Anomalous transport from gluons

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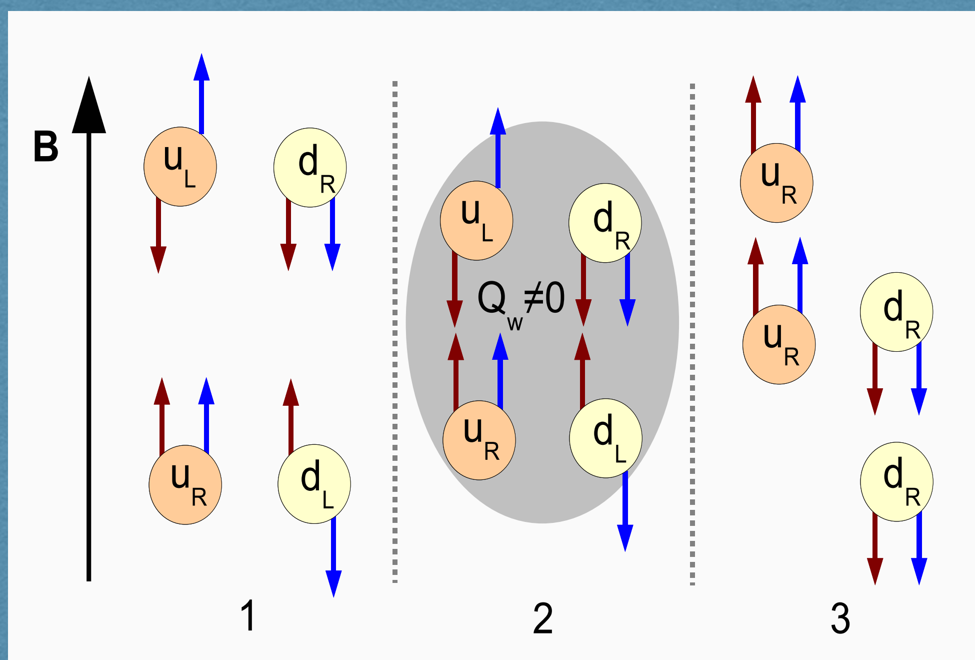
Based on: D. Gallegos, UG arXiv:1806.07138, ongoing.

Anomalous transport

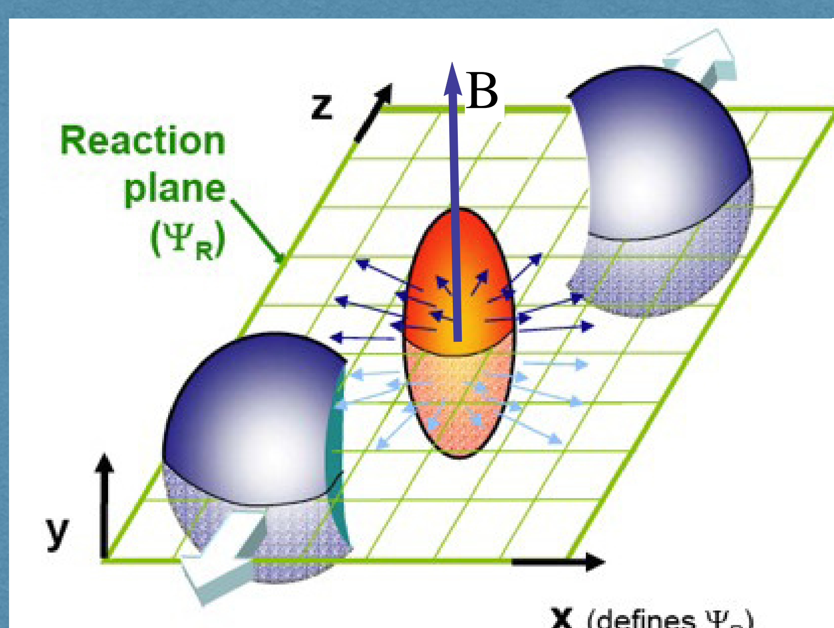
$$\nabla_\mu J^\mu_A = \frac{\epsilon^{\mu\nu\rho\sigma}}{4}\left[a_1 F^V_{\mu\nu}F^V_{\rho\sigma} + a_2 F^A_{\mu\nu}F^A_{\rho\sigma} + a_3 \mathrm{Tr}\left(G_{\mu\nu}G_{\rho\sigma}\right) + a_4 R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\sigma}\right]$$

