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Geometrical fluxes and Scherk–Schwarz deformations

Costas Kounnas

Laboratoire de Physique Théorique

Ecole Normale Supérieure, Paris

In collaboration with

J.-P. Derendinger, P.M. Petropoulos and F. Zwirner

1. Introduction

• **Superstrings and M-theory compactifications can give 4d vacua with exact or (spontaneously) broken supersymmetries.**

• **Phenomenologically interesting are those with**

$$N = 8, 4 \to N = 1 \to N = 0$$

• **The underlying D = 10 theories encode $N \geq 4$ constrained structure which can be used to obtain useful information on the effective $N = 1$ supergravity.**

• **The 4d $N = 1$ theories, typically include moduli fields whose vacuum expectation values are undetermined.**

Some of these moduli are the: dilaton field Φ, internal metric fields G_{IJ}, and p-form fields F^p

Generating a potential for some of the moduli is essential in order to :

- **reduce the number of massless scalars**
- **induce supersymmetry breaking**
- **determine the (3+1)d geometry**

In the $N \geq 4$ supergravity theories, the only available tool for generating a non-trivial potential is the "gauging".

"Gauging" $\rightarrow$ We introduce in the theory a gauge group G acting on the vector fields of the gravitational and the vector super-multiplets.

The important fact is: The kinetic terms of the fields in the gauged theory, remain the same as in the ungauged theory.

In the language of $N = 1 \quad (\longleftarrow N \geq 4)$

The gauging modifications are non-trivial for the the superpotential W.

The Kähler potential K remains the same as in the ungauged theory.

To be more precise consider the case of superstring constructions with an $N = 4$ **supersymmetry:**

- **Heterotic on** T^6
- **Type IIA or IIB on** $K3 \times T^2$
- **Type IIA, IIB on orientifolds**
- **Type IIA, IIB asymmetric (4,0)**
- $\ldots$

2. $N=4$ Gauging $\leftrightarrow$ $N=1$ Superpotential

Independently of our starting point, the scalar manifold M of the induced $N=4$ effective supergravity is identical for all superstring constructions.

$$M = \left(\frac{SU(1,1)}{U(1)}\right)_S \times \left(\frac{SO(6,6+n)}{SO(6)\times SO(6+n)}\right)_{T_A,U_A,Z_I}$$

After $Z^2 \times Z^2$ orbifold (CY) projections

$N=4 \longrightarrow N=1$ **and** $M \longrightarrow K$

$$K = \left(\frac{SU(1,1)}{U(1)}\right)_S \times \left(\frac{SO(2,2+n_1)}{SO(2)\times SO(2+n_1)}\right)_{T_1,U_1,Z_1^I}$$
$$\times \left(\frac{SO(2,2+n_2)}{SO(2)\times SO(2+n_2)}\right)_{T_2,U_2,Z_2^I}$$
$$\times \left(\frac{SO(2,2+n_3)}{SO(2)\times SO(2+n_3)}\right)_{T_3,U_3,Z_3^I}$$

$$K = -\log\left(S+\bar{S}\right)$$

$$-\log\left((T_1+\bar{T}_1)(U_1+\bar{U}_1)-(Z_1+\bar{Z}_1)^2\right)$$

$$-\log\left((T_2+\bar{T}_2)(U_2+\bar{U}_2)-(Z_2+\bar{Z}_2)^2\right)$$

$$-\log\left((T_3+\bar{T}_3)(U_3+\bar{U}_3)-(Z_3+\bar{Z}_3)^2\right).$$

The above choice of parameterization is a solution to the $N=4$ constraints after $Z^2 \times Z^2$ orbifold projections $N=4 \to N=1$:

S-manifold

$$|\phi_0|^2 - |\phi_1|^2 = \frac{1}{2} \quad \longrightarrow$$

$$\phi_0 - \phi_1 = \frac{1}{(S+\bar{S})^{1/2}}, \qquad \phi_0 + \phi_1 = \frac{S}{(S+\bar{S})^{1/2}}$$

T_A, U_A, Z_A^I-manifolds

$$|\sigma_A^1|^2 + |\sigma_A^2|^2 - |\rho_A^1|^2 - |\rho_A^2|^2 - |\Phi_A^I|^2 = \frac{1}{2}$$
$$(\sigma_A^1)^2 + (\sigma_A^2)^2 - (\rho_A^1)^2 - (\rho_A^2)^2 - (\Phi_A^I)^2 = 0$$

$$\sigma_A^1 = \frac{1 + T_A U_A - (Z_A^I)^2}{2Y_A^{1/2}}, \qquad \sigma_A^2 = i\frac{T_A + U_A}{2Y_A^{1/2}}$$

$$\rho_A^1 = \frac{1 - T_A U_A - (Z_A^I)^2}{2Y_A^{1/2}}, \qquad \rho_A^2 = i\frac{T_A - U_A}{2Y_A^{1/2}}$$

$$\Phi_A^I = \frac{iZ_A^I}{2Y_A^{1/2}}, \qquad K_A = -log\ Y_A$$

The superpotential of the $N = 1$ supergravity is determined by the gravitino mass terms in $N = 4$ after the $Z^2 \times Z^2$ orbifold projections.

