Non-Relativistic Strings

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based on work:

1907.01663 (Harmark,Hartong,Menculini,NO,Oling)

& 1810.05560 (JHEP) (Harmark,Hartong,Menculini,NO,Yan)

1705.03535 (PRD) (Harmark,Hartong,NO),

also:

1807.04765 (PRL), 1904.13282 (essay), 1906.13723 (procs)

& 19zz.yyyyy (Hansen,Hartong,NO)

Why non-relativistic (NR) string theory ?

- rich limit of string theory
- limit of AdS/CFT
- non-relativistic gravity via beta functions
- certain NR strings contained in double field theory
- perhaps simpler (UV complete) theory
- novel classes of sigma models
  (e.g. cont. limit of Heisenberg spin chains are NR strings)

Examples

current state-of-art: quite a few (a priori different) NR strings

- Gomis-Ooguri string & string Newton-Cartan generalization

  Gomis,Ooguri(2000);Danielsson et al.(2000); Andriga et al (2012), Bergshoeff,Gomis,Yan(2018); Gomis,Oh,Yan(2019),Bergshoeff,Gomis,Rosseel,Simsek,Yan(2019); Kluson (2018/19)

- null-reduced strings (on torsional Newton-Cartan)
  & further limits thereof

  Harmark,Hartong,NO(2017); Harmark,Hartong,Menculini,NO,Yan(2018); Gallegos,Gursoy,Zinnato(2019), Harmark,Hartong,Menculini,NO,Oling(2019); Kluson(2018/19), ….

- tensionless strings e.g. Bagchi,Gopakumar(2009) Bagchi,Banerjee,Parekh(2019)
- Galilean strings Battle,Gomis,Not(2016))
- relation to double field theory

  Morand,Park(2017);Berman,Blair,Otsuki(2019);Blair(2019)

Outline

- TNC (torsional Newton-Cartan) strings from null-reduction
- SNC (string Newton-Cartan) strings from 1/c expansion
- map between actions and remarks on beta functions
- outlook

Non-relativistic strings from null reduction

- start from Polyakov action (including NSNS) and reduce along null isometry

- implement conservation of string momentum along null isometry using Lagrange multipliers

- go to dual formulation that exchanges the (fixed) momentum along null direction for fixed winding of string along compact dual direction

→ action of non-relativistic strings moving in torsional Newton-Cartan (TNC) target space

intermezzo: TNC geometry from null reduction

Lorentzian metric with null isometry

$$ds^2 = G_{\mathcal{M}\mathcal{N}} dx^{\mathcal{M}} dx^{\mathcal{N}} = 2\tau(du - m) + h_{\mu\nu} dx^\mu dx^\nu$$

