Tiny Graviton Matrix Theory

DLCQ of tpye IIB strings on
the $AdS_5 \times S^5$ or the plane-wave background

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Based on:

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Plan of the Talk

■ Brief Review of BFSS Matrix Model ideas

■ DLCQ of string/M- theory on $AdS_p \times S^q$ bg's.

■ Some facts about the ten dim. Max. SUSic plane-wave,

■ Lessons from DLCQ of M-theory on the $11d$. plane-wave bg., the BMN matrix model:

- A quick review on giant gravitons and introduction of: the Tiny Gravitons

■ The proposal for DLCQ of strings on $AdS_5 \times S^5$ or plane-wave background, the Tiny Graviton Matrix Theory (TGMT).

■ Analysis of and Evidence for the Model.

■ Summary, works in progress and a to-do-list.

- According to BFSS conjecture

The Discrete Light-Cone Quantization (DLCQ) of M-theory on the flat 11 dim. space in the sector with J units of the light-cone momentum is described by a

$U(J)$ SUSic Quantum Mechanics, i.e. a $U(J)$ $0+1$ dim. SYM theory with 16 SUSY.

- This theory is describing or described by a dynamics of J $D0$-branes.

Remarks:

- $D0$-branes are 1/2 BPS objects.
- SUSY is a crucial ingredient for the consistency of the conjecture.
- The BFSS matrix Model has been extended to describe DLCQ of M-theory on weakly curved backgrounds. It is done by adding proper deformations to the $0+1$ SYM action. There is a one-to-one relation between the background and the deformations [W. Taylor & M. van Raamsdonk '98, '99].

▶ Q1: What about the strongly curved backgrounds? Namely, $AdS_{4,7} \times S^{7,4}$ or the 11 dim. plane-wave, as the max. SUSic 11 dim. bg's.)

▶ Q2 Does DLCQ of M-theory on the above bg's admit a Matrix theory formulation?
Challenge: $D0$-branes are NOT 1/2 BPS objects on the above bg's.

■ Ingredients of DLCQ:

- (Globally defined) Light-like Killing vector.
- Compactification along light-like direction.

Consider $AdS_{p+2} \times S^{q+2}$ geometry:

$$ds^2 = R_A^2\Big(-\cosh^2\rho d\tau^2 + d\rho^2 + \sinh^2\rho d\Omega_p^2\Big) + R_S^2\Big(\cos^2\theta d\psi + d\theta^2 + \sin^2\theta d\Omega_q^2\Big)$$

light-like geodesics
i) Insdie AdS along the radial direction ρ.
ii) Inside sphere and along the ψ direction, at

$$\rho = \theta = 0,\ \ R_A\tau = \pm R_S\psi.$$

Only ii) is appropriate for the purpose of DLCQ and the light-like compactification.

• Next, let us follow the light-like observer and elaborate on the geometry seen by this observer ....
Systematically this geometry is known as the Penrose limit of the original background.

For the $AdS_{p+2} \times S^{q+2}$ background, that is

$$ds^2 = -2dx^+ dx^- - \mu^2(\vec{x}_p^2 + \kappa^2 \vec{x}_q^2)(dx^+)^2 + d\vec{x}_p d\vec{x}_p + d\vec{x}_q d\vec{x}_q$$

where $\kappa = \frac{R_s}{R_A}$ and
μ is an arbitrary parameter of dimension of energy (length^{-1}).

This is a PLANE-WAVE geometry.

It has a globally defined light-like Killing vector:

$$p^+ = \frac{\partial}{\partial x^-}.$$

- For $(p,q) = (2,5)$ or $(5,2)$:

$$ds^2 = -2dx^+dx^- - \mu^2(X_i^2 + \frac{1}{4}X_a^2)(dx^+)^2$$
$$+ dX_i dX_i + dX_a dX_a$$

$i = 1,2,3$ and $a = 1,2,\cdots,6$. This is the max. SUSic 11 dim. plane-wave bg. (There is of course also an $11d$ four-form: $F_{+ijk} = \mu\epsilon_{ijk}$.)

Therefore, DLCQ of M-theory

on the $AdS_{4,7} \times S^{7,4}$ is the same as the one on the 11 dim. plane-wave background, which is the plane-wave (or BMN) Matrix theory.

- For $(p,q) = (3,3)$:

$$ds^2 = -2dx^+dx^- - \mu^2(X_i^2 + X_a^2)(dx^+)^2$$
$$+ dX_i dX_i + dX_a dX_a$$

$i, a = 1,2,3,4$. This metric supplemented with

$$F_{+ijkl} = \frac{\mu}{4!g_s}\epsilon_{ijkl} \ , \ F_{+abcd} = \frac{\mu}{4!g_s}\epsilon_{abcd}$$
$$e^\phi = g_s = const.$$

is the max. SUSic type IIB 10 dim. bg.

