# Eikonal, black holes and conformal bootstrap

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# Introduction

Heavy states $\simeq$ black holes in dual CFT.

- Thermalization of heavy states in CFTs
- New (and better) handle on strongly coupled finite temperature CFTs
- Correlators in CFTs with large central charge
- Inelastic gravitational scattering.

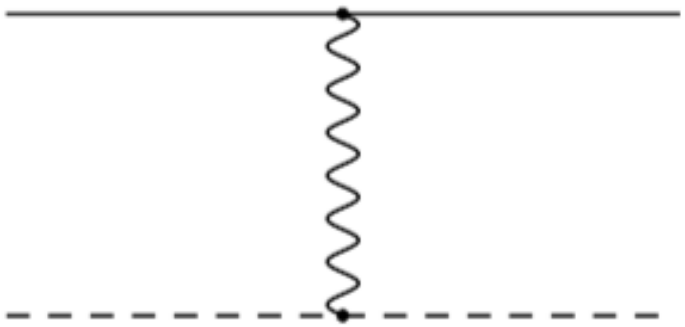
Based on work with R. Karlsson, M.Kulaxizi, G.S. Ng, P. Tadic (Dublin): 1812.03120, 1904.00060, 1907.00867, 1909.05775

# Introduction: the eikonal phase

High energy gravitational scattering amplitude (large $s$, finite $t$) is described by the eikonal phase (a.k.a. the phase shift)

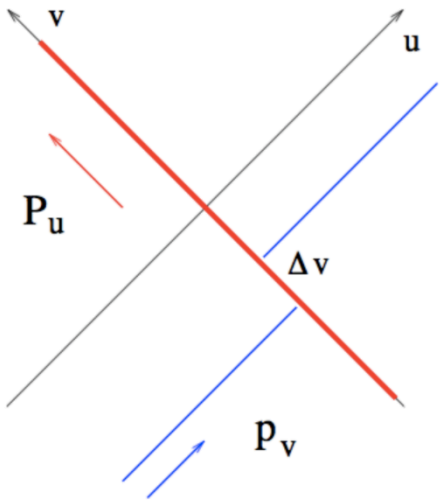
$$\mathcal{A} \sim e^{i\delta(s,b)}$$

where $b$ is the impact parameter. This is the result of the summation of infinite number of diagrams, but comes from the exponentiation of



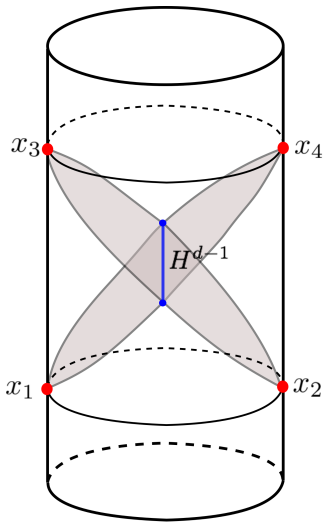
# Introduction: shock wave

The phase shift can also be obtained by studying the propagation of a null geodesic in a shock wave background ($v = x^-, u = x^+$)




$$\delta(s,b) = -P_1 \cdot \Delta x \simeq (P_1^+)(P_2^-)\ell_P^2 \left(\frac{\ell_P}{b}\right)^{D-4}, \quad G_N = \ell_P^{D-2}$$

# Introduction: scattering in Anti de Sitter




Similar expressions exist for scattering in $\mathrm{AdS}_{d+1}$ ($D = d + 1$). A factor of $b^{-D+4}$ becomes a propagator in $H_{d-1}$. AdS/CFT correspondence relates scattering amplitude to

$$\int_{x_3} \int_{x_4} e^{ip_3 x_3 + ip_4 x_4} \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

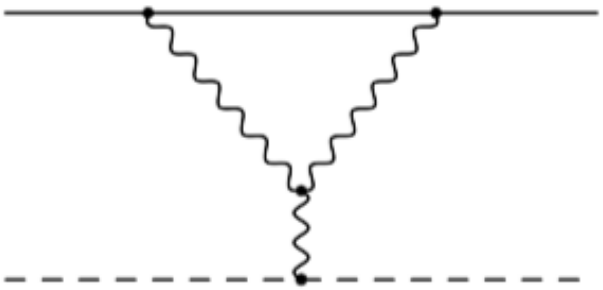
# Introduction: heavy-light scattering

Consider light particle of mass $m$ and energy $E$ scattering off a heavy particle of mass $M$. Assume $M \gg E \gg m$.