signature $(0, 1, \ldots, 1)$

TNC fields

τ_μ $h_{\mu\nu}$ m_μ

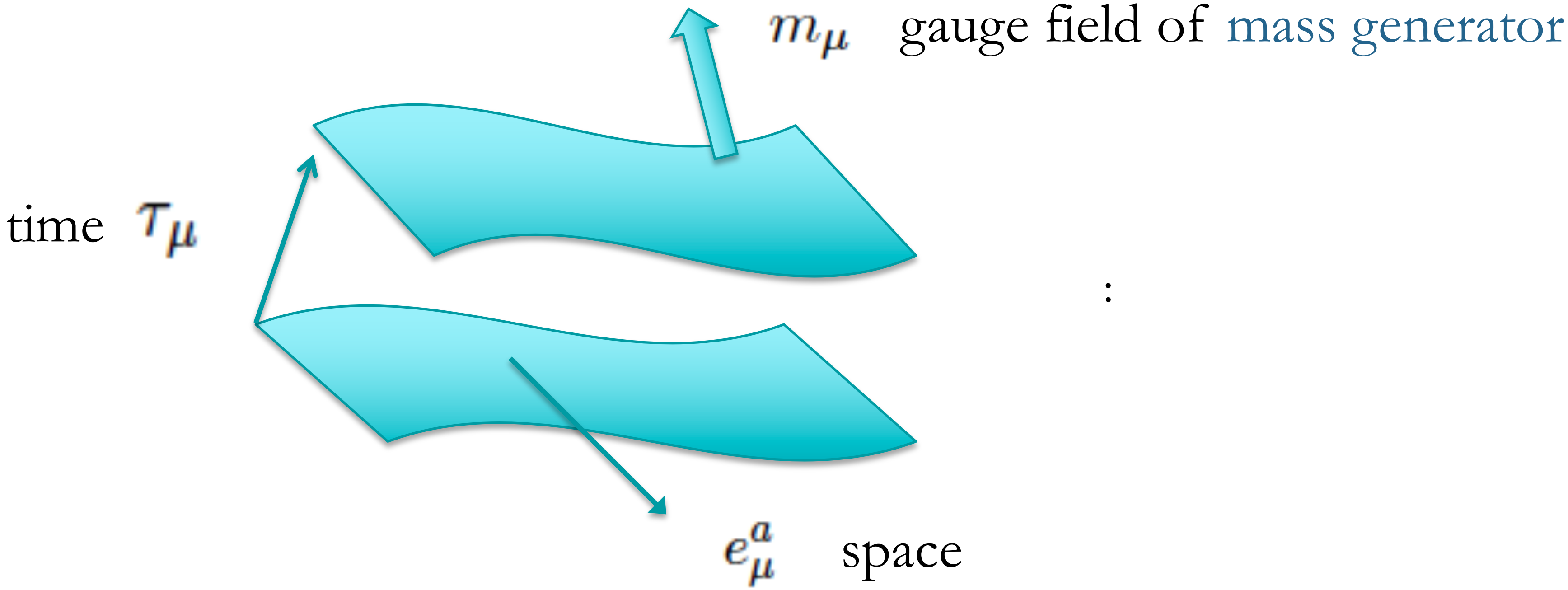
local syms:

$$\delta\tau_\mu = \mathcal{L}_\xi \tau_\mu \,, \quad \delta h_{\mu\nu} = \mathcal{L}_\xi h_{\mu\nu} + \lambda_\mu \tau_\nu + \lambda_\nu \tau_\mu \,,$$
$$\delta m_\mu = \mathcal{L}_\xi m_\mu + \lambda_\mu + \partial_\mu \sigma \,,$$

$v^\mu \lambda_\mu = 0$. Galilean (Milne) boosts

σ U(1) (mass) parameter

torsional Newton-Cartan geometry



NC = no torsion → $\tau_\mu = \partial_\mu t$ absolute time

TTNC = twistless torsion → $\tau_\mu = \text{HSO}$ preferred foliation equal time slices

TNC no condition on τ_μ

Action of strings on TNC geometry

$$S[\gamma_{\alpha\beta}, X^\mu, \eta, A_\alpha] = -\frac{T}{2}\int_\Sigma d^2\sigma\left(\sqrt{|\gamma|}\gamma^{\alpha\beta}\bar{h}_{\alpha\beta} + \varepsilon^{\alpha\beta}\mathcal{B}_{\alpha\beta}\right)$$
$$-T\int_\Sigma d^2\sigma\left(\sqrt{|\gamma|}\gamma^{\alpha\beta}\tau_\beta + \varepsilon^{\alpha\beta}(b_\beta + \partial_\beta\eta)\right)A_\alpha + \frac{1}{4\pi}\int_\Sigma d^2\sigma\sqrt{|\gamma|}R^{(2)}\phi\,.$$

momentum in null-direction: $P^\alpha_u = \frac{\partial\mathcal{L}}{\partial(\partial_\alpha X^u)} = -T\sqrt{|\gamma|}\gamma^{\alpha\beta}\tau_\beta - T\varepsilon^{\alpha\beta}b_\beta$

exchanged X^u for η and A_α

$$b_\mu = \mathcal{B}_{u\mu}$$

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \tau_\mu m_\nu - \tau_\nu m_\mu$$

worldsheet pullbacks:

$$\bar{h}_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \bar{h}_{\mu\nu}\,,\quad \tau_\alpha = \partial_\alpha X^\mu \tau_\mu\,,\quad \mathcal{B}_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \mathcal{B}_{\mu\nu}\,,\quad b_\alpha = \partial_\alpha X^\mu b_\mu$$