Similarly, DLCQ of type IIB strings on the $AdS_5 \times S^5$ geometry is the same as DCLQ of strings on the $10d$ plane-wave background.

Bosonic Isometries of the 10d plane-wave

- Translation along x^- and x^+:

$$H = P_- = i\frac{\partial}{\partial x^+}$$
$$p^+ = -i\frac{\partial}{\partial x^-}$$

- $SO(4)_i \times SO(4)_a$ rotations, generated by:

$$J_{ij}, \ \ J_{ab}.$$

- There are 16 other isometries not manifest in the above coordinate system, (K_i, L_i) and (K_a, L_a):

$$[K_i, L_j] = \mu p^+ \delta ij \ ; \ \ [K_a, L_b] = \mu p^+ \delta ab$$
$$[K_i, K_a] = [L_a, L_b] = [K_i, L_a] = [K_a, L_i] = 0$$

Altogether, #isometries$=2 + 12 + 16 = 30$.

Note:

$$dim\,(so(4,2) \times so(6)) = 30$$

$$dim\,(Iso(9,1)) = 55$$

$$dim\,(Iso(8)) = 36.$$

Fermionic Isometries of the 10d plane-wave

$SO(8)$ fermions can be decomposed into the $SO(4)\times SO(4)$ spinors as:

- (Complexified) $\mathbf{8_s} \to (\psi_{\alpha\beta},\ \psi_{\dot\alpha\dot\beta})$
- (Complexified) $\mathbf{8_c} \to (\psi_{\alpha\dot\beta},\ \psi_{\dot\alpha\beta})$

$\alpha, \dot\alpha = 1, 2$ are the $SO(4)$ Weyl indices.

■ Supercharges:

▶ Kinematical supercharges: $q_{\alpha\beta}, q_{\dot\alpha\dot\beta}$, and their complex conjugates, $\# = 16$.

▶ Dymanical supercharges: $Q_{\alpha\dot\beta}, Q_{\dot\alpha\beta}$, and their complex conjugates, $\# = 16$.

■ Kinematical SUSY:

$$\{q_{\alpha\beta}, q^{\dagger\rho\lambda}\} = 2\delta^{\rho}_{\alpha}\delta^{\lambda}_{\beta}P^{+}$$

$$\{q_{\dot\alpha\dot\beta}, q^{\dagger\dot\rho\dot\lambda}\} = 2\delta^{\dot\rho}_{\dot\alpha}\delta^{\dot\lambda}_{\dot\beta}P^{+}$$

$$[q_{\alpha\beta}, H] = \mu q_{\alpha\beta}\ ,\ [q_{\dot\alpha\dot\beta}, H] = -\mu q_{\dot\alpha\dot\beta}$$

$$[q_{\alpha\beta}, P^{+}] = 0$$

■ Dynamical SUSY:

$$\{Q_{\alpha\dot\beta}, Q^{\dagger\rho\dot\lambda}\} = 2\delta^\rho_\alpha\delta^{\dot\lambda}_{\dot\beta}H + 2\mu\delta^{\dot\lambda}_{\dot\beta}(\sigma^{ij})^\rho_\alpha J_{ij} + 2\mu\delta^\rho_\alpha(\sigma^{ab})^{\dot\lambda}_{\dot\beta}J_{ab}$$

$$\{Q_{\dot\alpha\beta}, Q^{\dagger\rho\dot\lambda}\} = 0$$

$$\{Q_{\dot\alpha\beta}, Q^{\dagger\dot\rho\lambda}\} = \delta^{\dot\rho}_{\dot\alpha}\delta^{\lambda}_{\beta}H + 2\mu\delta^{\lambda}_{\beta}(\sigma^{ij})^{\dot\rho}_{\dot\alpha} J_{ij} + 2\mu\delta^{\dot\rho}_{\dot\alpha}(\sigma^{ab})^{\lambda}_{\beta}J_{ab}$$

$$[Q_{\alpha\dot\beta}, H] = [Q_{\dot\alpha\beta}, H] = 0$$

$$[Q_{\alpha\dot\beta}, P^+] = [Q_{\alpha\dot\beta}, P^+] = 0$$

For the full SUSY algebra see [D. Sadri, M.M. Sh-J, hep-th/0310119, RMP **76** (2004) 853].

Remarks

• The plane-wave SUSY algebra can be obtained as the Penrose contract of $PSU(2,2|4)$.

• The dynamical part of the SUSY algebra is

$$PSU(2|2) \times PSU(2|2) \times U(1)_H \times U(1)_{p^+}$$

which is a subalgebra of $psu(2,2|4)$ w/ 16 susy.