Gravitino mass term: $e^{K/2}\, W =$

$$(\phi_0 - \phi_1) f_{IJK} \Phi_1^I \Phi_2^J \Phi_3^K + (\phi_0 + \phi_1) \bar{f}_{IJK} \Phi_1^I \Phi_2^J \Phi_3^K$$

$$\Phi_A^I = \left\{ \sigma_A^1,\ \sigma_A^2;\ \rho_A^1,\ \rho_A^2,\ \Phi_A^I \right\}$$

Both f_{IJK} $\bar{f}_{IJK}$ are the gauge structure constants of the $N=4$ "mother" theory.

In the heterotic, the term proportional to f_{IJK} give rise to a perturbative "electric gauging". The term proportional to $\bar{f}_{IJK}$ provide the non-perturbative "magnetic gauging".

- **What is the origin of f_{IJK} $\bar{f}_{IJK}$ in the superstrings and M-theory?**
- **What are the deformation parameters of the 2d σ-model in correspondence with the $N=4$ gauging coefficients f_{IJK} $\bar{f}_{IJK}$?**

3. Fluxes and $N = 4$ Gauging

In general, the breaking of SUSY requires a gauging with non-zero f_{IJK} involving the fields

$\sigma^1_A,\ \sigma^2_A;\ \rho^1_A,\ \rho^2_A$ $\longrightarrow$ **gauging involving the $N = 4$ graviphotons**

$\longrightarrow$ **gauging of the R-symmetry**

In string and M-theory, f_{IJK} and $\bar{f}_{IJK}$ are generated by non-zero FLUXES: Electric and Magnetic fluxes, RR and fundamental p-form fields:

- **3-form fluxes H^3, in the NS-sector of heterotic, type IIA and type IIB**

- **F^p, p-form fluxes, in M-theory and in the RR sector of type IIA and type IIB**

• F^2 **2-form fluxes**, in heterotic ($E_8 \times E_8$ or $SO(32)$) as well as in type I

• ω^3 **3-form geometrical fluxes**, in all strings and M-theory

Special cases have already been studied:

• H^3 **in heterotic**

Derendinger, Ibanez, Nilles, 85, 86;
Dine, Rohm, Seiberg, Witten, 85;
Strominger, 86; Rohm, Witten, 86.

• **Simultaneous presence of NS, RR** H^3 **and** F^3 **in Type IIB.**

Frey, Polchinski, 02;
Giddings, Kachru, Polchinski, 02;
Kachru, Schulz, Trivedi, 03;
Kachru, Schulz, Tripathy, Trivedi, 03;
Derendinger, Kounnas,
Petropoulos, Zwirner, 04.

• ω^3, H^3, F^2, **exact string solution via freely acting orbifold.**

$\longrightarrow$

Generalization of the Scherk–Schwarz deformation to superstring theory.

Rohm, 84;
Kounnas, Porrati, 88;
Ferrara, Kounnas,
Porrati, Zwirner, 89;
Kounnas, Rostand, 90;
Kiritsis, Kounnas, 96;
Kiritsis, Kounnas,
Petropoulos, Rizos, 99;
Antoniadis, Dudas, Sagnotti, 99;
Antoniadis, Derendinger, Kounnas, 99;
Derendinger, Kounnas,
Petropoulos, Zwirner, 04;
. . . . ,

See also F. Zwirner, J. Louis, . . . talks

4. Some examples of Geometrical Fluxes

• **Breaking of supersymmetry a la Scherk-Schwarz**

In the language of freely acting orbifolds, this corresponds to a twist induced by an R-symmetry operator and a shift in one internal coordinate.

The gravitino becomes massive due to the modification of the boundary conditions (in $D = 4$ Planck mass units)

$$m_{3/2}^2 = g^2 \, \frac{Q^2}{R^2}$$

Q is the R-symmetry charge
g_s is the string coupling constant
R is the compactification radius of the shifted coordinate.