Alternate forms

augment the TNC target space with the extra compact direction

$x^M = (v, x^\mu)$ $\quad\quad \bar{\mathcal{B}}_{MN} = \mathcal{B}_{MN} + m_M b_N - m_N b_M$ (and more)

$$S[\gamma_{\alpha\beta}, X^M, \lambda_\pm] = -\frac{T}{2}\int_\Sigma d^2\sigma \left(e\eta^{ab}e^\alpha{}_a e^\beta{}_b h_{MN} + \varepsilon^{\alpha\beta}\bar{\mathcal{B}}_{MN}\right)\partial_\alpha X^M \partial_\beta X^N$$
$$-\frac{T}{2}\int_\Sigma d^2\sigma \left[\lambda_+ \varepsilon^{\alpha\beta} e_\alpha{}^+ (\tau_M + b_M) + \lambda_- \varepsilon^{\alpha\beta} e_\alpha{}^- (\tau_M - b_M)\right]\partial_\beta X^M$$
$$+\frac{1}{4\pi}\int_\Sigma d^2\sigma \sqrt{|\gamma|}R^{(2)}\phi\,.$$

- lambdas = redefinition of Lagrange multipliers A
- exhibits local Lorentz/Weyl symmetry (2D CFT)
- Nambu-Goto form exists as well

→ reduces to GO action on flat target space

$$S_{GO} = \int \frac{d^2z}{2\pi}\left(\frac{1}{\alpha'_{eff}}\partial X^i \bar\partial X^i + \beta\bar\partial\gamma + \bar\beta\partial\bar\gamma\right)$$

Non-relativistic strings from 1/c expansion

- start from Nambu-Goto action (including NSNS) and do a (careful) 1/c expansion

- resulting action is that of strings on string Newton-Cartan (SNC) geometry
  (uses accidental symmetry from interplay between metric and B-field coupling)

- Polyakov form of SNC can be mapped to TNC
  (needs one assumption: compact spatial longitudinal direction of SNC is an isometry)

Expanding Nambu-Goto

$$S_{\mathrm{NG}}[X^M] = -Tc \int_\Sigma d^2\sigma \sqrt{-\det G_{\alpha\beta}}$$

flatten the lightcone in transverse directions

$$G_{MN} = c^2 \left(-E_M{}^0 E_N{}^0 + E_M{}^1 E_N{}^1\right) + \Pi^\perp_{MN} = -c^2 \eta_{AB} E_M{}^A E_N{}^B + \Pi^\perp_{MN}$$

large speed of light expansion of metric:

$$E_M{}^A = \tau_M{}^A + \frac{1}{c^2} m_M{}^A + \mathcal{O}\left(c^{-4}\right)$$
$$\Pi^\perp_{MN} = H^\perp_{MN} + \mathcal{O}\left(c^{-2}\right) .$$

string embedding coords also need to be expanded:

$$X^M = x^M + \frac{1}{c^2} y^M + \mathcal{O}\left(c^{-4}\right)$$

Expanding Nambu-Goto (cont'd)

$$\mathcal{L}_{\mathrm{NG}} = c^3 \mathcal{L}_{\mathrm{NG,LO}} + c\mathcal{L}_{\mathrm{NG,NLO}} + \mathcal{O}\left(c^{-1}\right)$$

$$\mathcal{L}_{\mathrm{NG,LO}} = -T\sqrt{-\tau}\,,$$

divergent term

$$\mathcal{L}_{\mathrm{NG,NLO}} = -T\sqrt{-\tau}\tau^{\alpha\beta}H_{\alpha\beta} + y^M \frac{\delta\mathcal{L}_{\mathrm{NG,LO}}}{\delta x^M}$$

no y-dependence: $d\left(\tau^0 \wedge \tau^1\right) = 0\,.$

constraint

→Including B-field coupling will:

