_{+}, Q_{\dot{+}}$ (or $Q_{-}, Q_{\dot{-}}$) we will construct a superspace with two fermionic coordinates θ^{+} and $\theta^{\dot{+}}$ (or θ^{-} and $\theta^{\dot{-}}$). We will follow the hybrid formalism in order to construct the covariant description of the strings in this superspace.

Covariant Non-Critical Superstrings

- We construct superspace variables as the dimension zero combinations

$$\theta^{+} = c\xi e^{-\frac{3}{2}\phi+\frac{i}{2}H+ix}, \qquad \theta^{\dot{+}} = e^{\frac{1}{2}\phi+\frac{i}{2}H-ix} \tag{18}$$

The variables θ^{+} and $\theta^{\dot{+}}$ have regular OPE, and

$$q_{+}(z)\theta^{+}(w) \sim \frac{1}{(z-w)}, \quad q_{\dot{+}}(z)\theta^{\dot{+}}(w) \sim \frac{1}{(z-w)} \tag{19}$$

- The conjugate momenta to θ^{+} and $\theta^{\dot{+}}$ are the dimension one objects

$$p_{+} = b\eta e^{\frac{3}{2}\phi-\frac{i}{2}H-ix}, \quad p_{\dot{+}} = e^{-\frac{1}{2}\phi-\frac{i}{2}H+ix} \tag{20}$$

and

$$p_{+}(z)\theta^{+}(w) \sim \frac{1}{(z-w)}, \quad p_{\dot{+}}(z)\theta^{\dot{+}}(w) \sim \frac{1}{(z-w)} \tag{21}$$

- The fermionic fields $\theta_{+}, \theta_{\dot{+}}, p_{+}, p_{\dot{+}}$ have singular OPE's with the field x. A way to solve this problem is to redefine the variable x:

$$x' = x + 2i(\phi + \kappa) \tag{22}$$

such that

$$\{Q_{+}, Q_{\dot{+}}\} = \oint \partial(\varphi - ix') \tag{23}$$

- The next step we rewrite the ghost fields in terms of new chiral bosons ω and ρ by imposing the following two equations

$$b = p_+ \epsilon^{\omega-\rho}\,, \qquad -\gamma^2 b = p_{\dot{+}} e^{\omega+\rho} \tag{24}$$

The conformal spin of the combinations $e^{\omega-\rho}$ and $e^{\omega+\rho}$ is 1 and 0, respectively.

- Their stress energy tensor

$$T_{\omega,\rho} = (\partial\omega)^2 - \partial^2\omega - (\partial\rho)^2 - \partial^2\rho \tag{25}$$

- To summarize: we replaced the four bosons variables $(x, \varphi, \beta, \gamma)$ and four fermion variables $(\psi, \psi_\varphi, b, c)$ in the NSR formulation by four bosons variables $(x', \varphi, \omega, \rho)$ and four fermion variables $(p_+, \theta^+, p_{\dot{+}}, \theta^{\dot{+}})$. Let us now compute the total conformal charge to check the consistency of the above manipulations. We have the following contributions

$$\begin{aligned} &(-2)_{p_{\dot{+}}\theta^{\dot{+}}} + (-2)_{p_+\theta^+} + (1-6)_\omega + \\ &(1+6)_\rho + (1-12)_{x'} + (1+12)_\varphi = 0 \end{aligned} \tag{26}$$

- There is another set of variables that will be usefull when considering curved target spaces with RR background fields. It is given by

$$\begin{aligned} \Pi_{+\dot{+}} &= \partial(\varphi - ix')\,, \qquad \Pi^{+\dot{+}} = \partial(\varphi + ix') - 2\theta^+\partial\theta^{\dot{+}} \\ d_+ &= p_+\,, \qquad d_{\dot{+}} = p_{\dot{+}} + \theta^+\Pi_{+\dot{+}} \end{aligned} \tag{27}$$

which satisfy the algebra

$$
\begin{aligned}
d_+(z) d_{\dot+}(w) &\sim \frac{\Pi_{+\dot+}}{(z-w)} \\
\Pi^{+\dot+}(z)\Pi_{+\dot+}(w) &\sim \frac{-2}{(z-w)^2} \\
d_+(z)\Pi_{+\dot+}(w) &\sim 0 \\
d_{\dot+}(z)\Pi_{+\dot+}(w) &\sim 0 \\
d_+(z)\Pi^{+\dot+}(w) &\sim -2\frac{\partial\theta^{\dot+}}{(z-w)} \\
d_{\dot+}(z)\Pi^{+\dot+}(w) &\sim -2\frac{\partial\theta^{+}}{(z-w)}
\end{aligned} \tag{28}
$$