$$\delta^{(1)} = Eb\left(\frac{R_s}{b}\right)^{D-4}, \qquad R_s^{D-3} \sim G_N M$$

Higher order terms can in principle be computed. $R/b$ is the expansion parameter. E.g.

$$\delta^{(2)} = Eb\left(\frac{R_s}{b}\right)^{2D-6}$$



# Introduction: scattering in AdS and holography

We study heavy-light scattering in AdS of radius $R$. Two dimensionless parameters: $R_s/R$ and $b/R$.

We compute the phase shift to all orders in both $R_s/R$ and $b/R$. The resulting formula reduces to the flat space result in the $R \to \infty$ limit.

CFT object is the (heavy-light)$^2$ $\langle \mathcal{O}_H \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \rangle$ correlator.

# Outline

# Phase shift in AdS

Consider heavy-light scattering in AdS of radius $R$. Study null geodesics in the asymptotically $\mathrm{AdS}_{d+1}$ black hole of mass $M$.

$$ds^2 = -f\, dt^2 + f^{-1} dr^2 + r^2 d\Omega_{d-1}^2$$

where

$$f = 1 + \frac{r^2}{R^2} - \frac{\mu R^{d-2}}{r^{d-2}}$$

and

$$\mu \simeq \frac{G_N M}{R^{d-2}} = \frac{\ell_P^{d-1} M}{R^{d-2}}$$

$\mu \approx (R_s/R)^{d-2}$ in the small $\mu$ regime.

# Phase shift in AdS

Killing vectors $\partial_t$ and $\partial_\varphi$ (isometries w.r.t. time translation and rotation of $S^{d-1}$) give rise to conserved quantities $p^t$ and $p^\varphi$:

$$p^t = \left(1 + \frac{r^2}{R^2} - \frac{\mu R^{d-2}}{r^{d-2}}\right)\frac{\partial t}{\partial \lambda}, \qquad p^\varphi = r^2 \frac{\partial \varphi}{\partial \lambda},$$

where $\lambda$ is an affine parameter. Null geodesics are labeled by the impact parameter $L$;

$$e^{2L} = \frac{p^t + p^\varphi}{p^t - p^\varphi}$$

Minkowski results are recovered in the flat space limit: $R_s/R \ll 1$ ($\mu \ll 1$) and also $L \approx b/R \ll 1$.

# Phase shift in AdS

In the following set $R = 1$. Null geodesic equation $g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = 0$ (derivatives w.r.t. affine parameter $\lambda$) takes the form

$$\frac{1}{2}\left(\frac{\partial r}{\partial \lambda}\right)^2 + V_{eff}(r) = \frac{1}{2}(p^t)^2$$

where

$$V_{eff}(r) = \frac{(p^\varphi)^2}{2}\left(1 + \frac{1}{r^2} - \frac{\mu}{r^d}\right)$$

For $\mu = 0$ all null geodesic reimerge at the same point $t = \pi, \varphi = \pi$. Deviation from it parameterized by $\Delta x(L) = (\Delta t, \Delta\varphi)$.

# Phase shift in AdS

The resulting phase shift $\delta = -p^\mu \Delta x_\mu$ can be expanded

$$\delta = \sum_{k=1}^{\infty} \delta^{(k)} \mu^k$$

We computed all $\delta^{(k)}$. For example,

$$\delta^{(1)} \simeq e^{-(d-1)L} \; {}_2F_1(\frac{d}{2} - 1, d-2, \frac{d}{2} + 1, e^{-2L})$$

In the small impact parameter regime $L \ll 1$ we recover the Minkowski result. The radius of convergence of the series corresponds to the null geodesic entering the black hole horizon.