- cancel the divergence
- lift the constraint (y-term cancels)

Expanding WZ term

expand B-field:

$$B_{MN} = -c^2 \left(E_M{}^0 E_N{}^1 - E_M{}^1 E_N{}^0 \right) + \bar{B}_{MN}$$

expand action:

$$\mathcal{L}_{\mathrm{WZ}} = -c\frac{T}{2}\varepsilon^{\alpha\beta} B_{\alpha\beta} = c^3 \mathcal{L}_{\mathrm{WZ,LO}} + c\mathcal{L}_{\mathrm{WZ,NLO}} + \mathcal{O}\left(c^{-1}\right),$$

$$\mathcal{L}_{\mathrm{WZ,LO}} = T\sqrt{-\tau}\,,$$

$$\mathcal{L}_{\mathrm{WZ,NLO}} = -\frac{T}{2}\sqrt{-\tau}\varepsilon^{\alpha\beta}\bar{B}_{\alpha\beta} - T\sqrt{-\tau}\varepsilon^{\alpha\beta}\left(\tau_\alpha{}^0 m_\beta{}^1 - \tau_\alpha{}^1 m_\beta{}^0\right) + y^M \frac{\delta\mathcal{L}_{\mathrm{WZ,LO}}}{\delta x^M}$$

Total action

$$S[X^M] = -\frac{T}{2}\int_\Sigma d^2\sigma \left(\sqrt{-\tau}\tau^{\alpha\beta}H^{\perp}_{\alpha\beta} + \varepsilon^{\alpha\beta}\bar{B}_{\alpha\beta}\right)$$

possesses accidental symmetry:

$$H^{\perp}_{MN} \to H^{\perp}_{MN} + 2C_{(M}{}^{A}\tau_{N)}{}^{B}\eta_{AB}\,, \qquad \bar{B}_{MN} \to \bar{B}_{MN} - 2C_{[M}{}^{A}\tau_{N]}{}^{B}\epsilon_{AB}\,.$$

can be used to write:

$$S[X^M] = -\frac{T}{2}\int_\Sigma d^2\sigma \left(\sqrt{-\tau}\tau^{\alpha\beta}H_{\alpha\beta} + \varepsilon^{\alpha\beta}B_{\alpha\beta}\right)$$

= Nambu-Goto form of string Newton-Cartan action

$$H_{MN} = H^{\perp}_{MN} + 2\eta_{AB}\tau_{(M}{}^{A}m_{N)}{}^{B}$$

Map between TNC and SNC actions

use Polyakov form of SNC action

$$\mathcal{L} = -\frac{T}{2}\Big[\sqrt{-\gamma}\,\gamma^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N H_{MN} + \varepsilon^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N B_{MN}$$
$$+ \lambda\varepsilon^{\alpha\beta} e_\alpha{}^{+}\tau_M{}^{+}\partial_\beta X^M + \bar{\lambda}\varepsilon^{\alpha\beta} e_\alpha{}^{-}\tau_M{}^{-}\partial_\beta X^M\Big]$$

background fields are:

generalized clock form $\tau_M{}^A$, the transverse vielbeine $E_M{}^{A'}$ and $m_M{}^A$

$M, N = 0, 1..., D-1$ (spacetime) $\quad A = 0, 1$ (longitudinal) $\quad A' = 2, ..., D-1$ (transverse)

can find map from SNC to TNC (Polyakov):

$$X^v = \eta\,, \qquad \tau_\mu{}^0 = \tau_\mu\,, \qquad \tau_\mu{}^1 = b_\mu\,, \qquad H^\perp_{\mu\nu} = h_{\mu\nu}$$
$$\bar{B}_{MN} = \bar{\mathcal{B}}_{MN}\,, \qquad \Phi = \phi\,,$$
$$\lambda' = \lambda_+\,, \qquad \bar{\lambda}' = \lambda_-\,.$$