What is the induced superpotential in the effective $N = 1$ description?

What is the flux interpretation of this specific model in the heterotic or type IIA orientifolds?

Choose the R-symmetry operator which induces the rotation in the ij plane

$$Q_{ij} = \oint \, dz \, [\Psi_i \Psi_j + x_i \partial x_j - (i \leftrightarrow j)]$$

$\Psi_i \rightarrow$ **2-d world sheet left-handed fermions**
x_i **the internal compactified coordinates.**

Strictly speaking, the operator Q is not well defined, since the internal coordinates are compactified $\rightarrow$ only discrete rotations are permitted $\leftrightarrow$ the crystallographic symmetries of the momentum lattice.

Switching on the deformation on the world sheet

$$\delta S_{ws} = F^{(k)}_{ij}\ Q_{ij}\ \bar{\partial} x_k \, ,$$

corresponds to switch on a non-zero $F^{(k)}_{ij}$ $\rightarrow$ a magnetic flux of the graviphotons

$$A^{(k)}_M = G^k_M + B^k_M, \qquad M = i, j$$

G^k_M and B^k_M are the $D = 10$ metric and antisymmetric tensor fields compactified on a S^1 cycle associated with x^k.

Only discrete rotations make sense $\rightarrow$ quantization of the magnetic fluxes.

The structure constant coefficients f^K_{IJ} of the $N = 4$ gauged supergravity are given in terms of the magnetic fluxes $F^{(k)}_{ij}$.

The induced superpotential in the $N = 1$ language (after the $Z_2 \times Z_2$ projections) reads

$$W = e^{-K/2}\ F^1_{2,3}\ (\sigma^1_1 + \rho^1_1)\ \sigma^2_2\ \sigma^2_3$$
$$= N_{flux}\ 1\ (T_2 + U_2)\ (T_3 + U_3)$$

x^k **is taken in the 1st complex plane**
x^i **and** x^j **in the second and third planes**

Some comments are in order:

- **The shifted direction has to be taken left-right symmetric; that is the reason of the** $\sigma^l_1 + \rho^l_1$ **combination**

- **The choice of** $l = 1, 2$ **corresponds to the two directions of the 1st complex plane. The two choices are equivalent via** $U_1 \leftrightarrow 1/U_1$ **duality transformation**

• **The twisted directions are taken only left-moving. The R-symmetry operators in heterotic are left-moving. This is the reason that only the σ_i^l appear in the superpotential. Here also the choice of $l = 2$ is equivalent to the $l = 1$ by means of U_i-duality transformations**

Having the $N = 1$ superpotential and the Kähler potential

$$K = -\log(S+\bar{S}) - \sum_{A=1}^{3} [\log(T_A+\bar{T}_A) + \log(U_A+\bar{U}_A)]$$

we can determine the potential.

The potential is flat in the field directions S, T and U with broken supersymmetry. (no-scale model)

$$\frac{G_S G_{\bar{S}}}{G_{S\bar{S}}} = \frac{G_{T_1} G_{\bar{T}_1}}{G_{T_1\bar{T}_1}} = \frac{G_{U_1} G_{\bar{U}_1}}{G_{U_1\bar{U}_1}} = 1$$

$$\frac{V}{N^2} = \frac{|T_2 - \bar{U}_2|^2 |T_3 + U_3|^2 + |T_3 - \bar{U}_3|^2 |T_2 + U_2|^2}{2^6 ReS ReU_1 ReT_1 ReU_2 ReT_2 ReU_3 ReT_3}$$

$$T_A = \bar{U}_A, \quad A = 2, 3 \qquad \text{at the minimum}$$

The gravitino mass is *independent* **of the moduli** $T_A, U_A,\ A = 2, 3$

$$m^2_{3/2} = \frac{N^2}{(S + \bar{S})(U_1 + \bar{U}_1)(T_1 + \bar{T}_1)} = g_s^2\ \frac{Q^2}{R_1^2}$$

