Three Dimensional Black Holes with Fluxes

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Outline

1. Techniques and General Background
2. Asymmetric Deformations
3. The 3d Black String
4. 3d Black String with Fluxes
5. Spectra
6. Conclusions

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Wess-Zumino-Witten Models

WZW action on a group G

- Action:

$$S_k(g) = \frac{k}{2\pi} \int_\Sigma \langle g^{-1}\partial g, g^{-1}\bar{\partial} g \rangle + \frac{k}{12\pi} \int_B \operatorname{Tr} \omega^3$$

where $g \in G$, $\partial B = \Sigma$ and $\omega = g^{-1} \mathrm{d} g$.

- Invariant under:

$$g(\xi) \to \Omega(z) g(\xi) \bar{\Omega}^{-1}(\bar{z})$$

The manifold has $G \times G$ symmetry.

- Conserved currents J and $\bar{J}$ generating the affine $\hat{g}_k$ algebra

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Gauged Wess-Zumino-Witten Models

Reducing the symmetry

- Gauge away an anomaly-free subgroup $H \subset G$
- Promote H to a local symmetry
- Integrate out the gauge fields

Geometry

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- G/H coset under the adjoint action

$$G/\{g \sim \epsilon_l(h^{-1}) g \epsilon_r(h) | h \in H\}$$

- Non-constant curvature
- Presence of a dilaton
- In general α' corrections

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What's there in between?

Basics

- Introduce a current-current deformation

$$S_k(g) + \int \mathrm{d}^2 z\, c_{ik} J^i \tilde{J}^k$$

- Truly marginal deformation if and only if

$$c_{im} c_{jn} f_{ijk} = 0 \qquad c_{mi} c_{nj} \tilde{f}_{ijk} = 0$$

- Exact cft for any value of the deformation parameters c_{ij}
- Boundaries for the moduli, corresponding to the gauged model.

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cft point of view

- $O(d,\bar{d})$ transformation of the charge lattice $\Lambda \subset \mathfrak{h} \times \bar{\mathfrak{h}}$

$$D_{\mathfrak{h},\bar{\mathfrak{h}}} \sim O(d,\bar{d}) / \left(O(d) \times O(\bar{d}) \right) .$$

- Parafermion decomposition. The undeformed wzw model is:

$$\hat{\mathfrak{g}}_k \simeq \left(\hat{\mathfrak{g}}_k / \hat{\mathfrak{h}} \otimes t_{\Lambda_k} \right) / \Gamma_k,$$

and the deformed one becomes

$$\hat{\mathfrak{g}}_k(\mathcal{O}) \simeq \left(\hat{\mathfrak{g}}_k / \hat{\mathfrak{h}} \otimes t_{\mathcal{O}\Lambda_k} \right) / \Gamma_k,$$

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Lagrangian point of view

- Implement the $O(d,d)$ rotation (first order in α')

$$M \to M' = \Omega M \Omega^t, \quad \Phi \to \Phi' = \Phi + \frac{1}{2} \log \left(\frac{\det \hat{g}}{\det \hat{g}'} \right)$$

  where M is a $2d \times 2d$ matrix containing g and B.
- Sum of the G/H parafermion and a deformed H with a suitable T-duality.
- Identify the deformed model with the coset

$$\frac{G \times H}{H}$$

  where H is embedded in both groups.

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Asymmetric Deformations

The Action

$$S = S_{wzw}(G) + \sum_a \mathsf{h}_a \int \mathrm{d}z^2 \, J^a(z) \bar{I}_G(z)$$

- $J(z)^a$ is a holomorphic current in the torus of the G group
- $\bar{I}_G(z)$ is an external right-moving current (in heterotic strings comes from the gauge sector).

Highlights

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- Simple geometric description.
- cft interpretation.
- Exact all-order solution.
- The limiting geometry is the coset by left action.

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Asymmetric Deformations

The Geometry

- Metric of the squashed group

$$\langle \mathrm{d}g, \mathrm{d}g \rangle_{\mathsf{h}} = \sum_{\mu} J_\mu \otimes J_\mu + \sum_a \left(1 - 2\mathsf{h}_a^2\right) J_a \otimes J_a$$

- Kalb--Ramond field

$$H_{[3]} = \frac{1}{2} f_{\mathsf{mnp}} J^{\mathsf{m}} \wedge J^{\mathsf{n}} \wedge J^{\mathsf{p}} - 2 \sum_a \mathsf{h}_a^2 f_{a\mathsf{np}} J^a \wedge J^{\mathsf{n}} \wedge J^{\mathsf{p}},$$

- a $U(1)^{\mathrm{rank}(G)}$ gauge field:

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Asymmetric Deformations

Limit case

- Limit value for the deformation parameter h $= 1/\sqrt{2}$
- $\mathrm{rank}(G)$ directions decompactify
- Metric of the left action coset G/H
- Most simple case: $\mathrm{AdS}_2 \times S^2$
- Flag spaces
- Asymmetric coset (mathematically)
- Kähler geometry. No B field.
- Just metric and $U(1)^{\mathrm{rank}(G)}$ electric/magnetic field.

