Entanglement interpretation of BH entropy

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אוניברסיטת בן-גוריון

R.B., M. Einhorn, A. Yarom,
to appear
Series of papers with
Amos Yarom, BGU
(also David Oaknin)

- Entanglement in space-time
- Entanglement and Gravity/FT duality
- The eternal AdS-BH

The idea

- Thermodynamics of space-times with a causal boundary is due to the lack of information about the inaccessible region

(“Microstates are due to entanglement”)

- Properties
  - Observer dependent
  - Area scaling
  - UV sensitive

What’s new ? Extension of methods to more space-times (BHs,dS)
Application to FT/Gravity duals (Maldacena)

Entanglement

$$\mathbf{H} = \mathbf{H}_{in} \otimes \mathbf{H}_{out}$$

$$|\psi> = \sum_{\alpha a} A_{\alpha a} |a> \otimes |\alpha>$$

$$O_{out} = I \otimes O$$
$$O_{in} = O \otimes I$$

$$\rho_{in} = Tr_{out} \rho$$
$$\rho_{out} = Tr_{in} \rho$$

$$\langle \psi | O_{in} | \psi \rangle = Tr(\rho_{in} O_{in})$$

If : thermal & time translation invariance then TFD:

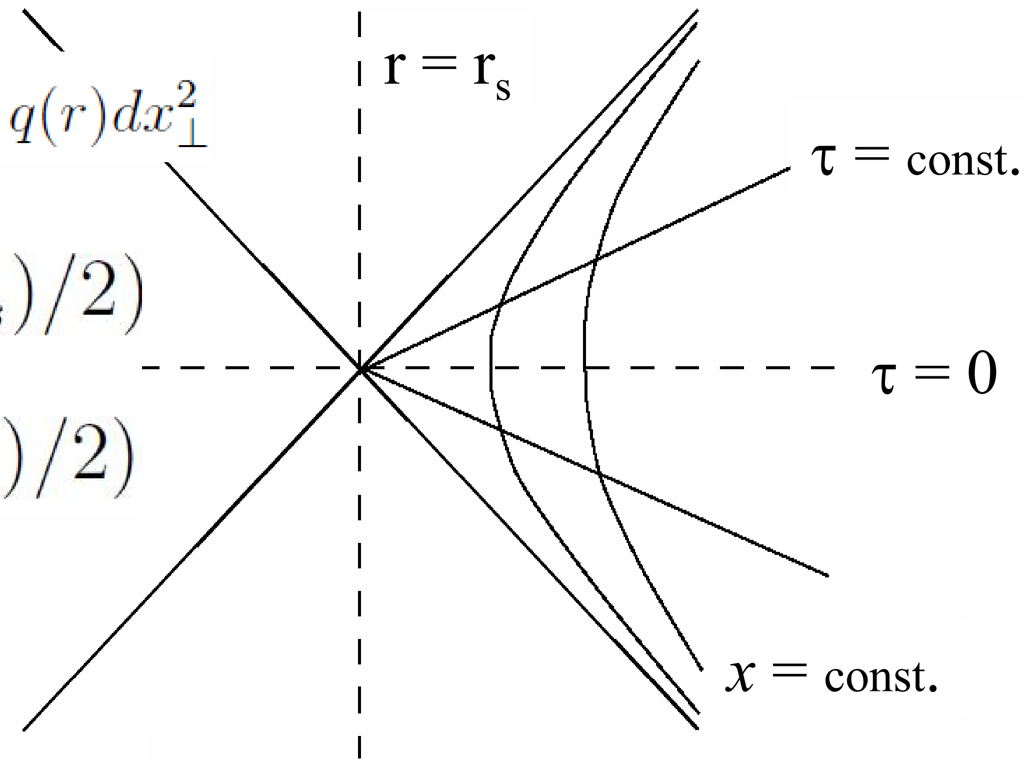
$$H_{in} = -H_{out} \quad \& \quad |\psi\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\frac{\beta}{2} E_i} |E_i\rangle |E_i\rangle$$