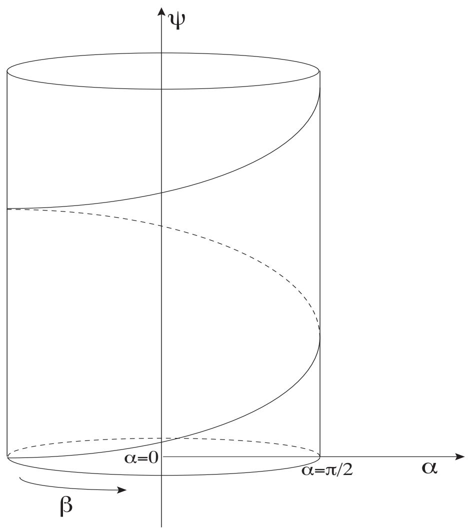
• p^+ is at the center of the whole plane-wave SUSY, i.e. it commutes with all supercharges. This should be contrasted with the flat space.

Penrose diagram of the plane-wave

By a series of coordinate transformations and analytic extension on the range of coordinates, the plane-wave metric can be brought to a form conformal to the Einstein static universe (conformal to $R \times S^9$):

$$ds^2 = \frac{1}{\mu^2} \frac{1}{|e^{i\psi} - \sin\alpha e^{i\beta}|^2} \Big(- d\psi^2 + \sin^2 \alpha d\beta^2 \\ + d\alpha^2 + \cos^2 \alpha d\Omega_7^2 \Big)$$

$\alpha \in [0, \pi/2], \ \beta \in [0, 2\pi], \ \psi \in \mathbb{R}.$



The $\psi = \beta, \ \alpha = \pi/2$ is the casual boundary of the plane-wave, which is one dimensional light-like.

Lessons From the BMN matrix Model

BMN Matrix model is a deformation of $0+1$ dim. $U(J)$ SYM by bosonic and fermionic mass terms plus a cubic CS term:

$$H = R_- Tr\Big[\Pi_I^2 - \frac{1}{4}[X^I, X^J]^2 + \psi^\dagger \Gamma^I [X^I, \psi]$$
$$+ (\frac{\mu}{R_-})^2 X_i^2 + \frac{1}{4}(\frac{\mu}{R_-})^2 X_a^2 + \frac{\mu}{R_-}\psi^\dagger\psi$$
$$+ \frac{\mu}{3! R_-}\epsilon_{ijk} X^i X^j X^k\Big]$$

where $I = \{i, a\} = 1, 2, \cdots 9$.

- BMN Matrix model can be obtained from

quantization (discretization) of super-membrane
in the light-cone gauge
on the $11d$ plane-wave bg.

This is done by replacing Poisson brackets with commutators and super-embedding coordinates with $J \times J$ matrices in the LC gauge fixed super-membrane action.

• Zero energy solutions, 1/2 BPS configurations, of the BMN matrix model are concentric

Fuzzy two spheres, S_F^2

which in the continuum (M-theory) limit, i.e. $J, R_- \to \infty, p^+ = J/R_- = fixed$, go over to

Membrane Giant Gravitons.

■ Giant Gravitons, A quick review

• Dp-branes are objects carrying $p+1$-form RR-charges proportional to their volume form.

• (Topologically) spherical D-brane can't carry the corresponding RR charge. It can, however, carry electric dipole moment of the RR form.

• In the absence of any other force a spherical brane would collapse under its own tension.

• The electric dipole can be used to stabilize the brane, iff we have a moving brane in the corresponding background RR flux.

• Such a magnetic form-field flux exists in $AdS_{p+2} \times S^{q+2}$, $(p,q) = (2,5), (3,3), (5,2)$ soln's.

• It turns out that it is possible to stabilize spherical p or q branes in $AdS_{p+2} \times S^{q+2}$ spaces,

that is,
spherical D3-branes in $AdS_5 \times S^5$ geometry and spherical M2 and M5 branes in $AdS_{4,7} \times S^{7,4}$.

- This is possible only when the branes are moving with the speed of light, i.e. when they are following a light-like geodesic and hence they are like graviton, the Giant Gravitons.
- Giant Gravitons are 1/2 BPS objects in the above backgrounds.
- Their size is then fixed by their angular momentum J as:

$$\left(\frac{R_{giant}}{R_{AdS}}\right)^{p-1} = \frac{J}{N}$$

where

$$(R_{AdS})^{p+1} = (l_p)^{p+1} N.$$

Therefore,

$$\left(\frac{R_{giant}}{l_p}\right)^{p-1} = \left(\frac{l_p}{R_{AdS}}\right)^2 J$$

What if J takes it minimal value $J = 1$?

In this case we call them Tiny Gravitons, as we'll momentarily see, they can become very small in the Planck units.