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There are more things in heaven and earth...

Other deformations

- Current-current deformations are in general the only truly marginal ones.
- Other operators are possible. At best marginally relevant or almost relevant.

Renormalization Group flow

- Compact groups: Zamolodchikov's c-theorem
- Degrees of freedom decrease with the flow

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Horne and Horowitz' Black String

History

- Witten's $SL(2,\mathbb{R})/U(1)$ two-dimensional black hole
- Generalization to three dimensions as an axial $SL(2,\mathbb{R}) \times \mathbb{R}/\mathbb{R}$ gauging (Horowitz and Horne)
- Low energy fields:

$$ds^2 = -\left(1 - \frac{M}{r}\right) dt^2 + \left(1 - \frac{Q^2}{Mr}\right) dx^2 + \frac{dr^2}{\left(1 - \frac{M}{r}\right)\left(1 - \frac{Q^2}{Mr}\right) 8r^2}$$

$$H = \frac{Q}{r^2} dt \wedge dx \wedge dr$$

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- Generalization to three dimensions as an axial $SL(2,\mathbb{R}) \times \mathbb{R}/\mathbb{R}$ gauging (Horowitz and Horne)
- Low energy fields:

$$\mathrm{d}s^2 = -\left(1 - \frac{M}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{Q^2}{Mr}\right)\mathrm{d}x^2 + \frac{\mathrm{d}r^2}{\left(1-\frac{M}{r}\right)\left(1-\frac{Q^2}{Mr}\right)8r^2}$$

$$H = \frac{Q}{r^2}\,\mathrm{d}t \wedge \mathrm{d}x \wedge \mathrm{d}r$$

$$\Phi = \log r$$

- M is the mass, Q the charge (with respect to the Kalb--Ramond field)

Horne and Horowitz' Black String

The Black String as a Deformation

- The axial gauging is one of the ways to read the effect of a current-current deformation.
- The black-string is a $J\bar{J}$ deformation of the $SL(2,\mathbb{R})$ wzw model
- Only the combination $\mu = Q/M$ is physical.

In detail

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- For $\mu > 1$ the black string is the $J_3\bar{J}_3$ deformation
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More defomations

Double Deformations

- After the $J\bar{J}$ deformation, the $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ isometry is broken to $U(1) \times U(1)$
- Room for another asymmetric deformation

$$\delta S = \delta\kappa \int \mathrm{d}z^2\, J\bar{J} + \mathsf{h} \int \mathrm{d}z^2\, J\bar{I}$$

Nothing new under the sun

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- For $SL(2,\mathbb{R})$ with J_3 deformation.
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Electric Black Brane

Local coordinates

$$\frac{1}{k}\,\mathrm{d}s^2 = -\,\mathrm{d}t^2 + \cos^2 t\,\frac{\left(\kappa^2 - 2\mathsf{h}^2\right)\cos^2 t + \kappa^4 \sin^2 t}{\Delta_\kappa(t)^2}\,\mathrm{d}\psi^2 + $$
$$- 4\mathsf{h}^2 \frac{\cos^2 t \sin^2 t}{\Delta_\kappa(t)^2}\,\mathrm{d}\psi\,\mathrm{d}\varphi + \sin^2 t\,\frac{\cos^2 t + \left(\kappa^2 - 2\mathsf{h}^2\right)\sin^2 t}{\Delta_\kappa(t)^2}\,\mathrm{d}\varphi^2$$

$$\frac{1}{k}B = \frac{\kappa^2 - 2\mathsf{h}^2}{\kappa^2}\frac{\cos^2 t}{\Delta_\kappa(t)}\,\mathrm{d}\varphi \wedge \mathrm{d}\psi$$

$$F = 2\mathsf{h}\sqrt{\frac{2k}{k_g}}\frac{\sin(2t)}{\Delta_\kappa(t)^2}\left(\kappa^2\,\mathrm{d}\psi \wedge \mathrm{d}t + \mathrm{d}t \wedge \mathrm{d}\varphi\right)$$

$$\mathrm{e}^{-\Phi} = \frac{\sqrt{\kappa^2 - 2\mathsf{h}^2}}{\Delta_\kappa(t)}$$

where $\Delta_\kappa(t) = \cos^2 t + \kappa^2 \sin^2 t$

Coordinates, coordinates...