Entanglement in space-time

$$ds^2 = -f(r)d\tau^2 + dr^2/f(r) + q(r)dx_\perp^2$$

$$x = \sqrt{g(r)}\cosh(\tau f'(r_s)/2)$$

$$t = \sqrt{g(r)}\sinh(\tau f'(r_s)/2)$$

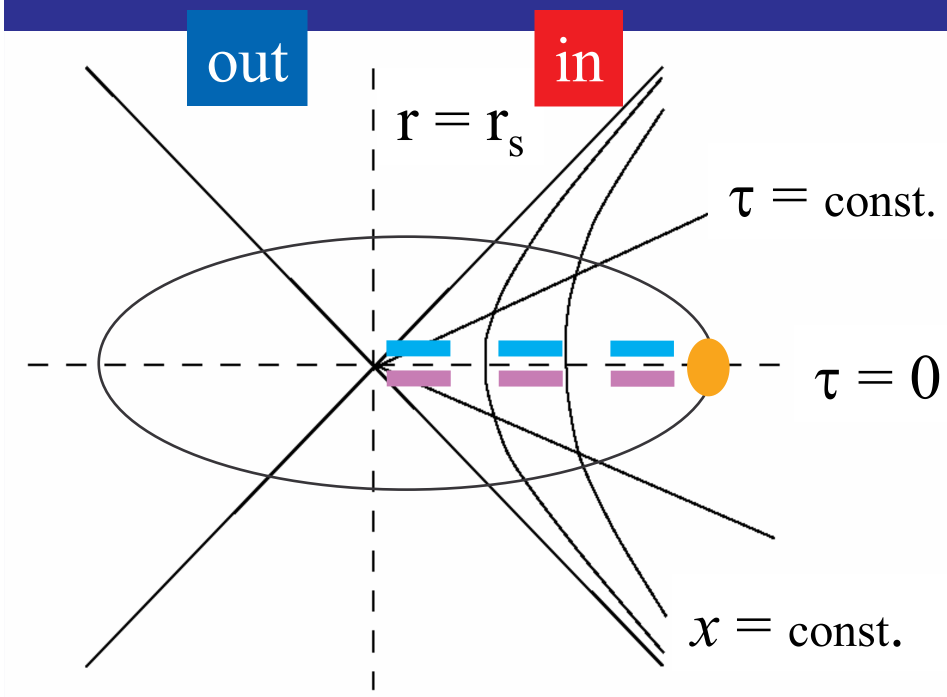


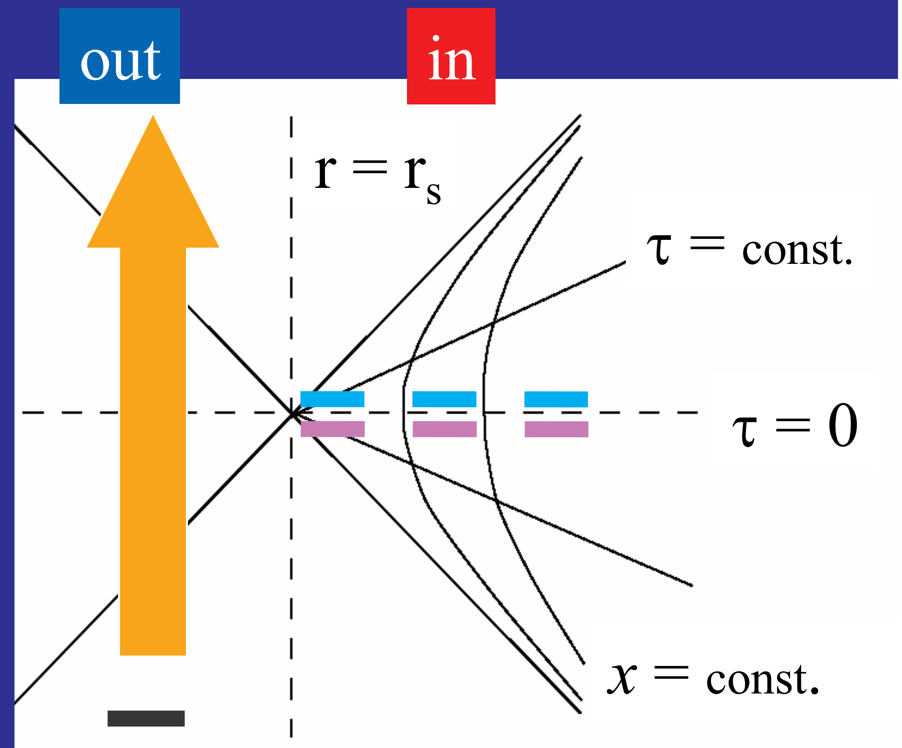
$$ds^2 = h(r)(-dt^2 + dx^2) + q(r)dx_\perp^2$$

$$h(r) = \frac{1}{\xi_0} f(r) e^{-f'(r_s)\int^r \frac{dr'}{f(r')}}$$

Examples: Minkowski, de Sitter, Shwartzschild, non-rotating BTZ BH, can be extended to rotating, charged, non-extremal BHs