Remarks

SNC action comes with foliation constraint $D_{[M}\tau_{N]}{}^{A} = 0$.
at classical level
 - but not seen in derivation above
(but: restriction on target space can also come from beta functions)

recent results on beta-functions:

- SNC string — Gomis,Oh,Yan(2019); Bergshoeff,Gomis,Rosseel,Simsek,Yan(2019)
- TNC string — Gallegos,Gursoy,Zinnato(2019)

→ describe the dynamics of (versions of) non-relativistic gravity

Non-relativistic world-sheet theories

can take a further world-sheet NR limit

-→ new class of sigma models that are
also non-relativistic on worldsheet:

exhibits 2D GCA

$$[L_n\,, L_m] = (n-m)L_{n+m}\,, \qquad [L_n\,, M_m] = (n-m)M_{n+m}\,.$$

- NR WS theories directly related to
near-BPS limits of AdS/CFT (spin-matrix theory (SMT))

simplest example: LL model appearing from continuum limit of Heisenberg spin chains

Outlook

- completion of full set of TNC beta functions
  & comparison to SNC using the relation between the two

- inclusion of WS fermions, ws/target space SUSY
  recent results using connection
  to double field theory Blair(2019)

- non-relativistic open strings and D-branes
- role of RR backgrounds

- strings in type II TNC backgrounds ?
  (i.e. related to NR gravity from 1/c expansion of GR)
  Hansen,Hartong,NO(2028/2019)

- further study (quantization) of NR world-sheet theories

The end

Intermezzo: Lagrangian expansions

- Expanding Lagrangians: $\mathcal{L}(c, \phi, \partial_\mu \phi)$ where $\phi = \phi_{(0)} + c^{-2}\phi_{(1)} + \cdots$

- Assuming the overall power of the Lagrangian is c^N we define $\tilde{\mathcal{L}}(\sigma) = c^{-N}\mathcal{L}(c, \phi, \partial_\mu \phi)$ where $\sigma = c^{-2}$

- Taylor expand $\tilde{\mathcal{L}}(\sigma)$ around $\sigma = 0$, i.e.

$$\tilde{\mathcal{L}}(\sigma) = \tilde{\mathcal{L}}(0) + \sigma \left(\frac{\partial \tilde{\mathcal{L}}}{\partial \sigma}\Big|_{\sigma=0} + \phi_{(1)} \left[\frac{\partial \tilde{\mathcal{L}}(0)}{\partial \phi_{(0)}} - \partial_\mu \left(\frac{\partial \tilde{\mathcal{L}}(0)}{\partial \partial_\mu \phi_{(0)}} \right) \right] \right) + \cdots$$

- The eom of the NLO field of the NLO Lagrangian is the eom of the LO field of the LO Lagrangian.

Local symmetries and connection

local syms:

$$\delta\tau_\mu = \mathcal{L}_\xi \tau_\mu\,, \quad \delta h_{\mu\nu} = \mathcal{L}_\xi h_{\mu\nu} + \lambda_\mu \tau_\nu + \lambda_\nu \tau_\mu\,,$$
$$\delta m_\mu = \mathcal{L}_\xi m_\mu + \lambda_\mu + \partial_\mu \sigma\,,$$

$v^\mu \lambda_\mu = 0.$ Galilean (Milne) boosts

σ U(1) parameter

$$\bar{\nabla}_\mu \tau_\nu = \bar{\nabla}_\mu h^{\nu\rho}.$$

connection: $$\bar{\Gamma}^\lambda_{\mu\nu} \equiv -\hat{v}^\lambda \partial_\mu \tau_\nu + \frac{1}{2} h^{\lambda\sigma}\left(\partial_\mu \bar{h}_{\nu\sigma} + \partial_\nu \bar{h}_{\mu\sigma} - \partial_\sigma \bar{h}_{\mu\nu}\right)$$

→ impossible to find action on type I TNC that gives Poisson eq.