▶ Tiny Membrane Gravitons:

$$R_{tiny} = \frac{l_p^3}{R_{Ads_4}^2}$$

Remarks:
M5branes do not become tiny in $AdS_4 \times S^7$ bg, while they do become tiny in $AdS_7 \times S^4$ bg.

As tiny membranes, by definition, carry one unit of the angular momentum, and they are 1/2 BPS objects, they may be used to give a DLCQ description of M-theory on the $AdS_4 \times S^7$ bg, or the 11d plane-wave. That is,

tiny membrane gravitons play the role of $D0$-branes of BFSS in this DLCQ.

In fact the BMN matrix model is nothing but the theory of J tiny gravitons, i.e.

11d plane-wave matrix theory
$\equiv$
tiny (membrane) graviton matrix theory.

In this viewpoint a giant membrane of radius $R_{giant} \sim K$ is a bound state of K tiny membrane gravitons blown up onto a fuzzy two sphere.

- Tiny 3-brane Gravitons:

$$R_{tiny} = \frac{l_p^2}{R_{Ads_5}} \quad OR \quad R_{tiny}^4 = \frac{1}{N} l_p^4$$

Q: Can we use the same observation, but now with tiny three branes, for DLCQ formulation of type IIB strings on the 10d plane-wave bg?

The Tiny Graviton Matrix Theory conjecture:

The DLCQ of type IIB strings on the Max. SUSic 10d plane-wave and/or the $AdS_5 \times S^5$ background in the sector with J units of the light-cone momentum is a theory of J tiny three-brane gravitons, a $0+1$ dim. $U(J)$ SUSY gauge theory with $PSU(2|2) \times PSU(2|2) \times U(1)_H$ SUSY

In other words, similarly to the 11d case, Nonperturbative formulation of type IIB strings on the 10d plane-wave is described by a quantized three brane theory.

Q: How do we quantize a three brane theory? What is the Hamiltonian of the TGMT?

DBI action for a 3-brane on the plane-wave bg

$$S = \frac{1}{l_p^4}\int d\tau d^3\sigma \sqrt{-\det\left(G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu\right)} + \int C_4$$

where $X^\mu = X^\mu(\sigma^r, \tau)$, $r = 1, 2, 3$ and

$$\mu \in \{+, -, I\},\ I = 1, 2, \cdots, 8.$$

Note that we have set the gauge field on the brane to zero. To be discussed further later.....

Fixing the light-cone gauge:

$$X^+ = \tau$$

$$g_{0r} = G_{\mu\nu}\partial_0 X^\mu \partial_r X^\nu = 0$$

Note: the latter leads to "level matching" condition.

- The momenta:

$$p^+ = \frac{\partial L_{BI}}{\partial(\partial_\tau X^+)}\ ;\ P^I = \frac{\partial L_{BI}}{\partial(\partial_\tau X^I)}$$

$$H_{l.c.} = -\frac{\partial L_{BI}}{\partial(\partial_\tau X^-)}$$

- p^+ is a constant of motion and may be used to eliminate $\partial_\tau X^-$ for other d.o.f, the X^I's.

$$H=\frac{1}{2p^+}\int d^3\sigma\Big[P_I^2+\frac{1}{g_s^2}\det g_{rs}+(\mu p^+)^2X_I^2$$
$$-\frac{\mu p^+}{3g_s}\Big(\epsilon_{ijkl}X^i\{X^j,X^k,X^l\}$$
$$+\epsilon_{abcd}X^a\{X^b,X^c,X^d\}\Big)\Big]$$

where

$$g_{rs}=\partial_r X^I\partial_r X^I=\partial_r X^i\partial_r X^i+\partial_r X^a\partial_r X^a$$

$$\{F,G,H\}=\epsilon_{rps}\partial_r F\ \partial_p G\ \partial_s H$$

are the Nambu 3-brackets, a direct generalization of the Poisson bracket, and

$$\det g_{rs}=\frac{1}{3!}\Big(\{X^I,X^J,X^K\}\{X_I,X_J,X_K\}\Big)\,.$$

(One may add fermionic terms as well. For more details of LC gauge fixing see [D. Sadri & M.M.Sh-J, hep-th/0312155,].)

After adding the fermions and the gauge fields, the full Hamiltonian enjoys the $PSU(2|2)\times PSU(2|2)\times U(1)_H$ invariance.

To quantize the above action, similarly to the the membrane case, it is enough to quantize the Nambu brackets.

De tour on Nambu brackets:

A Nambu p-bracket is defined as:

$$\{A_1, A_2, \cdots, A_p\} = \epsilon^{r_1 r_2 \cdots r_p} \frac{\partial A_1}{\partial \sigma^{r_1}} \frac{\partial A_2}{\partial \sigma^{r_2}} \cdots \frac{\partial A_p}{\partial \sigma^{r_p}}$$

where $A_i = A_i(\sigma^r)$, $r = 1, 2, \cdots, p$.