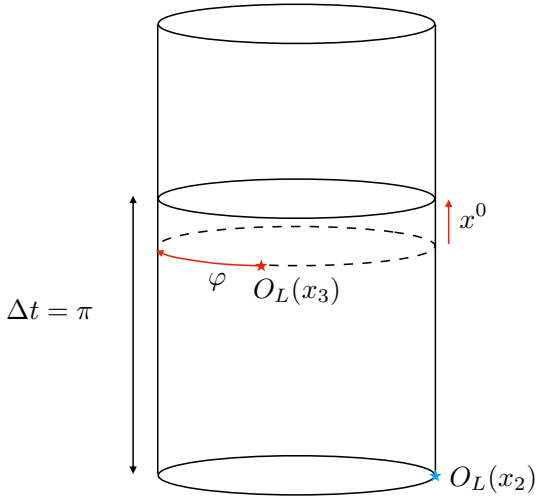
# (Heavy-light)$^2$ correlator in CFT

State-operator correspondence in CFT. Conformal dimension of the operator $\Delta$ is the energy of the state on a sphere.

Recall $\mu \simeq (\ell_P/R)^{d-2} MR = \Delta_H / C_T$ where $C_T$ is the central charge of the CFT: $\langle T_{\mu\nu} T_{\alpha\beta} \rangle \sim C_T$ (here $T_{\mu\nu}$ is the stress tensor)

We will consider the limit $\Delta_H, C_T \to \infty$, $\mu$ fixed and will study expansion in powers of $\mu$. This corresponds to considering a heavy state in the CFT – does it thermalize? (looks like a black hole in the dual gravity)

# (Heavy-light)$^2$ correlator in CFT




Similar to the shock wave situation, the phase shift can be obtained from a CFT amplitude

$$\int_x e^{ipx} \langle \mathcal{O}_H(0) \mathcal{O}_H(\infty) \mathcal{O}_L(1) \mathcal{O}_L(z, \bar{z}) \rangle, \qquad z, \bar{z} = e^{i(\Delta t \pm \varphi)}$$

# CFT in $d = 2$

In the $d = 2$ case the stress-tensor sector of the correlator is known to all orders in $\mu$ (Virasoro vacuum block):

$$\langle \mathcal{O}_H(0)\mathcal{O}_H(\infty)\mathcal{O}_L(1)\mathcal{O}_L(z)\rangle \simeq \frac{1}{\left(\sin\left[\bar{\alpha}\pi + \frac{\bar{\alpha}}{2}(\Delta t \pm \varphi)\right]\right)^{\Delta_L}}$$

where $\bar{\alpha} = \sqrt{1-\mu}$. The integral picks up the pole where the argument of the sin vanishes and the phase shift is simply

$$\delta = \frac{1}{2}p^-(\Delta t + \varphi) = \pi\sqrt{-p^2}e^{-L}\left(\frac{1}{\sqrt{1-\mu}} - 1\right)$$

which agrees exactly with the gravity result.

# CFT in $d > 2$

In $d > 2$ the exact CFT answer for the correlator is not known. The correlator can be decomposed into *conformal blocks*.

$\mathcal{O}(\mu)$ term comes from the conformal block of $T_{\mu\nu}$ in the direct channel

$$\mathcal{O}_H \times \mathcal{O}_H - T_{\mu\nu} - \mathcal{O}_L \times \mathcal{O}_L$$

In the cross-channel

$$\mathcal{O}_H \times \mathcal{O}_L - [\mathcal{O}_H \mathcal{O}_L]_{n,\ell} - \mathcal{O}_H \times \mathcal{O}_L$$

where $[\mathcal{O}_H \mathcal{O}_L]_{n,\ell} = \mathcal{O}_H \; \partial^\ell \; \Box^n \mathcal{O}_L$ are heavy-light double trace operators.

# CFT in $d > 2$

The stress-tensor exchange at $\mathcal{O}(\mu)$ fixed by Ward identity and reproduces $\mathcal{O}(\mu)$ phase shift.

Correlator at $\mathcal{O}(\mu^2)$ is a result of an infinite sum over *double stress tensor operators*: $T_{\mu\nu}\partial_\alpha \ldots \partial_\beta \Box^n T_{\gamma\delta}$. Higher orders involve summing over multi stress operators.

