Asymptotic charges in gravity

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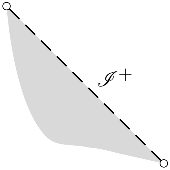
We are interested in defining asymptotic charges in
asymptotically flat (AF) spacetimes.

AF spacetimes model isolated gravitational systems, *e.g. astrophysical black holes, formation/collapse of stars etc.*

Physics of process measured by an external observer far away, where the gravitational field is negligible.

Roughly, the Penrose diagram (that reflects the causal structure) looks like that of Minkowski spacetime.



Bondi coordinates

AF spacetimes: there exist Bondi coordinates $(u, r, x^I = \{\theta, \phi\})$ such that the metric takes the form

$$ds^2 = -Fe^{2\beta}du^2 - 2e^{2\beta}dudr + r^2 h_{IJ}(dx^I - C^I du)(dx^J - C^J du)$$

$$\begin{aligned}
F(u,r,x) &= 1 + \frac{F_0(u,x)}{r} + \dots \\
\beta(u,r,x) &= \frac{\beta_0(u,x)}{r^2} + \dots \\
C^I(u,r,x) &= \frac{C_0^I(u,x)}{r^2} + \dots \\
h_{IJ}(u,r,x) &= \omega_{IJ}(x) + \frac{C_{IJ}(u,x)}{r} + \dots, \quad \det(h_{IJ}) = \det(\omega_{IJ}) = \sin^2\theta
\end{aligned}$$

N.B. Strictly defined in terms of constraints on initial data.

Motivations

- New quantity, useful for data analysis/simulations?
  C.f. Newman-Penrose charges [NP 1968].

- Quantum gravity: what are the proper degrees of freedom? What sort of charges can be defined and how are they organised?

- dS/AdS case? Important for cosmology and string theory.

Charges

[Barnich, Brandt 2002]

[ADM, Regge-Teitelboim, Abbott-Deser, Julia-Silva, Brown-Henneaux, Iyer-Wald, Wald-Zoupas, Ashtekar, ...]

Generally, we define a charge, or current, using a Killing isometry generated by k

$$J^a = T^{ab} k_b, \qquad \nabla_a J^a = 0.$$

Unsatisfactory: generically no Killing isometries!

If $\exists\, \xi$ such that

$$T^{ab}\xi_b \longrightarrow \nabla_b H^{ab}, \qquad H^{ab} = H^{[ab]}$$

then asymptotically

$$\nabla_a \nabla_b H^{ab} = \nabla_{[a} \nabla_{b]} H^{ab} = 0 \implies \text{asymptotic charge.}$$

Such a ξ generates an <u>asymptotic symmetry</u>.

Working on a particular background $\bar{g}$ and denoting the linear transformation of the metric by δg so that

$$g = \bar{g} + \delta g$$

the (variation of the) asymptotic charge defined by 2-form H is

$$\not{\delta}\mathcal{Q}_\xi[g, \delta g] = \frac{1}{8\pi G} \lim_{r \to \infty} \int_S \star H[\delta g, g, \xi].$$

$\not{\delta}$: not necessarily integrable. (This is in fact related to conservation! [Wald, Zoupas 1999])

Asymptotic symmetries: BMS group

[Bondi, van der Burg, Metzner 1962; Sachs 1962]

The asymptotic symmetries of asymptotically flat spacetimes, the BMS group, is

$$\mathrm{BMS} = \mathrm{SL}(2,\mathbb{C}) \ltimes \text{supertranslations}.$$

$\mathrm{SL}(2,\mathbb{C}) \approx \mathrm{SO}(3,1)$: conformal isometries, generated by Y^I.

supertranslations: angle-dependent translations along null infinity, generated by $s(x)$.

In this case [Barnich, Brandt 2002]

$$H[\delta g, g, \xi] = \frac{1}{2}\Big\{ \xi_b g^{cd}\nabla_a \delta g_{cd} - \xi_b \nabla^c \delta g_{ac} + \xi^c \nabla_b \delta g_{ac} + \frac{1}{2} g^{cd} \delta g_{cd} \nabla_b \xi_a + \frac{1}{2}\delta g_{bc}(\nabla_a \xi^c - \nabla^c \xi_a) \Big\} dx^a \wedge dx^b$$

Computing the variation of the asymptotic, BMS (supertranslation), charge then gives [Barnich, Troessaert 2011]

$$\delta\!\!\!/ \mathcal{Q} = \delta \mathcal{Q}^{(int)} + \mathcal{N}$$

$$\mathcal{Q}^{(int)} = -\frac{1}{8\pi G}\int_S s\, F_0, \qquad \mathcal{N} = \frac{1}{32\pi G}\int_S s\partial_u C_{IJ}\delta C^{IJ}$$

Integrable part, $\mathcal{Q}^{(int)}$: BMS charge.