- Properties of N.B.

1) Cyclicity & Exchange property:

$$\{A_1, A_2, \cdots, A_p\} = -\{A_2, A_1, \cdots, A_p\}$$

$$\{A_p, A_1, \cdots, A_{p-1}\} = (-1)^{p-1}\{A_2, A_1, \cdots, A_p\}$$

(Note that $\epsilon^{i_1 i_2 \cdots i_p} = (-1)^{p-1} \epsilon^{i_p i_1 \cdots i_{p-1}}$.)

2) Jacobi Identity:

$$\epsilon^{i_1 i_2 \cdots i_{2p-1}} \times$$

$$\times \Big\{ F_{i_1}, F_{i_2}, \cdots, F_{i_{p-1}}, \{F_{i_p}, F_{i_{p+1}}, \cdots, F_{i_{2p-1}}\} \Big\} = 0$$

3) Associativity:

$$\{F_1, F_2, \cdots, F_{p-1}, F_p G_p\} =$$

$$\{F_1, F_2, \cdots, F_{p-1}, F_p\} G_p + \{F_1, F_2, \cdots, F_{p-1}, G_p\} F_p$$

4) Trace property:

$$\int d^p\sigma \ \{F_1, F_2, \cdots, F_{p-1}, F_p\} = 0$$

5) By-part Integration:

$$\int d^p\sigma\{F_1, F_2, \cdots, F_{p-1}, F_p\}G_p =$$
$$-\int d^p\sigma\{F_1, F_2, \cdots, F_{p-1}, G_p\}F_p$$

Note: 5) is a result of 3)+4).

Quantization of Nambu Brackets

- Nambu EVEN brackets

i) $A(\sigma_i) \longleftrightarrow \hat{A}$ (matrices or operators).

ii)

$$\{F_1, F_2, \cdots, F_{2p}\} \longleftrightarrow [\hat{F}_1, \hat{F}_2, \cdots, \hat{F}_{2p}]$$
$$\equiv \frac{i^p}{(2p)!}\epsilon^{i_1 i_2 \cdots i_{2p}}\ \hat{F}_{i_1}\hat{F}_{i_2}\cdots\hat{F}_{i_{2p}}.$$

iii) $\int d^{2p}\sigma\ \star \longleftrightarrow Tr\hat{\star}.$

- Remarks:

It is not possible to perform quantization for $p \geq 2$ and maintain all the five properties of the classical Nambu brackets. In particular, with the above prescription associativity is lost. BUT, trace and by-part properties survive, because

$$\epsilon^{i_1 i_2 \cdots i_{2p}} = -\epsilon^{i_{2p} i_1 \cdots i_{2p-1}}.$$

- Nambu ODD brackets

Obviously the above procedure for even N.B. cannot be extended to odd N.B. while keeping the trace property......

The way out [hep-th/0406214]

Replace the Nambu $(2p-1)$-bracket with a Nambu $2p$-bracket:

$$\{F_1, F_2, \cdots, F_{2p-1}\} \longleftrightarrow [\widehat{F}_1, \widehat{F}_2, \cdots, \widehat{F}_{2p-1}, \mathcal{L}_{2p+1}]$$

$\mathcal{L}_{2p+1}$ is a given matrix (operator) closely related to the chirality operator in $2p$ dimensions. In particular, for the case of our interest, $p = 2$:

$$\begin{aligned}\{A, B, C\} \longleftrightarrow & [A, B, C, \mathcal{L}_5] = \\ & \frac{1}{4!}\Big([A, B][C, \mathcal{L}_5] + [C, \mathcal{L}_5][A, B] \\ & - [B, \mathcal{L}_5][A, C] - [A, C][B, \mathcal{L}_5] \\ & + [A, \mathcal{L}_5][B, C] + [B, C][A, \mathcal{L}_5]\Big).\end{aligned}$$

Now we can quantize the 3-brane action.......

End of De tour on Nambu brackets

Quantization of the LC Hamiltonian

- Replace:

$$p^+ \longleftrightarrow \frac{J}{R_-}$$

$$X_I(\sigma),\ P_I(\sigma) \longleftrightarrow X^I,\ J\Pi^I\ (J \times J \text{ matrices})$$

$$\psi_{\alpha\beta}(\sigma),\ \psi_{\dot\alpha\dot\beta}(\sigma) \longleftrightarrow \sqrt{J}\psi_{\alpha\beta},\ \sqrt{J}\psi_{\dot\alpha\dot\beta}$$

$$\frac{1}{p^+}\int d^3\sigma \star \longleftrightarrow R_-\ Tr\hat{\star}$$

$$\{F, G, K\} \longleftrightarrow \frac{1}{J}[\hat{F}, \hat{G}, \hat{K}, \mathcal{L}_5]$$

