D-branes in Non-Critical Superstrings and Pure Super Yang-Mills.

hep-th/0504079 (S.Ashok, S.M, J.Troost)

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$O\rho\theta o\delta o\xi o\varsigma\ A\kappa\alpha\delta\eta\mu\iota\alpha,\ K\rho\eta\tau\eta\varsigma.$

Motivation

• String description of pure (Super) Yang-Mills theory, or more generally of confining gauge theories.

• Understand string theory, D-branes in highly curved backgrounds. Supergravity not a priori a good approximation, need exact sigma model.

• Understand holography in linear dilaton backgrounds.

• Generalize the remarkable understanding of low dimensional string theory (Liouville theory $\Leftrightarrow$ Matrix models) to higher dimensions.

Plan

A) Closed string background and D-branes:

• A class of exact Superstring Backgrounds with target space $\mathbf{R}^{d-1,1}$($\times$ cigar) (Non-critical superstrings in d dimensions).

• A consistent class of $\frac{1}{2}$ BPS boundary states in these backgrounds.

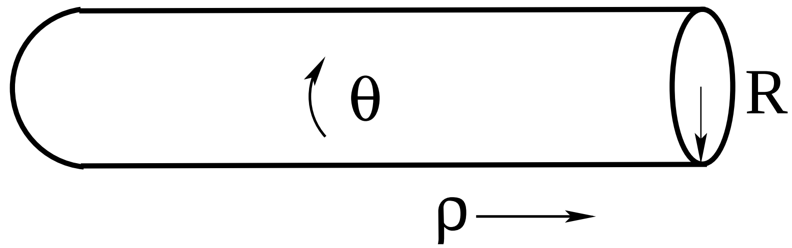
• These realize on their worldvolume pure SYM in $d = 2, 4, 6$ as the low-energy effective theory.

B) Modified geometry for closed strings:

• Compute backreaction of these boundary states onto the above background.

• Understand gauge theory quantities holographically.

The Cigar Background



$$ds^2 = d\rho^2 + \tanh^2\left(\frac{Q\rho}{2}\right) d\theta^2; \qquad \theta \sim \theta + \frac{4\pi}{Q}$$

$$\Phi = -\log\cosh(\frac{Q\rho}{2}); \quad g_s = e^{\Phi}.$$

Consistent String Background (at tree level)

Dhar, Mandal, Wadia; Elitzur, Forge, Rabinovic; Witten.

$$2D_aD_b\Phi + R_{ab} = 0$$

CFT: $SL_2(\mathbf{R})/U(1)$ supercoset at level $k = 2/Q^2$, $R = \sqrt{k\alpha'}$.

Conformal Field Theory

Asymptotically, $\mathcal{N}=2$ linear dilaton:

$$\begin{aligned} T_{\rm cig} &= -\frac{1}{2}(\partial\rho)^2 - \frac{1}{2}(\partial\theta)^2 - \frac{1}{2}(\psi_\rho\partial\psi_\rho + \psi_\theta\partial\psi_\theta) - \frac{1}{2}Q\partial^2\rho \\ G^\pm_{\rm cig} &= \frac{i}{2}(\psi_\rho \pm i\psi_\theta)\partial(\rho \mp i\theta) + \frac{i}{2}Q\partial(\psi_\rho \pm i\psi_\theta) \\ J_{\rm cig} &= -i\psi_\rho\psi_\theta + iQ\partial\theta \equiv i\partial H + iQ\partial\theta \equiv i\partial\phi \end{aligned}$$

String Background: Cigar $\times\mathbf{R}^d \Rightarrow$

$$c = 3(1+Q^2) + \frac{3}{2}d = 15.$$

$$\Rightarrow Q = \sqrt{\frac{8-d}{2}}.$$

$\mathcal{N}=2$ **Super Liouville Theory**

$$S_{int} = \mu \int d^2z \psi\tilde{\psi}\, e^{-\frac{1}{Q}(\rho+\tilde{\rho}+i(\theta-\tilde{\theta}))} + c.c$$

where $\psi = \psi_\rho + i\psi_\theta$ is the superpartner of $\rho + i\theta$ and $\tilde{\psi}$ is its rightmoving counterpart.

• Chiral $U(1)$ R symmetry on the worldsheet;

$$\psi \to \psi e^{i\alpha},\ \theta \to \theta + Q\alpha;$$

Involves bosonic spacetime coordinate.

Symmetries of the theory

- Chiral R symmetries on the worldsheet

$$R = \oint dz \partial \phi,$$

$$\bar{R} = \oint d\bar{z} \bar{\partial} \phi,$$

- Momentum

$$P^{\theta} = \oint dz \partial \theta + \oint d\bar{z} \bar{\partial} \theta.$$

- Winding broken by condensate.