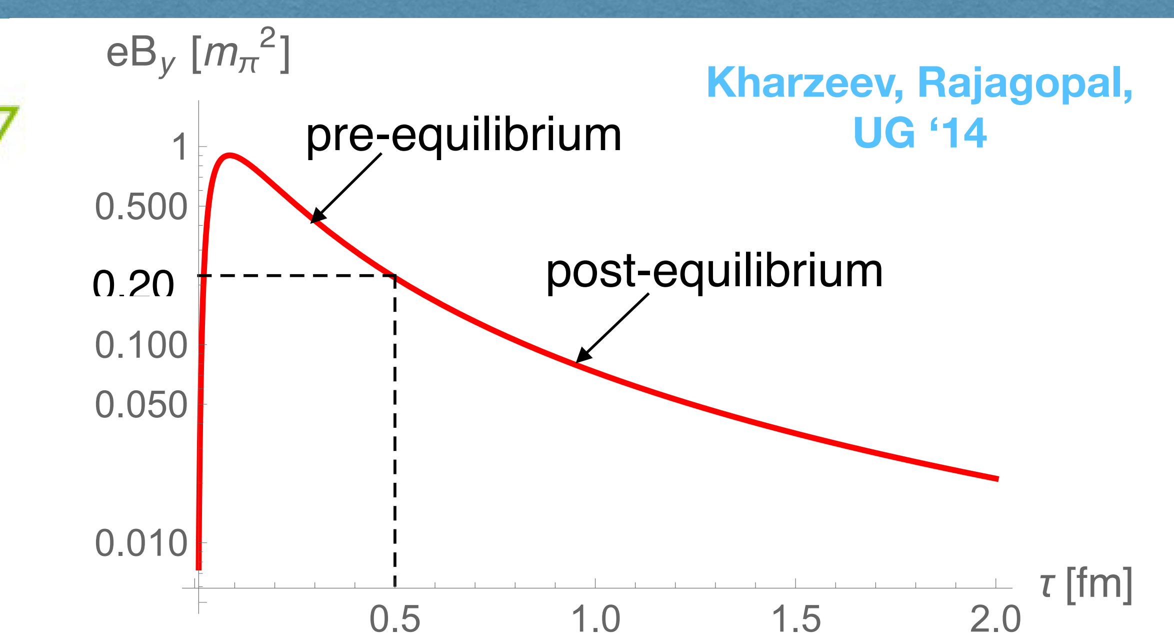


Kharzeev, McLerran, Warringa ‘08

$$\mathbf{J}_{\mathrm{V}} = \sigma_{\mathrm{VV}}\mathbf{B} + \sigma_{\mathrm{VA}}\mathbf{B}_5 + \sigma_{V\Omega}\boldsymbol{\omega},$$

$$\mathbf{J}_{\mathrm{A}} = \sigma_{\mathrm{AV}}\mathbf{B} + \sigma_{\mathrm{AA}}\mathbf{B}_5 + \sigma_{A\Omega}\boldsymbol{\omega},$$





Kharzeev, Rajagopal, UG ‘14

Also neutron stars, magnetars, early universe cosmology, Weyl/Dirac semimetals etc.

Anomalous conductivities at strong coupling

✳ Non-renormalization in the absence of dynamical gluons (QFT, hydro, holography):

Fukushima, Kharzeev, Warringa '08; Son, Surowka '09; Haack, Erdmenger, Kaminski, Yarom '09;
Amado, Landsteiner, Megias, Pena-Benitez 11; Jensen, Logayanagam, Yarom '13;
Tarrio, UG '15; Grozdanov, Poovuttikul '16

$$\sigma_{VV} = a_1\,\mu_5\,, \qquad \sigma_{VA} = \sigma_{AV} = a_1\,\mu$$

$$\sigma_{V\Omega} = a_1\,\mu\mu_5\,, \qquad \sigma_{A\Omega} = \frac{a_1}{2}\,\left(\mu^2 + \mu_5^2\right) + CT^2$$

✳ Coefficient C fixed most probably by global GR anomaly : Glorioso, Liu '18

✳ No universal form known in the presence of dynamical gluons.