- Definition of $\mathcal{L}_5$:

$\mathcal{L}_5$ is a hermitian $J \times J$ matrix and

$$Tr\mathcal{L}_5 = 0$$

$$\mathcal{L}_5{}^2 = \mathbf{1_{J\times J}}.$$

- The string theory (continuum) limit is then:

$$J, R_- \to \infty,\ p^+ = \frac{J}{R_-}, \mu, g_s = fixed$$

Full Hamiltonian of TGMT

$$
\begin{aligned}
H = R_- Tr \Bigg[& \frac{1}{2}\Pi_I^2 + \frac{1}{2}\left(\frac{\mu}{R_-}\right)^2 X_I^2 \\
& + \frac{1}{2\cdot 3! g_s^2}[X^I, X^J, X^K, \mathcal{L}_5][X_I, X_J, X_K, \mathcal{L}_5] \\
& - \frac{\mu}{3! R_- g_s}\Big(\epsilon_{ijkl} X^i [X_j, X_k, X_l, \mathcal{L}_5] \\
& \qquad + \epsilon_{abcd} X^a [X_b, X_c, X_d, \mathcal{L}_5]\Big) \\
& + \left(\frac{\mu}{R_-}\right)\left(\psi^{\dagger\alpha\beta}\psi_{\alpha\beta} - \psi^{\dagger\dot{\alpha}\dot{\beta}}\psi_{\dot{\alpha}\dot{\beta}}\right) \\
& + \frac{2}{g_s}\Big(\psi^{\dagger\alpha\beta}(\sigma^{ij})_\alpha^{\ \rho}\ [X_i, X_j, \psi_{\rho\beta}, \mathcal{L}_5] \\
& \qquad - \psi^{\dagger\alpha\beta}(\sigma^{ab})_\beta^{\ \lambda}\ [X_a, X_b, \psi_{\alpha\lambda}, \mathcal{L}_5]\Big) \\
& - \frac{2}{g_s}\Big(\psi^{\dagger\dot{\alpha}\dot{\beta}}(\sigma^{ij})_{\dot{\alpha}}^{\ \dot{\rho}}\ [X_i, X_j, \psi_{\dot{\rho}\dot{\beta}}, \mathcal{L}_5] \\
& \qquad - \psi^{\dagger\dot{\alpha}\dot{\beta}}(\sigma^{ab})_{\dot{\beta}}^{\ \dot{\lambda}}\ [X_a, X_b, \psi_{\dot{\alpha}\dot{\lambda}}, \mathcal{L}_5]\Big)\Bigg].
\end{aligned}
$$

- The above action is invariant under

$$PSU(2|2) \times PSU(2|2) \times U(1)_H .$$

For the representation of the superalgebra generators in terms of the $J \times J$ matrices see [hep-th/0406214].

- The above action has an extra $\mathbb{Z}_2$ symmetry which exchanges the two $PSU(2|2)$ factors. Or equivalently it exchanges X_i and X_a directions.
- The action is the Hamiltonian for a

0 + 1 dim. $U(J)$ gauge theory,

in the temporal gauge,

under which all the fields transform

in the adjoint of $U(J)$:

$$\Phi \to U \Phi U^{-1}, \quad U \in \ U(J)$$

and $\Phi = \{X_I, \Pi_I, \psi\}$.

- $\mathcal{L}_5$ is also transforming in the adjoint.

- Although a gauge theory, the TGMT is not a Yang-Mills theory.

• To obtain the physical states we should impose the Gauss law constraint:

$$\left(i[X^I, \Pi^I] + 2\psi^{\dagger\alpha\beta}\psi_{\alpha\beta} + 2\psi^{\dagger\dot{\alpha}\dot{\beta}}\psi_{\dot{\alpha}\dot{\beta}}\right)|phys\rangle = 0$$

This is the e.o.m for the only component of the gauge field A_0.
The Gauss law constraint is the quantized version of the $g_{0r} = 0$ condition.

• DLCQ vs. Covariant formulation

$dim(\text{plane} - \text{wave Isometries}) = dim(AdS_5 \times S^5)$.
unlike the flat or the BFSS case.

In the plane-wave we do not have J^{+-} and J^{+I} boosts and p^+ commutes with all the SUSY generators.

Therefore, compared with the BFSS case, TGMT has a better chance of capturing the covariant information of string on $AdS_5 \times S^5$.

Evidence for the TGMT conjecture

- The fact that the TGMT Hamiltonian is invariant under the expected SUSY.
and that the superalgebra is a big one (with 16 SUSY), puts severe restrictions on the form of the Hamiltonian. (To my knowledge, however, there is no no-go theorem on this.)