For Schwarzschild solution ($F_0 = -2m$), $\mathcal{Q}^{(int)}(s=1) = m$, the energy.

For $\ell = 0, 1$ spherical harmonics, $\mathcal{Q}^{(int)}$ corresponds to the Bondi 4-mtm.

Non-integrable part, $\mathcal{N}$: related to (non-)conservation of BMS charge.

Corresponds to Bondi news at null infinity: flux of energy leaving spacetime.

Vanishes iff Bondi news $\partial_u C_{IJ} = 0$.

$$ds^2 = -Fe^{2\beta}du^2 - 2e^{2\beta}dudr + r^2 h_{IJ}(dx^I - C^I du)(dx^J - C^J du)$$

$$F(u,r,x) = 1 + \frac{F_0(u,x)}{r} + \ldots, \quad \beta(u,r,x) = \frac{\beta_0(u,x)}{r^2} + \ldots$$

$$C^I(u,r,x) = \frac{C_0^I(u,x)}{r^2} + \ldots, \quad h_{IJ}(u,r,x) = \omega_{IJ}(x) + \frac{C_{IJ}(u,r,x)}{r} + \ldots$$

Generalising BMS charges

[H. Godazgar, M.G., Pope 2017–2019]

One can generalise BMS charges in two ways:

- Define <u>BMS charges away from infinity</u>: In Bondi scheme BMS group remains *relevant* away from infinity.
- Define <u>dual BMS charges</u>.

Tower of BMS charges

The Barnich-Brandt charge can be viewed in $1/r$ expansion depending on the $1/r$-expansion of the metric functions

$$\not{\delta}\mathcal{Q} = \not{\delta}\mathcal{Q}_0 + \frac{\not{\delta}\mathcal{Q}_1}{r} + \frac{\not{\delta}\mathcal{Q}_2}{r^2} + \frac{\not{\delta}\mathcal{Q}_3}{r^3} + \dots .$$

$\not{\delta}\mathcal{Q}_0$ is what we considered before. But now we have a whole set of them!

As before,

$$\not{\delta}\mathcal{Q}_i = \delta\mathcal{Q}_i^{(int)} + \mathcal{N}_i.$$

$\mathcal{Q}_i^{(int)}$ $(i > 0)$: subleading BMS charges.

$\mathcal{N}_i$ $(i > 0)$: prevents conservation of subleading BMS charge; *fake news*.

Dual BMS charges

We define new dual BMS charges using a different 2-form $\widetilde{H}$

$$\not{\delta}\widetilde{\mathcal{Q}} = \frac{1}{8\pi G}\int_S \widetilde{H}[\delta g, g, \xi]$$

$\widetilde{H} = \frac{1}{4}\delta g_{bc}(\nabla_a \xi^c + \nabla^c \xi_a)dx^a \wedge dx^b.$

As before,

$$\not{\delta}\widetilde{\mathcal{Q}} = \not{\delta}\widetilde{\mathcal{Q}}_0 + \frac{\not{\delta}\widetilde{\mathcal{Q}}_1}{r} + \frac{\not{\delta}\widetilde{\mathcal{Q}}_2}{r^2} + \frac{\not{\delta}\widetilde{\mathcal{Q}}_3}{r^3} + \dots.$$

E.g.

$$\not{\delta}\widetilde{\mathcal{Q}}_0 = \delta\widetilde{\mathcal{Q}}_0^{(int)} + \widetilde{\mathcal{N}}_0$$

$$\widetilde{\mathcal{Q}}_0^{(int)} = -\frac{1}{16\pi G}\int_S s\, D_I D_J \tilde{C}^{IJ}, \qquad \widetilde{\mathcal{N}}_0 = \frac{1}{32\pi G}\int_S s\partial_u C_{IJ}\delta\tilde{C}^{IJ}$$

Taking $s = 1$, $\widetilde{\mathcal{Q}}_0^{(int)}$ is total derivative. But, if we relax AF definition to allow non-regular function on S^2, it is the Taub-NUT charge.
C.f. superrotations [Barnich, Troessaert 2010]

Applications

- The subleading BMS charges at order $1/r^3$ give Newman-Penrose charges.[2,3]
- Connects Aretakis charges on extremal horizons to BMS charges via NP charges.[1]
- Resolves phase space problem for radiating modes.[4]
- Gives generalised soft theorem including soft NUT charges.[4]
- Identifies the Dirac monopole as the progenitor of Taub-NUT solution.[5]

1. GGP (2017) arXiv:1707.09804
2. GGP (2018) arXiv:1809.09076
3. GGP (2018) arXiv:1812.06935
4. GGP (2019) arXiv:1908.01164
5. GGP (2019) arXiv:1908.05962

Outlook

- Subleading charges for more physical polyhomogeneous spacetimes? Conserved quantities?
- Significance of dual 2-form $\widetilde{H}$?
- Subleading soft theorems?
- Relation of superrotations and dual charges?