Exact Dimensional Deconstruction

Costis Papageorgakis

10th Regional Meeting on String Theory

Kolymbari, 18^{th} September 2019

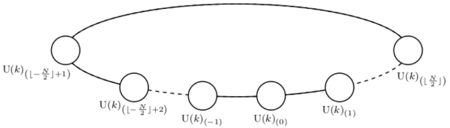
w/ J. Hayling, V. Niarchos, R. Panerai, E. Pomoni
and D. Rodríguez-Gómez [1809.10485, 1803.06177, 1704.02986]

Motivation

Dimensional deconstruction:

[Arkani-Hamed, Cohen, Georgi ’01]

- Limit of circular quiver-gauge theories reproduces the KK spectrum of extra dimensions
- The circle in theory space becomes geometric!



Long-term Goals

Interesting because we can learn about

- D > 4
- Non-Lagrangian QFTs
- Complicated quantum dynamics in any D

from (simpler) lower-dimensional theories

Elements of dimensional deconstruction

Start with N-noded D-dim $U(k)^N$ circular-quiver gauge theory in flat space with

$$G^{(\alpha)} = G$$

$\Rightarrow$ Higgs such that $U(k)^N \to U(k)$ by considering

$$\langle Q^{(\alpha)} \rangle = v$$

and identifying

$$\frac{G}{v} \equiv g_{\rm dec}^2 \qquad \frac{N}{Gv} \equiv 2\pi \widehat{R} \qquad \frac{1}{Gv} \equiv a$$

Recovers KK modes of D+1-dim theory on discretised $S^1_{\hat{R}}$

$$M_n^2 = 4v^2 G^2 \sin^2\left(\frac{\pi n}{N}\right) \simeq \left(\frac{2\pi n}{\widehat{R}}\right)^2$$

$\Rightarrow$ For $E \gg 1/a$ the D+1-dim theory completed by D-dim quiver

Q: Can we have $a \to 0$?

$\Rightarrow$ Yes if we take the limit

$$v \to \infty \qquad N \to \infty \qquad G \to \infty$$

$\Rightarrow$ Produces continuum $U(k)$ theory, with $g^2_{\rm dec}$ and $\widehat{R}$ fixed

Challenges

A number of conceptual and technical issues:

Challenges

A number of conceptual and technical issues:

- $\diamond$ All considerations are Lagrangian

$\Rightarrow$ Continuum limit requires strong coupling

Challenges

A number of conceptual and technical issues:

- ◇ All considerations are Lagrangian
- $\Rightarrow$ Continuum limit requires strong coupling
- ◇ Deconstruction of $\mathrm{QFT}_{\mathrm{D+1}}$ correlation functions?
- $\Rightarrow$ What is the precise $\mathrm{QFT}_{\mathrm{D}} \rightarrow \mathrm{QFT}_{\mathrm{D+1}}$ dictionary?

Challenges

A number of conceptual and technical issues:

- ◇ All considerations are Lagrangian
- $\Rightarrow$ Continuum limit requires strong coupling
- ◇ Deconstruction of $\mathrm{QFT}_{\mathrm{D+1}}$ correlation functions?
- $\Rightarrow$ What is the precise $\mathrm{QFT}_{\mathrm{D}} \to \mathrm{QFT}_{\mathrm{D+1}}$ dictionary?
- ◇ Rules for deconstructing S^1 (and partially S^2) understood
- $\Rightarrow$ How does one deconstruct general compact geometries?

Today

Focus on SUSY examples and deconstruct S^1

$\Rightarrow$ Continuum limit is more controlled

- $\diamond$ Moduli space is not lifted
- $\diamond$ Can use the power of supersymmetric localisation
- $\diamond$ Deconstruction leads to SUSY doubling

$\Rightarrow$ Deconstruct exact partition function of $\mathrm{QFT}_{\mathrm{D+1}}$

Deconstruction of exact PFs

Start with PF for the quiver-gauge theory (e.g. on S^D)

$\Rightarrow$ A “modular” quantity. Schematically:

$$\mathcal{Z} = \prod_{\alpha=-\frac{N}{2}}^{\frac{N}{2}} \mathcal{Z}^{(\alpha)}(a, b^{(\alpha)})$$

where

$$\mathcal{Z}^{(\alpha)} = \mathcal{Z}^{(\alpha)}_{\text{vec}} \mathcal{Z}^{(\alpha)}_{\text{mat}}$$

$\Rightarrow$ $\mathcal{Z}^{(\alpha)}_{\text{vec}}$, $\mathcal{Z}^{(\alpha)}_{\text{mat}}$ known from SUSY localisation

The exact-deconstruction prescription

The rules:

The exact-deconstruction prescription

The rules:

- Identify all parameters between nodes (couplings, Coulomb-branch parameters, mass deformations etc.)