Some technical stuff

- define

$$\kappa^2 = \frac{\lambda}{\lambda + 1}$$
$$r = \lambda + \cos\beta$$

- curvature

$$\mathcal{R} = 2\frac{2r\left(1+2\lambda\right) - 7\lambda\left(1+\lambda\right) - 2\mathrm{h}^2\left(1+\lambda\right)^2}{r^2}.$$

- AdS_3 doesn't appear any more ($\lambda \to \infty$)
- Constant-length Killing vector $k = \left(1+\lambda\right)\partial_\psi + \lambda\partial_\phi$:

$$\left\|k\right\|^2 = \lambda\left(1+\lambda\right) - 2\mathrm{h}^2\left(1+\lambda\right)^2 = \omega^2$$

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The metric

- Using the Killing vector and continuing on r:

$$ds^2 = \left(-1 + \frac{1+2\lambda}{r} - \frac{\lambda\left(1+\lambda\right) - \omega}{r^2}\right) dt^2 + \\ + 2\frac{\omega}{r}\, dx\, dt + \omega\, dx^2 + \frac{1}{4\left(r-\lambda\right)\left(r-\lambda-1\right)}\, dr^2$$

- Singular in $r = 0, \lambda, \lambda + 1$
- Volume form $\sqrt{\omega}/(2r)\, dt \wedge dx \wedge dr$
- Eddington-Finkelstein coordinates:

$$ds^2 = \left(-1 + \frac{1+2\lambda}{r} - \frac{\lambda\left(1+\lambda\right) - \omega}{r^2}\right) dT^2 + \\ + 2\frac{\omega}{r}\, dX\, dT + \omega\, dX^2 - \frac{1}{r}\, dT\, dr$$

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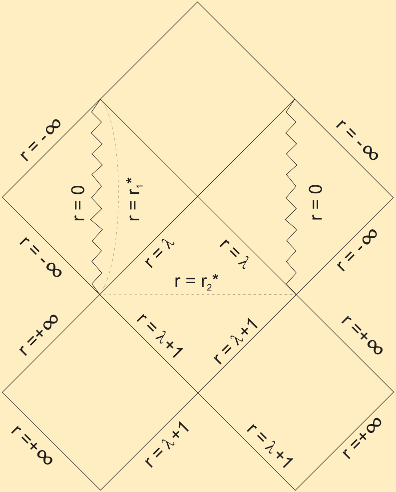
Penrose diagram



- Two horizons in $r = \lambda$ and $r = \lambda + 1$
- Time-like singularity in $r = 0$

Non-gravitational fields

Other fields

$$F = \pm \frac{\sqrt{2}\mathsf{h}\,(1+\lambda)}{r^2\sqrt{k_g}}\,\mathrm{d}t \wedge \mathrm{d}r$$

$$H = \mp \frac{\omega}{r^2}\,\mathrm{d}t \wedge \mathrm{d}x \wedge \mathrm{d}r$$

$$\Phi = \Phi_* - \frac{1}{2}\log r$$

Conserved charges

- Asymptotically is flat space with linear dilaton
- Choose Killing vectors at infinity that preserve the dilaton
- Density of mass and momentum

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Spectrum

Primary operators

- The costruction is motivated by the underlying CFT
- The deformation is a $O(2,2)$ rotation on the abelian subgroup
- Weights for these lattices are:

$$L_0 = \frac{1}{k}\left(\mu + n + \frac{a}{2}\right)^2,$$

$$\bar{L}_0 = \frac{\bar{\mu}^2}{k+2} + \frac{1}{k_g}\left(\bar{n} + \frac{\bar{a}}{2}\right)^2,$$

- Deformation driven the operator

$$\mathcal{O} = \kappa^2 \frac{\left(J^2 + \imath\psi_1\psi_3\right)}{\sqrt{k}} \frac{\bar{J}^2}{\sqrt{k+2}} + \mathsf{h} \frac{\left(J^2 + \imath\psi_1\psi_3\right)}{\sqrt{k}} \frac{\bar{I}}{\sqrt{k_g}}.$$

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Spectrum

Deformed Weights

- Rotation in the anti-holomorphic + boost

$$
L_0^{\mathsf{dd}} = \left\{ \frac{1}{\sqrt{k}} \left(\mu + n + \frac{a}{2} \right) \cosh x + \right. \\
\left. + \left(\frac{\bar{\mu}}{\sqrt{k+2}} \cos \alpha + \frac{1}{\sqrt{k_g}} \left(\bar{n} + \frac{\bar{a}}{2} \right) \sin \alpha \right) \sinh x \right\}^2 ,
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WZW deformations

- Current-current deformations
- Asymmetric deformations
- Exact CFT backgrounds with nice geometric properties.

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