Need to know OPE coefficients for all spins! Things simplify in the lightcone limit $\bar{z} \to 1$ .

# Lightcone limit

$d = 4$ correlator at $\mathcal{O}(\mu)$ :

$$\langle \mathcal{O}_H \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \rangle^{(1)} \simeq [(1-z)(1-\bar{z})]^{-\Delta_L} f_3(z)$$

Make an ansatz

$$\langle \mathcal{O}_H \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \rangle^{(2)} \sim b_{33} f_3^2 + b_{24} f_2 f_4 + b_{15} f_1 f_5$$

$$\langle \mathcal{O}_H \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \rangle^{(3)} \sim b_{333} f_3^3 + b_{234} f_2 f_3 f_4 + \dots$$

and similarly for all values of $k$ for $\mathcal{O}(\mu^k)$ coefficients. Here

$$f_a(z) = (1-z)\ {}_2F_1(a, a, 2a, 1-z)$$

Bootstrap and solve for $b_{i_1 \dots i_k}$ .

# Bootstrapping the lightcone correlator

Consider stress-tensor contribution ($k = 1$ term in the ansatz):

$$\langle \mathcal{O}_H \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \rangle^{(1)} \sim (\alpha_0 + \alpha_1 z + \ldots) + (\beta_0 + \beta_1 z + \ldots) \log z$$

with $\alpha_i, \beta_i$ known $\Delta_L$-dependent coefficients. The cross channel produces

$$\langle \mathcal{O}_H \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \rangle^{(1)} \sim (P_0^{(1)} + P_1^{(1)} z + \ldots) + (\gamma_0^{(1)} + \gamma_1^{(1)} z + \ldots) \log z$$

where $\gamma(n, \ell) = \gamma_n^{(1)}/\ell + \mu^2 \gamma_n^{(2)}/\ell^2 + \ldots$ and similarly $P(n, \ell)$.

# Bootstrapping the lightcone correlator

This determines $P_n^{(1)}, \gamma_n^{(1)}$ and all terms $\sim \log^2 z$ at $\mathcal{O}(\mu^2)$. This determines **all** coefficients in the ansatz at $\mathcal{O}(\mu^2)$:

$$\langle \mathcal{O}_H \mathcal{O}_H \mathcal{O}_L \mathcal{O}_L \rangle^{(2)} \simeq \frac{(1-\bar{z})^2}{[(1-z)(1-\bar{z})]^{\Delta_L}} \left( \frac{\Delta_L}{\Delta_L - 2} \right) \times \left[ (\Delta_L - 4)(\Delta_L - 3) f_3(z)^2 + \frac{15}{7}(\Delta_L - 8) f_2(z) f_4(z) + \frac{40}{7}(\Delta_L + 1) f_1(z) f_5(z) \right]$$

Agrees with the $\mathcal{O}(\mu^2)$ phase shift.

# Bootstrapping the lightcone correlator

One can continue this procedure and compute the lightcone correlator to any desired order in $\mu$

We can read off the OPE coefficients $C_{OPE}(\mathcal{O}_L, \mathcal{O}_L, T^k_{\mu\nu})$ to leading order in $1/C_T$. We did not use holography, but recover the OPE coefficients which have been computed using holography (and get many more).

# Summary

- Computed phase shift in AdS to all orders in $\mu$; reproduced Minkowski results in flat space limit.
- Leading and subleading terms (in $d = 2$ all terms) in the eikonal phase are reproduced by a CFT computation of a heavy-heavy-light-light correlator. This hints at thermalization of a generic heavy state.
- Lightcone limit of the (heavy-light)$^2$ correlator can be computed to any desired order. OPE coefficients computed.

# To do list

- Find closed form of heavy-heavy-light-light correlator.
- Symmetry of the lightcone correlator?
- Understand thermal CFTs from the OPE point of view. Role of double trace operators? Non-universality of subleading twist?
- Applications. Hydrodynamics, thermalization, quantum chaos...
- Inelastic scattering...

Thank you!