The exact-deconstruction prescription

The rules:

- Identify all parameters between nodes (couplings, Coulomb-branch parameters, mass deformations etc.)
- Include a mass parameter $m^{(\alpha)} \propto \frac{\alpha}{\hat{\hat{R}}}$ for the α-th node

The exact-deconstruction prescription

The rules:

- Identify all parameters between nodes (couplings, Coulomb-branch parameters, mass deformations etc.)
- Include a mass parameter $m^{(\alpha)} \propto \frac{\alpha}{\widehat{\widehat{R}}}$ for the α-th node
- Take $N \to \infty$

The exact-deconstruction prescription

The rules:

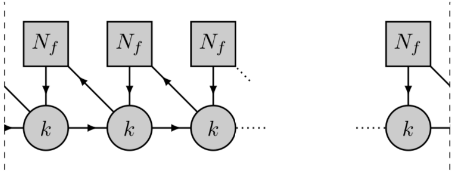
- ◇ Identify all parameters between nodes (couplings, Coulomb-branch parameters, mass deformations etc.)
- ◇ Include a mass parameter $m^{(\alpha)} \propto \frac{\alpha}{\hat{\hat{R}}}$ for the α-th node
- ◇ Take $N \to \infty$

$$\prod_{\alpha=-\frac{N}{2}}^{\frac{N}{2}} \mathcal{Z}^{(\alpha)}(a, b^{(\alpha)}) \to \prod_{\alpha=-\infty}^{\infty} \mathcal{Z}^{(\alpha)}(a, b, m^{(\alpha)})$$

Claim: This is the exact partition function of $\mathrm{QFT}_{\mathrm{D+1}}$ on extra S^1

Example: Deconstructing 4D $\mathcal{N} = 2$ SQCD

Start with the following 3D $\mathcal{N} = 2$ quiver



Its full squashed-S^3 PF is given by

$$\mathcal{Z}_{\text{3D}}^{\text{quiver}} = \prod_{\alpha} \frac{1}{k!} \int \prod_{b=1}^{k} \mathrm{d}\sigma_b^{(\alpha)} \Delta^{\text{Haar}} \left(\sigma^{(\alpha)}\right) \prod_{b,c=1}^{k} \frac{\Gamma_h \left(\omega_+ + \sigma_b^{(\alpha)} - \sigma_c^{(\alpha+1)} \middle| \omega_1, \omega_2\right)}{\widehat{\Gamma}_h \left(\sigma_b^{(\alpha)} - \sigma_c^{(\alpha)} \middle| \omega_1, \omega_2\right)}$$

$$\times \prod_{b=1}^{k} \prod_{j=1}^{N_f} \Gamma_h \left(\frac{1}{2}\omega_+ - \mu_j^{(\alpha)} + \sigma_b^{(\alpha)} \middle| \omega_1, \omega_2\right) \Gamma_h \left(\frac{1}{2}\omega_+ - \sigma_b^{(\alpha+1)} + \mu_j^{(\alpha)} \middle| \omega_1, \omega_2\right)$$

These are defined as

$$\Gamma_h(x|\omega_1,\omega_2) = \prod_{\boldsymbol{\ell}\in\mathbb{N}^2} \frac{-x+(\ell_1+1)\omega_1+(\ell_2+1)\omega_2}{x+\ell_1\omega_1+\ell_2\omega_2}$$