- $SU(2)_k \times SU(2)_{k'}$ **- gauging in heterotic**

The $N = 1$ **superpotential is determined from the left- and right- moving structure constants of the left- and right-moving** $SU(2)_k \times SU(2)_{k'}$**. This generates non trivial** σ_A **and** ρ_A **terms in the superpotential**

$$W = \ e^{-K/2}\ A_l\ (\ \sigma_1^l\ \sigma_2^l\ \sigma_3^l + \rho_1^l\ \rho_2^l\ \rho_3^l\ )$$

$$W = iN\ (T_1+U_1)(T_2+U_2)(T_3+U_3)$$

$$+iN\ (T_1-U_1)(T_2-U_2)(T_3-U_3)$$

$$+N'\ (T_1U_1+1)(T_2U_2+1)(T_3U_3+1)$$

$$+N'\ (T_1U_1-1)(T_2U_2-1)(T_3U_3-1)$$

After minimization of the potential:

$$\frac{G_S G_{\bar{S}}}{G_{S\bar{S}}} = 1$$

$$\frac{G_{T_A} G_{\bar{T}_A}}{G_{T_A\bar{T}_A}} = \frac{G_{U_A} G_{\bar{U}_A}}{G_{U_A\bar{U}_A}} = 0, \qquad A = 1,2,3$$

$$T_A = \bar{T}_A = U_A = \bar{U}_A = 1 \qquad A = 1,2,3$$

The potential is negative with runaway behavior in the S direction

$$V = -2\ m_{3/2}^2 = -2\ \frac{N^2 + N'^2}{(S+\bar{S})}$$

This is precisely the form of the Dilaton potential in the heterotic theory on $SU(2)_k \times SU(2)_{k'}$.

Indeed, because of the central charge deficit $\delta\hat{c}$ coming from the $SU(2)_k \times SU(2)_{k'}$ six - dimensional compactification

$$\delta\hat{c} = -\frac{4}{k+2} - \frac{4}{k'+2}$$

a negative potential is generated which in the Einstein frame takes precisely the above form with

$$N^2 = \frac{2}{k+2}, \qquad N'^2 = \frac{2}{k'+2}$$

- $SU(2)_k \times SU(2)_{k'}$ **perturbative and non-perturbative gauging in heterotic**

$$W = -iS\ W[SU(2)_k] + W[SU(2)_{k'}]$$

$$\begin{aligned} W = {} & S\ N\ (T_1 + U_1)(T_2 + U_2)(T_3 + U_3) \\ & + S\ N\ (T_1 - U_1)(T_2 - U_2)(T_3 - U_3) \\ & + N'\ (T_1 U_1 + 1)(T_2 U_2 + 1)(T_3 U_3 + 1) \\ & + N'\ (T_1 U_1 - 1)(T_2 U_2 - 1)(T_3 U_3 - 1) \end{aligned}$$

After minimization of the potential:

$$\frac{G_S G_{\bar{S}}}{G_{S\bar{S}}} = \frac{G_{T_A} G_{\bar{T}_A}}{G_{T_A \bar{T}_A}} = \frac{G_{U_A} G_{\bar{U}_A}}{G_{U_A \bar{U}_A}} = 0, \qquad A = 1, 2, 3$$

$$S = \frac{N'}{N}, \quad T_A = \bar{T}_A = U_A = \bar{U}_A = 1, \quad A = 1, 2, 3$$

Stabilization of all moduli $\rightarrow$ AdS$_4$-solution with unbroken supersymmetry

$$V = -3m_{3/2}^2$$

This is similar to the stabilization of all the moduli found recently in Type IIA, D_6 orientifold, by combining the RR-fluxes and the geometrical fluxes suitably.

The $N = 4$ gauging found in type IIA was based is based on $SU(2)_k \times E_{k'}^3$

Derendinger-Kounnas-Petropoulos- Zwirner

Conclusion

Illustration and application of a general method that relates the $N=1$ effective Kähler potential and the superpotential to a consistent orbifold and/or orientifold projections of gauged $N=4$ supergravity.

Derivation of the effective superpotential $N=4 \rightarrow N=1$ for the main moduli in the presence of general fluxes

$\omega^3, \quad H^3 \quad H^2$ **In heterotic**

$\omega^3, \quad H^3 \quad H^2$ **In Type II asymmetric**

$\omega^3, \quad H^3, F^6, \quad F^4, \quad F^2, \quad F^0$ **in Type IIA**

$F^1, \quad F^3, \quad H^3, \quad \omega^3$ **in Type IIB**

We identify the correspondence between various admissible fluxes, $N = 4$ gauging and $N = 1$ superpotential terms.

Construction of explicit examples with different features:

- **Stabilization of four moduli, $V \geq 0$: No-scale models.**

- **Stabilization of less than four moduli, $V > 0$: de Sitter like, runaway solutions with possible cosmological interest.**

- **Models based on compact "gaugings", $V < 0$: Domain-Wall Solutions, Five-brane solutions with non trivial Dilaton or else.**

- **Models which admit a supersymmetric AdS_4 vacuum with all moduli stabilized.**