- We will use the notation $({}_{+\dot+}, {}^{+\dot+}, +, \dot+)$ for the superspace indices

$$
x_{+\dot+} = \varphi - ix', x^{+\dot+} = \varphi + ix', \theta^+, \theta^{\dot+} \tag{29}
$$

- Let us compute the spectrum in the new variables. It is convenient to find the inverse map between the original variables and the new supersymmetric variables. Bosonizing the fermions $\theta^+, \theta^{\dot+}, p_+$ and $p_{\dot+}$ by

$$
\theta^+ = e^{\alpha}, p_+ = e^{-\alpha}, \theta^{\dot+} = e^{\beta}, p_{\dot+} = e^{-\beta} \tag{30}
$$

- The BRST cohomology consists of states at ghost numbers zero, one and two. At ghost number one there are two types of vertex operators. In the NS sector we have

$$T_k = e^{-\phi + ikx + p_l \varphi} \tag{31}$$

Locality with respect to the space-time supercharges Q_+ (and $Q_{\dot{+}}$) projects on half integer values of the momentum in the x-direction

$$x : k \in Z + \frac{1}{2} \tag{32}$$

The introduction of a second supercharge does not change the constraint on the spectrum and it is needed for the construction of the covariant formalism.

- In the Ramond sector we have the vertex operators

$$V_k = e^{-\frac{\phi}{2} + \frac{i}{2}\epsilon H + ikx + p_l \varphi} \tag{33}$$

where $\epsilon = \pm 1$. Locality with respect to the space-time supercharges Q_+ and $Q_{\dot{+}}$ implies $k \in Z + \frac{1}{2}$ for $\epsilon = 1$ and $k \in Z$ for $\epsilon = -1$.

- The Liouville dressing is determined by requiring conformal invariance of the integrated vertex operators. Thus, the coefficient p_l has to be a solution to the equation

$$\frac{k^2}{2} - \frac{1}{2} p_l \left(p_l - 2 \right) = \frac{1}{2} \tag{34}$$

This equation can be solved by $p_l = 1 \pm k$. Furthermore the locality constraint requires $p_l \leq \frac{Q}{2} = 1$.

- In supersymmetric variables we have

$$T_k = e^{\left(\alpha\left(-\frac{3}{2}+k\right)+\beta\left(\frac{1}{2}-k\right)+\omega+(-2+k)(2\rho+ix'_L)+p_l\varphi\right)} \tag{35}$$

and in the R sector, for $\epsilon = \pm$, we have

$$V_{k,\epsilon=+1} = e^{\left(\beta\left(\frac{1}{2}-k\right)+\alpha\left(-\frac{1}{2}+k\right)+\omega+(-1+k)\left(2\rho+ix'_L\right)+p_l\varphi\right)}$$

$$V_{k,\epsilon=-1} = e^{\left(\alpha(-1+k)-\beta k+(-1+k)\left(2\rho+i\,x'_L\right)+p_l\varphi\right)} \tag{36}$$

- From equations we immediately see that, in the case of NS vertex operators, we need that the momentum k must be half integer in order to rewrite it in terms of the new variables, and for the R sector we have that for $\epsilon = +1$ k must be half integer, while for $\epsilon = -1$, the momentum should be an integer. This is the way that the locality with respect to the space-time supercharges in the NSR formalism is realized in the hybrid variables.
- Let us examine the Liouville-independent states. By setting $p_l = 0$, the conformal invariance (and the Dirac constraint) implies that $k = \pm 1$ and we can have only the state

$$V_{+1,\epsilon=-1} = p_{\dot{+}} \tag{37}$$

- By the definition of the supercurrent $q_{\dot{+}} = p_{\dot{+}}$. The vertex operator $V_{+1,\epsilon=-1}$ describes the single fermion of the open string theory. It is massless and it does not depend on the coordinate x'.

- Combining left and right sectors, we have for constant RR fields of IIB/A string theories the vertex operators

$$V_{RR}^{A} = q_{\dot{+}}\bar{q}_{\dot{-}} \ , \qquad V_{RR}^{B} = q_{\dot{+}}\bar{q}_{\dot{+}} \tag{38}$$

where $\bar{q}_{\dot{+}}$ and $\bar{q}_{\dot{-}}$ are the right moving charges.