✳ Both QFT and lattice indicate large corrections

Hou, Liu, Ren '13; Golkar, Son '13
Jensen, Kovtun, Ritz '13
Yamamoto '11; Braguta et al '13

Does holography predict a universal form?

Dynamical gluons in holography

$$Z[A_5,\theta] = \int \mathcal{D}q\,\mathcal{D}A_g\, e^{-\int L[A_g,q]+A_5\cdot J_5+\theta\,\mathrm{Tr}G\wedge G}$$

$$A_5 \to A_5 + d\lambda_5, \quad \theta \to \theta - a_3\lambda_5$$

$$\nabla\cdot J_5 = a_3 \mathrm{Tr} G\wedge G$$

Chiral current is anomalous $\Longrightarrow$ anomalous dimension $\Delta \neq 3$

Holographic theory should include :

$C_0(r,x) \Longleftrightarrow$ Tr $G\wedge G$, $C_0 \to \theta$ as $r \to 0$

$A(r,x)$ with Stückelberg mass

Bulk action should involve $(m\,A - d\,C_0)^2$

Klebanov, Witten, Ouyang ’02; A. Jansen, UG ’14; Jimenez-Alba, Landsteiner, Melgar ‘14

Generic non-conformal holographic theory

$$16\pi G_N S = S_g + S_f + S_a + S_{CS} + S_{GH} + S_{ct}\,,$$

$$S_g = \int_{\mathcal{M}} \left[R \star 1 - \frac{1}{2} d\phi \wedge \star d\phi - V(\phi) \star 1 \right]\,,$$

$$S_f = -\frac{x}{2} \int_{\mathcal{M}} \left[Z_V(\phi) F^V \wedge \star F^V + Z_A(\phi) F^A \wedge \star F^A \right]\,,$$

$$S_a = -\frac{x^2 m^2}{2} \int_{\mathcal{M}} Z_0(\phi) \tilde{A} \wedge \star \tilde{A}\,,$$

$$S_{CS} = \int_{\mathcal{M}} A \wedge \left[\kappa F^V \wedge F^V + \gamma F^A \wedge F^A + \lambda \mathrm{Tr}\,(R \wedge R) \right]\,,$$

with Stückelberg combination $\tilde{A} \equiv A - \frac{dC_0}{Q_f} \equiv A - d\mathfrak{a}\,.$

External anomalies fix $\kappa = -16\pi G_N a_1, \qquad \gamma = -16\pi G_N a_2, \qquad \lambda = -16\pi G_N a_4.$

Anomalous dimension $\Delta = \sqrt{1 + \frac{Q_f^2}{x}} - 1 = \sqrt{1 + m^2 x} - 1\,.$ **Q_f=mx, x = N_f/N_c**

$\dim[J_V]=3$ and $\dim[J_A]=3+\Delta.$ **For relevance in the IR: Δ<1**

Holographic calculation of vector current

Read off the one-point function of the *consistent* current

$$\langle J_V^\nu \rangle = \frac{1}{16\pi G_N} \lim_{r\to\infty} \left[-x\sqrt{-G} Z_V F^{V,r\nu} + 2\kappa \tilde{\epsilon}^{\nu\mu\rho\sigma} A_\mu F^V_{\rho\sigma} \right] ,$$

Anomalous conductivities, from dependence of ⟨J$_V$⟩ on elect./axial B and vorticity:

$$B^{V,\mu} = \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho \tilde{v}_\sigma \qquad B^{A,\mu} = \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho \tilde{a}_\sigma \qquad \omega^\mu = \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma \, .$$