- 1/2 BPS (zero energy) solutions.

$$V_B = R_- Tr\Bigg[\frac{1}{2}\left(\frac{\mu}{R_-}X^i - \frac{1}{3!g_s}\epsilon_{ijkl}[X^j, X^k, X^l, \mathcal{L}_5]\right)^2 + \frac{1}{2}\left(\frac{\mu}{R_-}X^a - \frac{1}{3!g_s}\epsilon_{abcd}[X^b, X^c, X^d, \mathcal{L}_5]\right)^2 + \frac{1}{4g_s^2}\left([X^a, X^b, X_i, \mathcal{L}_5]^2 + [X^i, X^j, X_a, \mathcal{L}_5]^2\right)\Bigg].$$

All four terms are +ve definite and hence to have zero energy, they should all vanish:

$$[X^j, X^k, X^l, \mathcal{L}_5] = \epsilon_{ijkl}\frac{\mu g_s}{R_-}\ X^i$$

$$[X^b, X^c, X^d, \mathcal{L}_5] = \epsilon_{abcd}\frac{\mu g_s}{R_-}\ X^a$$

$$[X^a, X^b, X^i, \mathcal{L}_5] = [X^a, X^i, X^j, \mathcal{L}_5] = 0.$$

All the solutions of these equations has been classified in [M.M. Sh-J, M. Torabian, hep-th/0501001].

• They are all in the form of Fuzzy three Spheres S_F^3 either in X^i and/or X^a directions. As some particular examples, the solutions to

$$X^a = 0 \ , \quad [X^j, X^k, X^l, \mathcal{L}_5] = \epsilon_{ijkl} \frac{\mu g_s}{R_-} \ X^i$$

gives the concentric S_F^3 in the X^i direction, centered at $X^a = 0$. (There are a similar class of solutions with $X^i \leftrightarrow X^a$.)

• These soln's are classified by $J \times J$ representations of $SO(4)$. For the irreducible rep., that is a single fuzzy sphere of radius (in units of l_s):

$$R^2 = \frac{\mu g_s}{R_-} J = \mu p^+ g_s.$$

In the continuum (string theory) limit it recovers the commutative giant 3-brane graviton.

• The reducible reps, then generically give concentric giants.

• One should compare the above result with a generic S_F^3 result in which

$$R^2 = l^2 J$$

with l being the *fuzziness* scale. In our case

$$l^2_{fuzziness} = \frac{\mu g_s}{R_-} l_s^2 = l_p^2 \sqrt{\frac{1}{N}} = l^2_{tiny}.$$

The string theory limit can be understood as $l_{tiny} \to 0$ keeping l_p, R_{giant} fixed, that is

$$J, N \to \infty \ , \ \ g_s, l_p, J^2/N = fixed.$$

There is a one-to-one correspondence between the half BPS configurations of $\mathcal{N} = 4$ $U(N)$ SYM with R-charge J, the chiral primary opts, and the fuzzy sphere soln's of the TGMT. Both of them are labeled by representations of group of permutations of J objects, $\mathcal{S}_J$.

[hep-th/0501001].

• Spectrum of fluctuations about the single giant fuzzy sphere solution has been worked out in [hep-th/0406214] and shown that it exactly matches with that of a spherical D3-brane in the plane-wave (or $AdS_5 \times S^5$) background. (The latter has been worked out in [D. Sadri, M.M. Sh-J, hep-th/0312155].
This is a non-trivial test, because in the DBI action we started with, we had not included the gauge fields living on the brane.

• One can also work out the effective coupling of these fluctuation modes [hep-th/0406214]. The effective coupling about the single giant vacuum is:

$$g_{eff} = \frac{R_-}{\mu\sqrt{g_s}}\frac{1}{J} = \frac{1}{\mu p^+\sqrt{g_s}}$$

Expressed in terms of the $\mathcal{N} = 4$ $U(N)$ SYM parameters:

$$g_{eff}^2 = \frac{N}{J^2} = \frac{1}{g_2}$$

where g_2 is the effective coupling for strings on plane-wave S. Minwalla, et.al.[hep-th/0205089].

Q: What about the $X = 0$ vacuum?!
Where are type IIB fundamental strings?

■ 2nd part of the conjecture

In the string theory limit the $X = 0$ vacuum becomes strongly coupled and

Fundamental type IIB strings are non-perturbative objects about the $X = 0$ vacuum.

(Remark: This is very similar to the M5-brane giants in the BMN matrix model, [J. Maldacena, M.M. Sh-J, M. van Raamsdonk, hep-th/0211139].)

• Evidence: the spectrum of small BPS fluctuations (in a $\frac{R_-}{\mu}$ expansion) about the $X = 0$ vacuum, exactly matches with the spectrum of SUGRA modes (BPS spectrum of strings) in the plane-wave bg.

More works in this direction is under way .....