Type II theories: Spacetime Supersymmetry

Kutasov and Seiberg

• Exponentiate $U(1)$ R symmetry on worldsheet
⇒ Spacetime Supercharge.

$$S = e^{-\frac{\varphi}{2}+i\frac{\phi}{2}}.$$

• Algebra:

$$\{\mathcal{S}_\alpha, \bar{\mathcal{S}}_\beta\} = 2\gamma^\mu_{\alpha\beta}P_\mu, \qquad \text{or} \qquad \{\mathcal{S}_\alpha, \bar{\mathcal{S}}_{\dot\beta}\} = 2\gamma^\mu_{\alpha\dot\beta}P_\mu.$$

• Translation around the cigar P^θ is an R symmetry in spacetime:

$$[P^\theta, \mathcal{S}_\alpha] = \frac{1}{2}\mathcal{S}_\alpha, \qquad [P^\theta, \bar{\mathcal{S}}_{\dot\alpha}] = -\frac{1}{2}\bar{\mathcal{S}}_{\dot\alpha}.$$

Closed string Spectrum, IIB theory

S.M. hep/th 0305197

NSNS:

- Graviton mulitplet (G_{IJ}, B_{IJ}, Φ).
- (Non-tachyonic) scalar T with non-zero momentum/winding.

RR:

- form fields appropriate to dimension.

Eg: $d = 4$, IIB theory, C_0, C_2.

$+$

- Tower of states

$SL(2)_k/U(1)$: a few details.

- Primary fields on $SL(2)/U(1)$ are $V^j_{m,\bar{m}}$.

$$\Delta^j_{n,w} = -\frac{j(j-1)}{k} + \frac{m^2}{k}, \qquad Q^j_{n,w} = \frac{2m}{k}.$$

where the $U(1)$ quantum numbers are related to the momentum and "winding" by:

$$m = \frac{n+kw}{2}, \qquad \bar{m} = -\frac{n-kw}{2}$$

- Reflection:

$$\Phi^{-j+1}_{m,\bar{m}} = R(-j+1, m, \bar{m})\Phi^j_{m,\bar{m}}$$

$$R(j,m,\bar{m}) = \nu^{1-2j} \frac{\Gamma(-2j+1)\Gamma(1+\frac{1-2j}{k})}{\Gamma(-2j+1)\Gamma(1+\frac{1-2j}{k})} \frac{\Gamma(j+m)\Gamma(j-m)}{\Gamma(-j+1+\bar{m})\Gamma(-j+1-\bar{m})}$$

Supercoset Representations

- Continuous representations

$$j = \frac{1}{2} + iP, \qquad P \in \mathbf{R}_0^+.$$

- Discrete representations

$$\frac{1}{2} < j < \frac{k+1}{2}$$

with

$$j + r = m, \qquad j + \bar{r} = \bar{m}.$$

$r, \bar{r}$ are (half) integers for (R) NS sector.

- Asymptotically, $\Phi^j_{m\bar{m}}(\rho) \sim e^{(j-1)\rho}$.

D-branes and Open String Theory

Boundary State

- General form of boundary state:

$$|\mathbf{B}\rangle = \frac{T}{2} \sum_{\alpha} |B_{X^\mu,\psi^\mu}\rangle_\alpha \otimes |B_{cigar}\rangle_\alpha \otimes |B_{ghost}\rangle_\alpha$$

with $\alpha = NS, NS(-)^F, R, R(-)^F$.

• Branes filling $\mathbf{R}^{d-1,1}$ obey:

$$\begin{aligned} \partial_\tau X^\mu(\sigma,0)|B_{X^\mu,\psi^\mu}\rangle &= 0, \\ (\psi^\mu - i\eta\tilde{\psi}^\mu)|B_{X^\mu,\psi^\mu}\rangle &= 0, \\ \mu = 0, 1..d-1 & \end{aligned}$$

• Solution:

$$|B_{X^\mu,\psi^\mu}\rangle =$$
$$\exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}\alpha^\mu_{-n}\eta_{\mu\nu}\tilde{\alpha}^\nu_{-n} - i\eta\sum_{r>0}\psi^\mu_{-r}\eta_{\mu\nu}\tilde{\psi}^\nu_{-r}\right]|0,\eta,k^\mu=0\rangle_{NS/R}$$

r (half) integer for (R) NS sector.

• For $|B_{cig}\rangle$ localized on the cigar, use the $\mathcal{N}=2$ algebra.