Chiral magnetic effect
σ_{VV}

Chiral separation effect
$\sigma_{VA} = \sigma_{AV}$

Chiral vortical effect
$\sigma_{V\Omega}$

Ansatz for background and fluctuations:

$$ds^2 = \frac{dr^2}{gr^2} + r^2 \left[-f u_\mu u_\nu + \Delta_{\mu\nu} + \mathfrak{A} u_\mu \tilde{a}_\nu + \mathscr{A} \tilde{a}_\mu \tilde{a}_\nu \right] dx^\mu dx^\nu$$

$$+ r^2 \left[\gamma_I u_\mu B^I_\nu + \kappa_I \tilde{a}_\mu B^I_\nu \right] dx^\mu dx^\nu + \mathcal{O}\left(\partial^2\right) ,$$

$$V = -V_t u + \mathcal{V} \tilde{a} + \tilde{v} + \beta_I B^I + \mathcal{O}\left(\partial^2\right) \ ,$$

$$A = -\tilde{A}_t u + \mathcal{A} \tilde{a} + \alpha_I B^I + \mathcal{O}\left(\partial^2\right) \ ,$$

$$\phi = \phi(r, x), \qquad \mathfrak{a} = \mathfrak{a}(x, r) \, ,$$

Results

We solve perturbatively in x-derivatives and Q_f

$$\boxed{\begin{aligned}\sigma_{\rm CME} &= \sigma_{\rm CVE} = 0\,,\\ \sigma_{\rm CSE} &= 2a_1\left[\mu\mathcal{A}(r_h) + \frac{Q_f^2}{x}\int_{r_h}^{\infty} dr V_t'\left(L(r) - xA_tH(r)\right.\right.\\ &\qquad \left.\left.+x\int_{r_h}^{r} dr' A_t'(r')H(r')\right)\right] + \mathcal{O}\left(Q_f^4\right)\,,\end{aligned}}$$

$$H(r) = \int_\infty^r \frac{dr' A_t(r') D(r')}{r'^5 f(r')\sqrt{f(r')g(r')}},\qquad Q = \frac{\sqrt{-G}Z_V g V_t'}{f},\qquad Q_5 = \frac{\sqrt{-\mathcal{G}}Z_A g A_t'}{f},$$

$$L(r) = \int_{r_h}^r \frac{dr' D(r')}{r'^3\sqrt{f(r')g(r')}Z_A(\phi(r'))},\qquad \mathcal{A}(r_h) = 1 + \frac{Q_f^2}{2x}\lim_{r\to\infty}\left[\ln\left(\frac{r}{r_h}\right) - 2L(r) - 2x\int_{r_h}^r dr' A_t'(r')H(r')\right] + \mathcal{O}\left(Q_f^4\right).$$

$$D(r) = \int_{r_h}^r dr' Z_0(\phi(r'))r'\sqrt{\frac{f(r')}{g(r')}}.$$

Example I: AdS-Reissner-Nördstorm

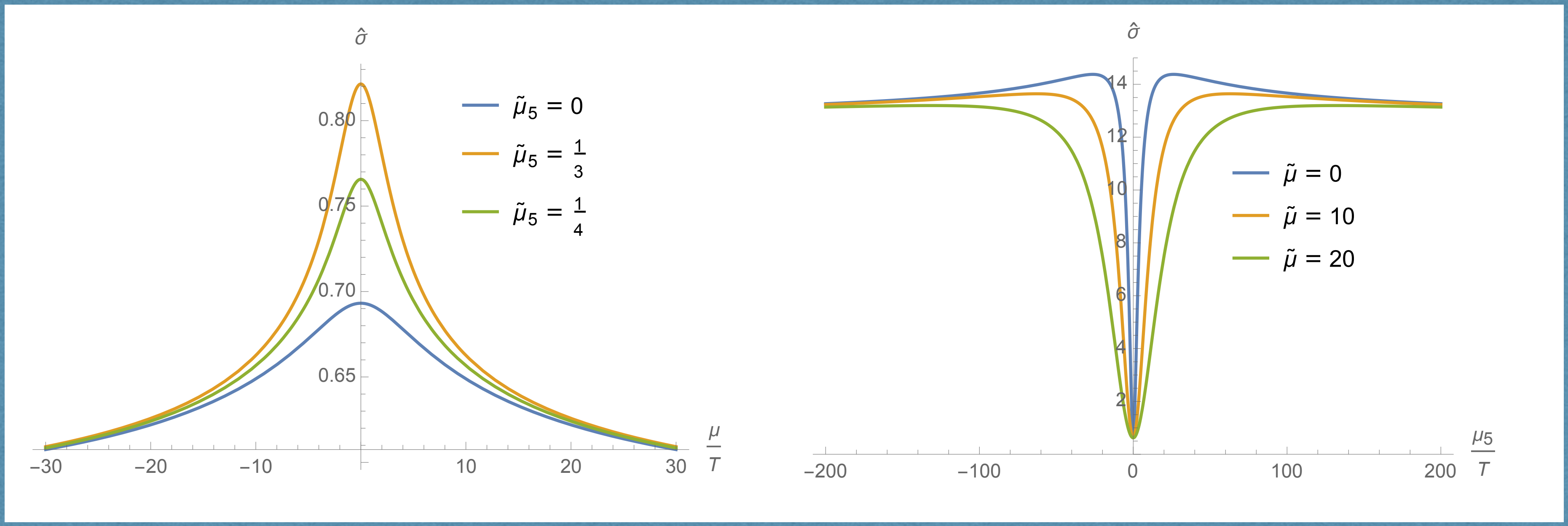
Conformal plasma with finite electric and axial charge