Two ways of calculating ρ_{in}





Kabat & Strassler (flat space)

Construct the HH vacuum: the invariant regular state

Results*:

* Method works for more general cases

$$< \psi'_{in}|\rho_{in}|\psi''_{in} >= \int_{\varphi(\vec{x},0)=\begin{cases} r>r_s, \tau=0^- & \psi'_{in}(\vec{x}) \\ r>r_s, \tau=0^+ & \psi''_{in}(\vec{x}) \end{cases}} \exp\left[-\int_{-\infty}^{\infty}\dots\int \mathcal{L} d^d x d\tau\right] D\varphi$$

$$< \psi'_{in}|e^{-\beta_0 H_{eff}}|\psi''_{in} >= \int_{\substack{\varphi(\xi,0)=\psi'_{in}(\xi) \\ \varphi(\xi,\beta_0)=\psi''_{in}(\xi)}} \exp\left[-\int_0^{\beta_0}\int \mathcal{L}\_ \, d^d\xi d\eta\right] |\Omega| \; |g|^{\frac{1}{4}} D\varphi$$

If

1. The boundary conditions are the same
2. The actions are equal
3. The measures are equal

Then

$$\rho_{in} = e^{-\beta_0 H_{eff}}$$

H_{eff} – generator of (Imτ) time translations

Entanglement entropy

$$S_{in} = -Tr\left(\rho_{in} \ln \rho_{in}\right)$$

$$T = \frac{f'(r_s)}{4\pi}$$

de Alwis & Ohta

$$S = \frac{q(r_s)^{\frac{d-1}{2}}(d+1)\Gamma\left(\frac{d+1}{2}\right)\zeta(d+1)}{2^{2d-1}\pi^{\frac{3d+1}{2}}(d-1)}\delta^{-(d-1)}\int d\vec{x}_\perp$$

δ – invariant short distance cutoff

S is divergent
Naïve origin:
divergence of the optical volume near the horizon
More later …*not* brick wall

Entanglement & FT/Gravity duality

Extension of Maldacena's "eternal BH in AdS"

- $Z_{string}(X) = Z_{FT}(M)$
- Hilbert spaces isomorphic

$$\boxed{\mathbf{H}_W \cong \tilde{\mathbf{H}}_W}$$

- $Z_{string}(X_W) = Z_{FT}(M_W)$
- Hilbert spaces isomorphic

$$\boxed{\mathbf{H} \cong \tilde{\mathbf{H}}}$$

Assume:

if one side of the duality is in a thermal state so is the other

(Periodic time on one side is mapped to periodic time on the other side)

$X \longleftrightarrow M \qquad X_W \longleftrightarrow M_W$

Entanglement & FT/Gravity duality

$X \longleftrightarrow M$ $\qquad$ $X_W \longleftrightarrow M_W$

$g_s \rightarrow 0,\ \alpha' \rightarrow 0$

- $Z_{string}(X) \sim e^{-I_{SUGRA}}$
- Some limit in the FT (In AdS/CFT $N \rightarrow \infty$, $\lambda_{YM} \rightarrow \infty$)

The duality holds in the limit

$$\boxed{\tilde{\mathbf{H}} = \tilde{\mathbf{H}}_W \otimes \tilde{\mathbf{H}}_W} \leftarrow \boxed{\mathbf{H} = \mathbf{H}_W \otimes \mathbf{H}_W}$$