SNC symmetry algebra

$$[M_{A'B'}, M_{C'D'}] = 4\delta_{[A'[C'} M_{B']D']} \,,$$
$$[M_{A'B'}, G_{CD'}] = 2\delta_{D'[A'|} G_{C|B']} \,,$$
$$[M_{AB}, H_C] = 2\eta_{C[A} H_{B]} \,,$$

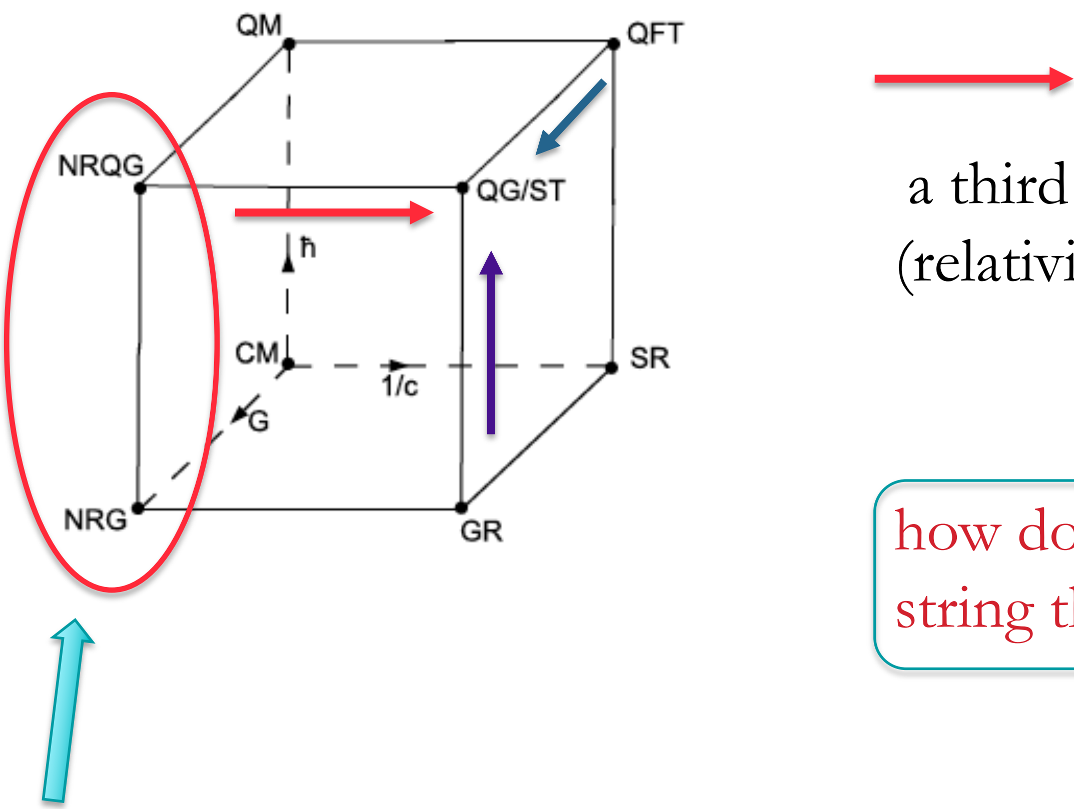
$$[M_{A'B'}, P_{C'}] = 2\delta_{C'[A'} P_{B']} \,,$$
$$[M_{AB}, G_{CD'}] = 2\eta_{C[A} G_{B]D'} \,,$$
$$[G_{AB'}, H_C] = \eta_{AC} P_{B'} \,.$$

$$[P_{A'}, G_{BC'}] = \delta_{A'C'} Z_B \,,$$
$$[G_{AB'}, G_{CD'}] = \delta_{B'D'} Z_{AC} \,,$$

$$[M_{AB}, Z_C] = 2\eta_{C[A} Z_{B]}$$
$$[Z_{AB}, H_C] = 2\eta_{C[A} H_{B]}$$

Cube of physical theories

$(\hbar, G_N, 1/c)$



a third route towards (relativistic) quantum gravity

how does this fit with string theory/holography ?

already (classical) non-relativistic gravity (NRG) is more than just Newtonian gravity