$$\sigma_{\text{CSE}} = 2a_1\mu\left\{1+\frac{Q_f^2}{2x}\left(\frac{1}{4q}\right)\left[3\ln\left(\frac{3+q}{3-q}\right)+q\ln\left(\frac{9-q^2}{4}\right)\right.\right.$$
$$\left.\left.+\frac{1}{q^2}\left(\frac{x\mu_5^2}{r_h^2}\right)\left[6q+(9-q^2)\ln\left(\frac{3+q}{3-q}\right)\right]\right]+\mathcal{O}\left(Q_f^3\right)\right\}.$$

$$q=\sqrt{1+\frac{16x\left(\tilde{\mu}^2+\tilde{\mu}_5^2\right)}{3\pi^2\left(1+\sqrt{1+\frac{2\left(\tilde{\mu}^2+\tilde{\mu}_5^2\right)}{3}}\right)^2}}.$$

Corrections to universal value

$$\sigma_{\text{CSE}}=\sigma_{\text{CSE}}^U\left[1+\Delta\,\hat{\sigma}\left(\tilde{\mu},\tilde{\mu}_5\right)+\mathcal{O}\left(Q_f^3\right)\right]$$

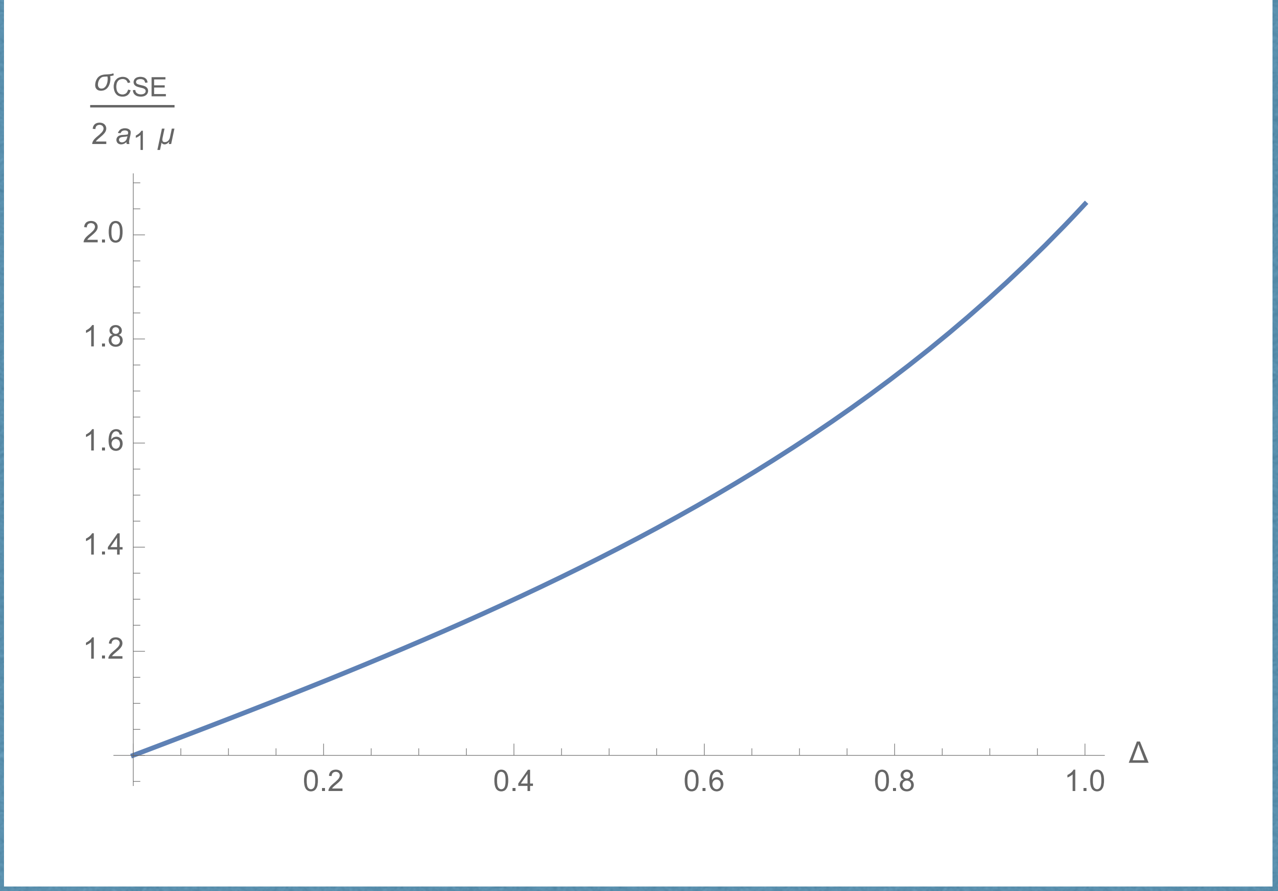


Exhibits bounds

$$\hat{\sigma}(\pm\infty,0)=\frac{\ln(3)}{2}\leq\hat{\sigma}\left(\tilde{\mu},\tilde{\mu}_5\right)\leq 14.3765=\tilde{\sigma}(0,\pm 26.271).$$

Example II: arbitrary Δ, probe limit

Ignore the back reaction of sources on metric in AdS-RN



Deviates from the linear solution $\sigma_{\rm CSE} = 2a_1\mu\left[1+\Delta\ln(2)\right]$ at around $\Delta\sim 0.35$.

Agrees perfectly with numerical result of Landsteiner et al.

Example III: Chiral separation effect at vanishing μ, μ_5

- Generation of chiral current due to vortices