• B type boundary conditions: $J^R = -\tilde{J}^R$, $G^{\pm} = i\eta\tilde{G}^{\pm}$.

Solution: $|B_{cig}\rangle_\alpha = \sum_{j,n,w} \Psi^{j,\alpha}_{n,w}\Phi^{j,\alpha}_{n,w}|0\rangle_\alpha$.
with

$$\begin{aligned}
\Psi^{j,NS}_{n,w} &= k^{-\frac{1}{2}}(-1)^w \delta_{n,0}\,\nu^{\frac{1}{2}-j}\frac{\Gamma(j+\frac{kw}{2})\Gamma(j-\frac{kw}{2})}{\Gamma(2j-1)\Gamma(1-\frac{1-2j}{k})} \\
\Psi^{j,\widetilde{NS}}_{n,w} &= i^w \Psi^{j,NS}_{n,w} \\
\Psi^{j,R\pm}_{n,w} &= k^{-\frac{1}{2}}(-1)^w \delta_{n,0}\,\nu^{\frac{1}{2}-j}\frac{\Gamma(j+\frac{kw}{2}\pm\frac{1}{2})\Gamma(j-\frac{kw}{2}\mp\frac{1}{2})}{\Gamma(2j-1)\Gamma(1-\frac{1-2j}{k})}
\end{aligned}$$

where $k\nu^{-1} \equiv (g_s^{tip})^{-2} = \left(\frac{\mu}{k}\right)^{\frac{2}{k}} = \tilde{\mu}\frac{\Gamma(1/k)}{\Gamma(1-1/k)}$.

Following

Zamolodchikov,Zamolodchikov, Fateev,Zamolodchikov,Zamolodchikov

for D-branes in Liouville,

Hosomichi; Ahn, Stanishkov,Yamamoto; Eguchi and Sugawara

D-branes in $\mathcal{N} = 2$ Liouville.

Following

e.g.: Ribault, Schomerus

for D-branes in bosonic cigar,

Israel,Pakman,Troost.

D-branes in $\mathcal{N} = 2$ cigar.

Annulus Partition function

$$
\begin{aligned}
Z(t) &= Tr\left(e^{-2\pi t L_0^{op}}\right) = \langle \mathbf{B}| e^{-sL_0^{cl}} |\mathbf{B}\rangle, \quad s = \pi/t, \\
&= V_d \int \frac{d^d k}{(2\pi)^d} e^{-2\pi t k^2} \frac{1}{2}\left(Z^{NS}(t) - Z^{\widetilde{NS}}(t) - Z^{R}(t) - Z^{\widetilde{R}}(t)\right) \\
&= 0.
\end{aligned}
$$

$$
\begin{aligned}
Z^{NS}_{Dp}(t) &= \left(\frac{\Theta_{00}(it)}{\eta^3(it)}\right)^{\frac{d-2}{2}} \times \frac{\Theta_{00}(it)}{\eta^3(it)} \sum_{s\in\mathbf{Z}+\frac{1}{2}} \frac{1}{1+q^s}\left(q^{\frac{s^2-s}{k}} - q^{\frac{s^2+s}{k}}\right) \\
Z^{\widetilde{NS}}_{Dp}(t) &= \left(\frac{\Theta_{01}(it)}{\eta^3(it)}\right)^{\frac{d-2}{2}} \times \frac{\Theta_{01}(it)}{\eta^3(it)} \sum_{s\in\mathbf{Z}+\frac{1}{2}} \frac{(-1)^{s-\frac{1}{2}}}{1-q^s}\left(q^{\frac{s^2-s}{k}} + q^{\frac{s^2+s}{k}}\right) \\
Z^{R}_{Dp}(t) &= \left(\frac{\Theta_{10}(it)}{\eta^3(it)}\right)^{\frac{d-2}{2}} \times \frac{\Theta_{10}(it)}{\eta^3(it)} \sum_{s\in\mathbf{Z}} \frac{1}{1+q^s}\left(q^{\frac{s^2-s}{k}} - q^{\frac{s^2+s}{k}}\right) \\
Z^{\widetilde{R}}_{Dp}(t) &= \left(\frac{\Theta_{11}(it)}{\eta^3(it)}\right)^{\frac{d-2}{2}} \times \frac{\Theta_{11}(it)}{\eta^3(it)} \sum_{s\in\mathbf{Z}} \frac{(-1)^{s}}{1-q^s}\left(q^{\frac{s^2-s}{k}} + q^{\frac{s^2+s}{k}}\right) .
\end{aligned}
$$

Open String theory on D3-branes and $\mathcal{N}=1$ SYM

• Massless modes:

$$\epsilon_\nu \psi^\nu_{-\frac{1}{2}} |k_\mu, NS\rangle, \qquad u^\alpha |k_\mu, \Sigma_\alpha, R\rangle.$$

BRST invariance $\Rightarrow$

$$k^\mu k_\mu = 0, \qquad k^\mu \gamma_\mu u = 0.$$

$$k^\mu \epsilon_\mu = 0, \qquad \epsilon^\mu \equiv \epsilon^\mu + k^\mu.$$

Gauge Boson A_μ, Gaugino λ_α.