- The coupling of the RR vertex operators to the space-time RR fields strength $F^{\alpha\beta}$ is

$$F^{\dot{+}\dot{-}} q_{\dot{+}}\bar{q}_{\dot{-}} \ , \qquad F^{\dot{+}\dot{+}} q_{\dot{+}}\bar{q}_{\dot{+}} \tag{39}$$

- Thus we find one RR scalar both for IIA and IIB. In Type IIB this corresponds to a self-dual 1-form field strength in two dimensions. In Type IIA this corresponds to a 2-form field strength, or alternatively, its scalar Hodge dual.
- Ramond-Ramond Fields: In $d = 4$ the there are 4 RR degrees of freedom. In type IIB it a 1-form (or its hodge dual 3-form) and in type IIA it is a self-dual 2-form.
- In $d = 6$ the there are 16 RR degrees of freedom. In type IIB these are a 1-form (or its hodge dual 5-form) and a self-dual 3-form, and in type IIA these are 0-form (or its hodge dual 6-form) and a 2-form (or its hodge dual 4-form).
- In $d = 8$ the there are 64 RR degrees of freedom. In type IIB these are a 1-form (or its hodge dual 7-form) and a 3-form (or its hodge dual 5-form), and in type IIA these are 0-form (or its hodge dual 8-form) and a 2-form (or its hodge dual 6-form) and a self-dual 4-form.
- The bosonic part of the target space effective action takes

the form

$$S = \frac{1}{2k_d^2}\int d^dx\sqrt{G}(e^{-2\Phi}(R + 4(\partial\Phi)^2 + \frac{10-d}{\alpha'} - \frac{1}{2\cdot 3!}H^2) - \frac{1}{2\cdot n!}F_n^2) \quad (40)$$

- For instance, consider curved backgrounds with RR fields, with constant dilaton and vanishing NS-NS field, which will be considered later. Then, the field equations imply that the scalar curvature is

$$l_s^2\mathcal{R} = d - 10 \quad (41)$$

- One class of such backgrounds of type IIA non-critical strings are AdS_d spaces with a constant dilaton $e^{2\Phi} = \frac{1}{N_c^2}$ and a d-form RR field F_d

$$l_s^2 F_d^2 = 2(10-d)d!N_c^2 \quad (42)$$

- Using the supersymmetric variables the classical action for IIB in the flat background is given by

$$S_{IIB} = \frac{1}{\alpha'}\int dzd\bar{z}\Big(\frac{1}{2}\Pi_{+\dot{+}}\overline{\Pi}^{+\dot{+}} + d_+\bar{\partial}\theta^+ + d_{\dot{+}}\bar{\partial}\theta^{\dot{+}} + \bar{d}_+\partial\bar{\theta}^+ + \bar{d}_{\dot{+}}\partial\bar{\theta}^{\dot{+}}\Big) + S_B^{flat} \quad (43)$$

where S_B is the action for the chiral bosons $\omega, \bar{\omega}$ and ρ, $\bar{\rho}$.

- The the classical action for IIA in the flat background takes the form

$$S_{IIA} = \frac{1}{\alpha'} \int dz d\bar{z} \Big(\frac{1}{2} \Pi_{+\dot{+}} \overline{\Pi}^{+\dot{+}} + d_+ \bar{\partial} \theta^+ + d_{\dot{+}} \bar{\partial} \theta^{\dot{+}} + \bar{d}_- \partial \bar{\theta}^- + \bar{d}_{\dot{-}} \partial \bar{\theta}^{\dot{-}} \Big) + S_B^{flat} \quad (44)$$

- In order to couple the system to the background, we introduce the curved vielbein E^A_M where the A are tangent superspace indices and M are curved superspace indices. We will use the notation $({}_{+\dot{+}}, {}^{+\dot{+}}, +, \dot{+})$ for the tangent superspace indices, and Z^M for the curved target superspace coordinates.
- The the new supersymmetric variables are given by

$$\Pi^A = E^A_M \partial Z^M, \quad \overline{\Pi}^A = E^A_M \bar{\partial} Z^M \quad (45)$$

- We have

$$(G+B)_{AB} = E^M_A E_{BM} \quad (46)$$

- The action for IIB in curved space can be written as

$$S_{IIB} = \frac{1}{\alpha'} \int dz d\bar{z} ((G+B)_{AB} \Pi^A \overline{\Pi}^B + d_+ \bar{\Pi}^+ + d_{\dot{+}} \bar{\Pi}^{\dot{+}} + \bar{d}_+ \Pi^+ + \bar{d}_{\dot{+}} \Pi^{\dot{+}} \Big) + d_{\dot{+}} \bar{d}_{\dot{+}} F^{\dot{+}\dot{+}} + S_B \quad (47)$$

- Similarly, the action for type IIA takes the form

$$S_{IIA} = \frac{1}{\alpha'} \int dz d\bar{z} ((G+B)_{AB} \Pi^A \overline{\Pi}^B + d_+ \bar{\Pi}^+$$

$$+d_{\dot{+}}\bar{\Pi}^{\dot{+}} + \bar{d}_{-}\Pi^{-} + \bar{d}_{\dot{-}}\Pi^{\dot{-}}) + d_{\dot{+}}\bar{d}_{\dot{-}}F^{\dot{+}\dot{-}} + S_B \quad (48)$$