Exact-deconstruction prescription gives:

$$\mathcal{Z}_{\rm 3D}^{\rm Dec} = \frac{1}{k!}\int\prod_{b=1}^{k}{\rm d}\sigma_b \prod_{\alpha=-\infty}^{\infty} \frac{\Gamma_h(\omega_+ + \frac{\alpha}{R}\Big|\omega_1,\omega_2)^k}{\widehat{\Gamma}_h(\frac{\alpha}{R}|\omega_1,\omega_2)^k} \prod_{b\neq c} \frac{\Gamma_h\left(\omega_+ + \sigma_b - \sigma_c + \frac{\alpha}{R}\Big|\omega_1,\omega_2\right)}{\Gamma_h\left(\sigma_b - \sigma_c + \frac{\alpha}{R}\Big|\omega_1,\omega_2\right)}$$

$$\times\prod_{b=1}^{k}\prod_{j=1}^{N_f}\Gamma_h\left(\frac{1}{2}\omega_+ \mp \sigma_b \pm \mu_j + \frac{\alpha}{R}\Big|\omega_1,\omega_2\right)\ .$$

$\Rightarrow$ The $S^3\times S^1$ PF of 4D $\mathcal{N}=2$ SQCD with N_f flavours?

Not obvious. Use the identity

$$\prod_{\alpha=-\infty}^{\infty} \Gamma_h\left(x + \frac{\alpha}{R}\middle|\omega_1, \omega_2\right) = \mathfrak{x}^2\left(\mathfrak{p}\mathfrak{q}\right)^{-\frac{1}{2}}\Gamma_e(\mathfrak{x}|\mathfrak{p},\mathfrak{q})$$

for $\mathfrak{x} = e^{2\pi i R x}$, $\mathfrak{p} = e^{2\pi i R\omega_1}$, $\mathfrak{q} = e^{2\pi i R\omega_2}$ and

$$\Gamma_e(z|\mathfrak{p},\mathfrak{q}) = \prod_{\boldsymbol{\ell}\in\mathbb{N}^2} \frac{1 - z^{-1}\mathfrak{p}^{\ell_1+1}\mathfrak{q}^{\ell_2+1}}{1 - z\,\mathfrak{p}^{\ell_1}\mathfrak{q}^{\ell_2}}$$

to arrive at

$$\mathcal{Z}_{\text{3D}}^{\text{Dec}} = \frac{1}{k!}(\mathfrak{p};\mathfrak{p})^k(\mathfrak{q};\mathfrak{q})^k \oint \prod_{b=1}^{k}\frac{\mathrm{d}v_b}{2\pi i v_b}\prod_{b\neq c}\Gamma_e\left(v_b v_c^{-1}\middle|\mathfrak{p},\mathfrak{q}\right)^{-1}$$

$$\times\prod_{b=1}^{k}\prod_{j=1}^{N_f}\Gamma_e\left((\mathfrak{p}\mathfrak{q})^{\frac{1}{4}}(v_b s_j^{-1})^{\pm}\middle|\mathfrak{p},\mathfrak{q}\right)$$

$\Rightarrow$ This is precisely the expected result for $\mathcal{Z}_{S^3\times S^1}^{\mathcal{N}=2\text{ SQCD}}$

Deconstructing 4D PFs with $\frac{1}{2}$-BPS defects

$\Rightarrow$ Bypass Lagrangian deconstruction and directly reproduce the $S^3 \times S^1$ PFs in the presence of defects from 3D

Exact deconstruction gives:

1) $S^3 \times S^1$ PFs with (2,2) surface operators
   [Gadde, Gukov '14]
2) $S^3 \times S^1$ PFs with (4,0) surface operators - new
3) $S^3 \times S^1$ PFs with codimension-one defects - new

More Exotic Applications

The following theories are also related by deconstruction:

- ◇ 4D $\mathcal{N} = 2$ circular quiver and 6D (2,0) theory on T^2
- ◇ 4D $\mathcal{N} = 1$ toroidal quiver and 6D (1,1) LST on T^2

[Arkani-Hamed, Cohen, Kaplan, Karch, Motl ’01]

$\Rightarrow$ Can recover the 6D (2,0) PF on $S^4 \times T^2$

$\Rightarrow$ Can recover part of 6D (1,1) LST PF on $S^4 \times T^2$

Summary and Outlook

- Deconstruction has been useful tool for some time
- Can now be applied to exact partition functions
- Apply to other setups where the deconstructed theory is non Lagrangian? (e.g. 4D $\mathcal{N} = 3$ SCFTs)
- Reformulate exact deconstruction for setups where both the starting point and the end-point is non Lagrangian?
- Beyond partition functions: correlators? (e.g. (2,0) in 6D)