- Typically much harder as requires axial perturbation fully
- Analytic solution possible at vanishing chemical potentials

$$\sigma_{\mathrm{CVSE}} = 8\pi^2 a_4 T^2 \left[1 + \frac{\Delta(\Delta+2)}{2}\left(\ln 2 - \frac{1}{4}\right) + \mathcal{O}\left(Q_f^3\right)\right]$$

An analytic universal correction to T^2 term!

A general formula

Beyond the small Q_f limit:

$$\boxed{\tilde{J}^i = 2\kappa\tilde{\epsilon}^{\nu\mu\rho\sigma}\int_{r_h}^{r} dr\left[A_t\mathcal{V}' - \mathcal{A}V_t'\right]\epsilon^{ijk}\partial_j\tilde{a}_k.}$$

$$V = -V_t u + \mathcal{V}\tilde{a} + \tilde{v} + \beta_I B^I + \mathcal{O}\left(\partial^2\right) ,$$

$$A = -\tilde{A}_t u + \mathcal{A}\tilde{a} + \alpha_I B^I + \mathcal{O}\left(\partial^2\right) ,$$

Conclusion and outlook

- ✴Obtained contribution of dynamical gluons to anomalous conductivities
- ✴Analytic results valid for large coupling, large N, small Δ
- ✴Universal bounds and corrections

- ✴Is the T^2 term really determined by global gravitational anomaly?
- ✴How to implement dynamical gluons contributions in hydro?
  Hints from Grozdanov, Hofman, Iqbal'18
- ✴Chiral vortical separation and chiral magnetic wave?
- ✴Can we identify distinct observables for the QGP and Dirac/Weyl semimetals?
- ✴How do our predictions compare with lattice?