- S_B is the action for the chiral bosons ρ and ω. In order to write the action S_B for the chiral bosons we notice that the field ρ depends on x and therefore couples to the $U(1)$ connection A^R_M of the R-symmetry. The action S_B reads

$$S_B = S_B^{flat} + \int dz d\bar{z} \left(\bar{\partial} Z^M A^R_M \partial\rho + \partial Z^M A^R_M \bar{\partial}\bar{\rho}\right) \quad (49)$$

- Consider the example of AdS_2 background of type IIA non-critical string (Verlinde) Let $Z, \bar{Z}$ denote the coordinates on AdS_2. The dilaton Φ, the metric G and the RR 2-form F take the form

$$e^{2\Phi} = \frac{1}{N_c^2}, \quad G_{Z\bar{Z}} = -\frac{1}{2(Z-\bar{Z})^2}, \quad F_{Z\bar{Z}} = \frac{8N_c}{(Z-\bar{Z})^2} \quad (50)$$

- We denote the curved superspace coordinates by $Z_M = (Z, \bar{Z}, \Theta^{\dot{+}}, \bar{\Theta}^{\dot{-}})$. In addition there are two free variables $(\Theta^{+}, \bar{\Theta}^{-})$ needed for the extension of the superspace. The tanget space coordinates are denoted by $z_A = (z, \bar{z}, \theta^{\dot{+}}, \bar{\theta}^{\dot{-}})$, and in addition we have $(\theta^{+}, \bar{\theta}^{-})$. For the simplicity of the notation we denote $\Theta = \Theta^{\dot{+}}, \bar{\Theta} = \bar{\Theta}^{\dot{-}}$ and $\theta = \theta^{\dot{+}}, \bar{\theta} = \bar{\theta}^{\dot{-}}$.
- The curved quantities Π^A are related to the flat ones by the vielbein $\Pi^A = E^A_M \partial Z^M$ where

$$E^z{}_Z = \frac{1}{(Z - \bar{Z} - \Theta\bar{\Theta})}$$

$$E^{z}{}_{\Theta} = \frac{\Theta}{(Z - \bar{Z} - \Theta\bar{\Theta})}$$

$$E^{\theta}{}_{Z} = \frac{\Theta - \bar{\Theta}}{(Z - \bar{Z} - \Theta\bar{\Theta})^{3/2}} \tag{51}$$

$$E^{\theta}{}_{\Theta} = \frac{1}{(Z - \bar{Z} - \Theta\bar{\Theta})^{1/2}} - \frac{\Theta\bar{\Theta}}{(Z - \bar{Z} - \Theta\bar{\Theta})^{3/2}} \tag{52}$$

- The action takes the form

$$S_{IIA} = \int d^2 z \Big(\Pi_z \bar{\Pi}_{\bar{z}} d_{\dot{+}} E^{\theta}_{M} \bar{\partial} Z^M + d_{+} \bar{\partial} \theta^{+} + \bar{d}_{\dot{-}} E^{\bar{\theta}}_{M} \partial \bar{Z}^M + \bar{d}_{-} \partial \bar{\theta}^{-} + F^{\dot{+}\dot{-}} d_{\dot{+}} \bar{d}_{\dot{-}} \Big) \tag{53}$$

  where $F^{\dot{+}\dot{-}} = 8N_c$ is the constant RR field strength.

- In order to enlarge the possible conformal backgrounds of non-critical strings, open strings can be included. In Klebanov-Maldacena a Born-Infeld type term corresponding to N_f branes-antibranes uncharged system has been added

$$S_{open} = \frac{-2N_f}{2k_d^2} \int d^d x \sqrt{G} e^{-\Phi} \tag{54}$$

  which allows for gravity solutions such as $AdS_5 \times S^1$. In our framework, such a term is generated by considering worldsheets with boundaries.

- The inclusion of open string vertex operator can be done in the same way as for the closed deformations. The only difference is that the vertex operator has to placed on the boundary of the worldsheet. The general form of the massless boundary vertex operator is

$$V_{open} = \oint dz(\partial\theta^{\alpha} A_{\alpha} + \Pi^{m} A_{m} + d_{\alpha} W^{\alpha} + \ldots) \quad (55)$$

where $A_{\alpha}, A_{m}, W^{\alpha}$ are superfields. The lowest component of the superfields A_m is represented by the gluon field and $W^{\alpha} = \psi^{\alpha} + \ldots$ has the gluino as the lowest component.

- In the D=1 case, the only massless vertex operator which is independent of Liouville field is a constant gluino field and the coupling reduces to

$$V_{open} = \oint \psi^{\alpha} q_{\alpha} \quad (56)$$

where q_{α} is the supercurrent.

Open Problems

- What backgrounds are consistent backgrounds of non-critical strings?
- Are there dual field theories?
- The D-branes of non-critical strings.
- How does the pure spinor formalism work for non-critical strings (Work in progress)