• Other modes have masses $m^2 \sim \alpha'^{-1}$.

SUSY

Exactly half of the supercharges in the bulk are preserved by the brane:

$$\mathcal{S}_\alpha + \widetilde{\mathcal{S}}_\alpha \equiv \mathcal{S}^{bdry}_\alpha, \qquad \bar{\mathcal{S}}_{\dot\alpha} + \widetilde{\bar{\mathcal{S}}}_{\dot\alpha} \equiv \bar{\mathcal{S}}^{bdry}_{\dot\alpha}.$$

$$\{\mathcal{S}^{bdry}_\alpha, \bar{\mathcal{S}}^{bdry}_{\dot\beta}\} \quad = \quad 2\gamma^\mu_{\alpha\dot\beta} P_\mu$$

$$\left[P^\theta, \mathcal{S}^{bdry}_\alpha\right] = \frac{1}{2}\mathcal{S}^{bdry}_\alpha, \qquad \left[P^\theta, \bar{\mathcal{S}}^{bdry}_{\dot\alpha}\right] = -\frac{1}{2}\bar{\mathcal{S}}^{bdry}_{\dot\alpha}$$

$\Rightarrow$ Low energy limit of the worldvolume theory is pure $\mathcal{N} = 1$ SYM:

$$S_{YM} = \frac{1}{g_{YM}^2} \int d^4x \, \mathrm{Tr} \, \left(\frac{1}{4} F^2 + \bar{\lambda} \partial \lambda \right) .$$

Closed String Physics

Two technical results

- Correct tachyon wavefunction:

$$
\begin{aligned}
T^{phys}(\rho) &= \mu \lim_{k\to 1} e^{\pm i\sqrt{\frac{k}{2}}(\theta-\tilde{\theta})}\left(e^{-k\rho}+R(k)e^{-(2-k)\rho}\right)\\
&= \mu \lim_{\epsilon\to 0} e^{\pm i\sqrt{\frac{1+\epsilon}{2}}(\theta-\tilde{\theta})}\left(e^{-(1+\epsilon)\rho}+R(1+\epsilon)e^{-(1-\epsilon)\rho}\right)\\
&= -(\mu\epsilon)\,\rho\, e^{\pm i\frac{1}{\sqrt{2}}(\theta-\tilde{\theta})}e^{-\rho} \equiv \mu_{ren}\,\rho\, e^{\pm i\frac{1}{\sqrt{2}}(\theta-\tilde{\theta})-\rho}
\end{aligned}
$$

- Renormalized couplings for $k=1$:

$$(g^{tip}_{s,ren})^{-2} = \mu^2_{ren} = \tilde{\mu}_{ren} \equiv \nu^{-1}.$$

Backreaction onto Closed String Modes

- Propagating mode (j, n, w):

$$|\mathcal{V}(k)\rangle = \left[\mathcal{V}^{flat}|0, k^\mu\rangle_{X,\psi}\right] \otimes \left[\Phi^j_{nw}|0\rangle_{Cig}\right].$$

- First order backreaction on the mode by $|\mathbf{D3}\rangle$

$$\delta\Phi(k) \equiv \langle\mathcal{V}(k)|L_0^{-1}|\mathbf{D3}\rangle = \frac{\delta^4(k^\mu)}{\frac{1}{2}k^\mu k_\mu - j(j+1) + \Delta_{int}}\Psi^j_{nw}.$$

Position space: Geometrical Approximation

• Find position space solutions to equation of motion (semiclassical).

• Note: Solution to Laplacian on the six dimensional background factorizes $V_{k,P}(x^\mu, \rho) = e^{ik_\mu X^\mu} \phi_0^P(\rho)$.

• "Fourier transform" backreaction in momentum space using the above solutions as basis.