- Bulk
  - Thermal state

$$\rho_R = \mathrm{Tr}_L |0\rangle\langle 0|$$

$$e^{-I(X)} \xrightarrow[Tr]{} e^{-I(X_W)}$$

- Boundary
  - Thermal state

$$\widetilde{\rho}_W = \frac{1}{\widetilde{Z}} \sum_i e^{-\beta \widetilde{E}_i} |\widetilde{E}_i\rangle\langle\widetilde{E}_i|$$

$$Z_{FT}(M) = e^{-I(X)} \xrightarrow[Tr]{} e^{-I(X_W)} = Z_{FT}(M_W)$$

$g_s \rightarrow 0,\ \alpha' \rightarrow 0$

$$\sum_i \langle \psi_{Li} | 0 \rangle \langle 0 | \psi_{Li} \rangle \cong \sum_i \langle \tilde{\psi}_{Li} | \tilde{0} \rangle \langle \tilde{0} | \tilde{\psi}_{Li} \rangle$$

$$\rho_W = \mathrm{Tr}_L |0\rangle\langle 0| \cong \mathrm{Tr}_L |\tilde{0}\rangle\langle \tilde{0}|$$

$$\tilde{\rho}_W = \mathrm{Tr}_L |\tilde{0}\rangle\langle \tilde{0}|$$

$|\tilde{0}\rangle$ is TFD on $\tilde{\mathbf{H}}_W \otimes \tilde{\mathbf{H}}_W$

$$\begin{tikzcd} X \arrow[r, leftrightarrow] \arrow[d, "\mathrm{Tr}"'] & M \arrow[d, "\mathrm{Tr}"] \\ X_W \arrow[r, leftrightarrow] & M_W \end{tikzcd}$$

If the trace on each side gives a thermal state
with the same temp. then the state is a TFD

Interpretation

$g_s \rightarrow 0, \alpha' \rightarrow 0$

$$\begin{array}{ccc} X & \longleftrightarrow & M \\ \downarrow{\scriptstyle Tr} & & \downarrow{\scriptstyle Tr} \\ X_W & \longleftrightarrow & M_W \end{array}$$

$$\boxed{S(M_W) = \frac{A}{4G_N}} \rightarrow \boxed{S(X_W) = \frac{A}{4G_N}}$$

<u>BH entropy is due to entanglement</u>

$S(M_W)$ is due to entanglement $\rightarrow$

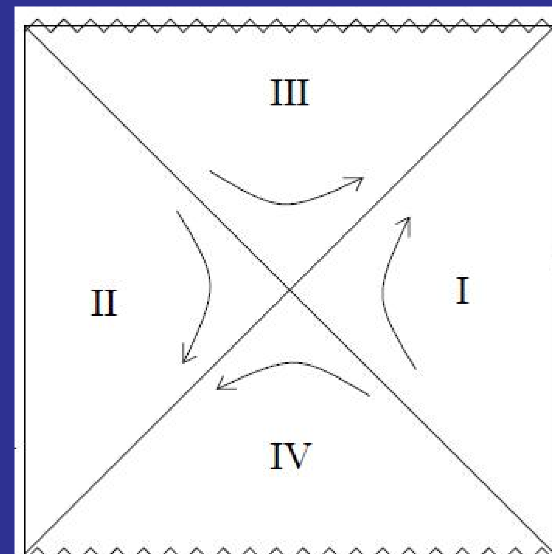
$S(X_W)$ is due to entanglement

Both entropies diverge in the limit $G_N \rightarrow 0$

Use CFT to define and compare at finite G_N

Eternal AdS-BH

Entanglement entropy is the entropy in a canonical ensemble of the CFT.



$$S_{BH} = \frac{A}{4G_5} = \frac{1}{4}\frac{CR^5 A}{\alpha'^4 g_s^2}$$

$$S_{ENT} = NA\delta^{-3}$$

(ofer: what is N)

For consistency $N\,\delta^{-3} \sim l_{Planck}^{-3}$

At the moment cannot compute the bulk regulator in terms of the boundary regulator → free constant

Number of fields

UV issues

The origin of the divergence:

- Looks like it is coming from the NH region ("brick wall"), however, exists also for free fields in flat-space

$$\lim_{\vec{x}_1 \to \vec{x}_2} \left\langle O(\vec{x}_1) O(\vec{x}_2) \right\rangle \sim \frac{1}{\left|\vec{x}_1 - \vec{x}_2\right|^{\Delta}}$$

- Expect a softer, finite expression in string theory
- To calculate need correlation functions in the coincidence limit

Summary

1. Method of calculating ρ in wedge
2. FT/Gravity duality: The dual state to the global HH vacuum is a TFD
3. Entropy is entanglement entropy
4. UV sensitivity