■ Summary and a short to-do-list

• Tiny Graitvons may be used as "D0-branes" to give a matrix theory, $0+1$ dim. gauge theory, formulation for string/M- theory on the curved backgrounds, such as AdS-spaces and the plane-waves.

• The plane-wave string theory, in the DLCQ description, admits a matrix theory formulation, the TGMT. It is a SUSY gauge theory (though not a SYM!) with SUSY

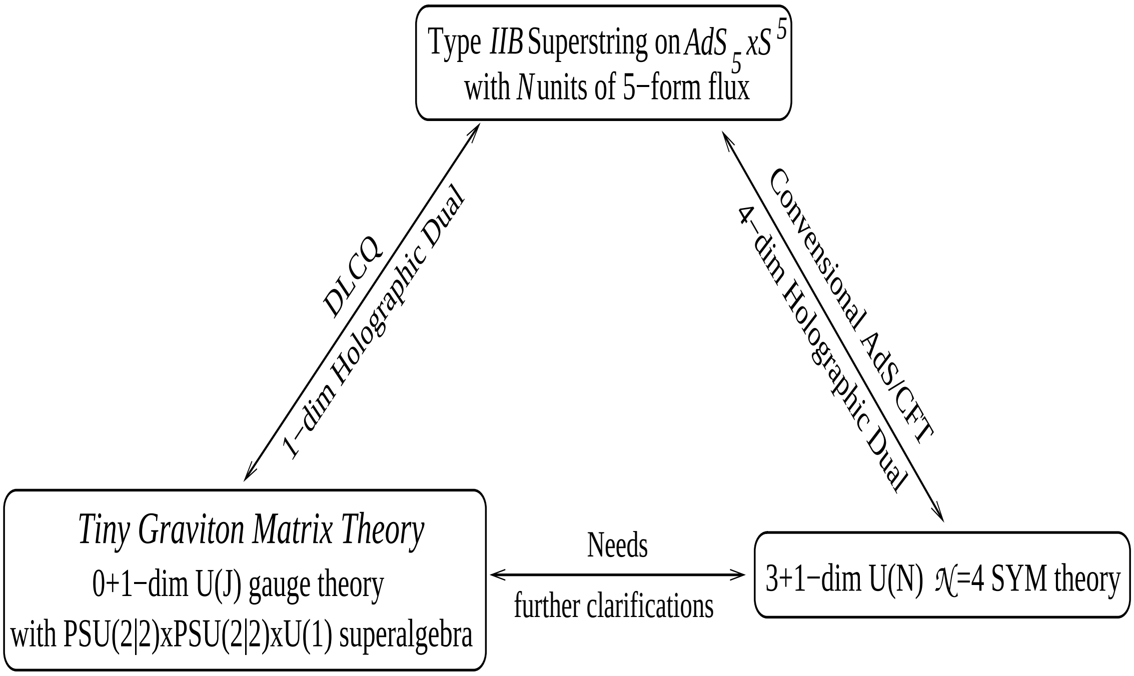
$$PSU(2|2) \times PSU(2|2) \times U(1)_H .$$

• In the formulation of TGMT, we introduced an extra traceless $J \times J$ matrix $\mathcal{L}_5$, which squares to identity.
The $\mathcal{L}_5$ is reminiscent of the eleven dimensional origin of the type IIB theory. It is related to the 11^{th} circle [hep-th/0501001].

• The $\mu \to 0$ (flat space) limit, is it a smooth one? Is the TGMT in that limit related to IKKT or (2+1) SYM/T^2 (Susskind-Sethi model)?

- As the causal boundary of the plane-wave is one-dimensional light-like, the DLCQ description is the holographic description. Therefore, TGMT is the holographic formulation of strings on plane-wave.

- How and where does TGMT fit in the AdS/CFT duality?!



- Interesting observation: fuzziness l, the "size of tiny gravitons" is

$$l_{tiny}^4 = \frac{1}{N} l_p^4 .$$

That is, the $1/N$ expansion has now a geometric meaning.

- $\frac{1}{J}$ vs. $\frac{1}{N}$ expansion?!

- Lower SUSY solutions?!
  Other D-brane solutions?!

- Connection to Verlinde's String Bit Model?!

- For finite R_- we expect strings to have winding modes, where are they?

- Does TGMT satisfy the duality requirements and in particular $SL(2,\mathbb{Z})$ duality of type IIB?

- Can we "quantize" an M5-brane theory in the same way we did for a 3-brane?
That is, by replacing the Nambu five brackets which appear in the M5-brane Hamiltonian [e.g. see, hep-th/0211139], by Nambu six brackets and introduction of $\mathcal{L}_7$?
Are the quantized giant M5-branes in the form of S^5_F?!
If the above is correct, can we have a matrix theory formulation of six dimensional $(0,2)$ theory (on $R\times S^5$)?

There are much more things to be done on the TGMT.......