Example: Backreaction on metric

- The delta-function normalized minisuperspace field is

$$\phi_0^{grav}(\rho,P)=$$
$$\left[(\sinh\rho)^{2iP-1}F\left(\frac{1}{2}-iP,\frac{1}{2}-iP;1-2iP;-\frac{1}{\sinh^2\rho}\right)\right.$$
$$\left.+R^{grav}(-P)(\sinh\rho)^{-2iP-1}F\left(\frac{1}{2}+iP,\frac{1}{2}+iP;1+2iP;-\frac{1}{\sinh^2\rho}\right)\right]$$

where

$$R^{grav}(P)\equiv\frac{\Gamma(2iP)\Gamma(\frac{1}{2}-iP)^2\Gamma(1+\frac{2iP}{k})\nu^{iP}}{\Gamma(-2iP)\Gamma(\frac{1}{2}+iP)^2\Gamma(1-\frac{2iP}{k})\nu^{-iP}}$$

In position space:

$$\frac{1}{N} h^{IJ}(x^\mu, \rho) = \eta^{IJ} \int_0^\infty dP\ \Psi^{grav}(P)\ \frac{1}{P^2 + M^2}\ \phi^{grav}(\rho, P)$$

$$= \eta^{IJ} \int_{-\infty}^\infty dP \frac{1}{P^2 + M^2} \nu^{iP} \frac{\Gamma(\frac{1}{2} - iP)^2}{(\Gamma(-2iP)\Gamma(1 - 2iP))} \times$$

$$(\sinh\rho)^{-2iP-1} F\left(\frac{1}{2} + iP, \frac{1}{2} + iP; 1 + 2iP; -\frac{1}{\sinh^2\rho}\right)$$

$$\equiv \eta^{IJ} I_{grav}$$

Complete the integral to a contour and use Cauchy's theorem

$\Rightarrow$ Pick residue of the integrand at the pole $P = +iM = \frac{i}{2}$.

$$\begin{aligned} I_{grav} &= \frac{2\pi i}{2iM}\nu^{-M}(\sinh\rho)^{-2M-1}F\left(\frac{1}{2}+M, \frac{1}{2}+M; 1+2M; -\frac{1}{\sinh^2\rho}\right) \\ &= 2\pi\nu^{-\frac{1}{2}}(\sinh\rho)^{-2}F\left(1,1;2;-\frac{1}{\sinh^2\rho}\right) \\ &= 2\pi\nu^{-\frac{1}{2}}\log\left(1+\frac{1}{\sinh^2\rho}\right). \end{aligned}$$

Summary of Backreaction Results

N $D3$-branes at the tip of the cigar source:

- Graviton-dilaton multiplet:

$$
\begin{aligned}
h^{IJ}(\rho) &= \eta^{IJ} 2\pi N \log\left(1 + \frac{1}{\sinh^2\rho}\right) \\
&\longrightarrow \eta^{IJ} \left[2\pi N e^{-2\rho}\right] \qquad \text{as} \qquad \rho \to \infty \\
&\longrightarrow \eta^{IJ} \left[4\pi N \log\rho\right] \qquad \text{as} \qquad \rho \to 0.
\end{aligned}
$$

- Tachyon ($\mathcal{N} = 2$ Liouville coupling)

$$
\begin{aligned}
\delta T^{phys} &= \cosh^{-1} \rho; \\
&\longrightarrow e^{-\rho}, \qquad \text{as} \qquad \rho \to \infty \\
&\longrightarrow 1, \qquad \text{as} \qquad \rho \to 0
\end{aligned}
$$

- RR axion with a constant field strength.

$$\chi(\rho, \theta) = N\theta$$

Running gauge coupling

• Near the tip of the cigar, we find:

$$\delta(e^{-\Phi}) \sim N \log \rho.$$

• Semiclassically,

$$S_{D3} = T \int d^4x \; (e^{-\Phi} TrF^2 + ...)$$

• $\Rightarrow$ Holographic "prediction" of running gauge coupling:

$$\frac{1}{g_{YM}^2} - \frac{1}{g_{YM,0}^2} \sim N \log(\rho/\Lambda)$$

• Liouville (radial) direction ρ plays the role of the scale in the gauge theory.

Instantons and Chiral $U(1)_R$ Symmetry Breaking.

• $U(1)_R$ symmetry of the Super Yang-Mills theory is realized in the string theory dual as the conserved $U(1)$ momentum around the cigar.

• $D3$-branes at the tip source $\chi(\rho,\theta) = N\theta$.
$\Rightarrow U(1)_R$ is broken to $\mathbf{Z_{2N}}$. Can be checked by a D-instanton probe.

• Not clear from our construction how the chiral symmetry is broken further to Z_2.

Conclusions/Lessons

• Found first order backreaction onto the geometry and the fluxes – first step towards string dual of pure Yang-Mills.

• String theory: Localized branes source *Normalizable modes on cigar*, asymptotics not destroyed.

• Gauge theory: Can extract *UV information in gauge theory*.