10th Regional String Meeting — Kolymbari

Back-reacting massless de Sitter QFTS via Holography

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Motivation

Understanding the interplay of a **QFT** and a **dynamical background space-time** highly relevant for a fundamental description of the evolution of our universe.

More tractable when considering backgrounds with **maximal symmetry**, with **de Sitter (dS) space** of particular phenomenological importance.

How do quantum fluctuations **back-react** on dS space?

Motivation

Long-standing set of hints that **dS space** may be **unstable against fluctuations**: [Mottola '85; Tsamis, Woodard '96, '97; Polyakov '12]

- Possible instability due to **graviton** fluctuations [Tsamis, Woodard '96, '97]
- and fluctuations of **massless scalars**.

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Could these potential instabilities (due to massless fields) be **artefacts of perturbation theory?**

Here: use techniques of **gauge-gravity duality** to study back-reaction of **holographic QFTs** on dS.

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To study back-reaction, consider the effective action for the metric $g_{\mu\nu}$ obtained by integrating out a QFT:

$$S_{\rm eff}[g] = S_0[g] - i \ln Z_{\rm QFT}[g]$$

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$$Z_{\rm QFT}[g] = \int d[\Phi]\, e^{iS_{\rm QFT}[g,\Phi]}$$

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$$\nabla_\rho R_{\mu\nu} = 0 \quad \Leftrightarrow \quad R_{\mu\nu} = \kappa g_{\mu\nu} \,, \quad R = 4\kappa$$

This includes the phenomenologically interesting cases of max. symmetric space-times (Minkowski, AdS, dS).

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So far, this analysis has been performed for special cases:

- **massive free scalar on de Sitter** [see e.g. Mazur, Mottola 1986]
- ϕ^4**-theory on de Sitter** via non-perturbative RG techniques. [Moreau, Serreau 2018]

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Here, we will employ **holography** to integrate out a QFT on an Einstein manifold and calculate $\ln Z_{\text{QFT}}[g]$.

Outline

1.) Setup:

- What types of QFTs are back-reacted?
- Integrating out via holography
- UV divergences & renormalisation

2.) Results for constant-curvature solutions:

- Case I: The physical system is UV-complete
- Case II: QFT with a UV cutoff.

3.) Conclusions and open questions

Setup

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In particular:

1.) CFTs

2.) RG flow QFTs:

RG flows driven by a relevant operator $\mathcal{O}$ of dimension Δ^{UV} from a UV fixed point to a IR fixed point.

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Setup: Integrating out via holography

Postulate that the **4d QFT** possesses a holographic dual given by a **5d gravitational theory**.

Duality: $$Z_{\mathrm{QFT},4d}[g] = Z_{\mathrm{grav},5d}[g]$$

with $$Z_{\mathrm{QFT},4d}[g] = \int d[\Phi]\, e^{iS_{\mathrm{QFT}}[g,\Phi]}$$

and $$Z_{\mathrm{grav},5d}[g] = \int_{G|_{\partial\mathcal{M}}=g} d[G]\, e^{iS_{\mathrm{grav}}[G]}$$

Setup: Integrating out via holography

We take the QFTs to be at **large** N_c and at **infinite coupling**.

The gravity dual is dominated by classical gravity, i.e.

$$Z_{\mathrm{grav},5d}[g] = \int_{G|_{\partial\mathcal{M}}=g} d[G]\, e^{iS_{\mathrm{grav}}[G]} = e^{iS_{\mathrm{grav}}^{\text{on-shell}}[g]}$$

$$\Rightarrow \quad -i \ln Z_{\mathrm{QFT}}[g] = S_{\mathrm{grav}}^{\text{on-shell}}[g]$$

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The limit $N_c \to \infty$ implies $\tilde{a}, \tilde{a}_{\mathrm{UV}}, \tilde{a}_{\mathrm{IR}} \to \infty$ but $\tilde{a}_{\mathrm{UV}}/\tilde{a}_{\mathrm{IR}}$ can be chosen finite.

Setup: the holographic dual

$$S_{\mathrm{grav},5d} = M^3 \int du\, d^4x \sqrt{|G|} \left(R^{(G)} - (\partial\varphi)^2 - V(\varphi) \right) + S_{\mathrm{GHY}}\,.$$

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Ansatz:

$$ds^2 = du^2 + e^{A(u)} g_{\mu\nu} dx^\mu dx^\nu \,, \qquad \varphi(u, x^\mu) = \varphi(u)$$

Einstein manifold

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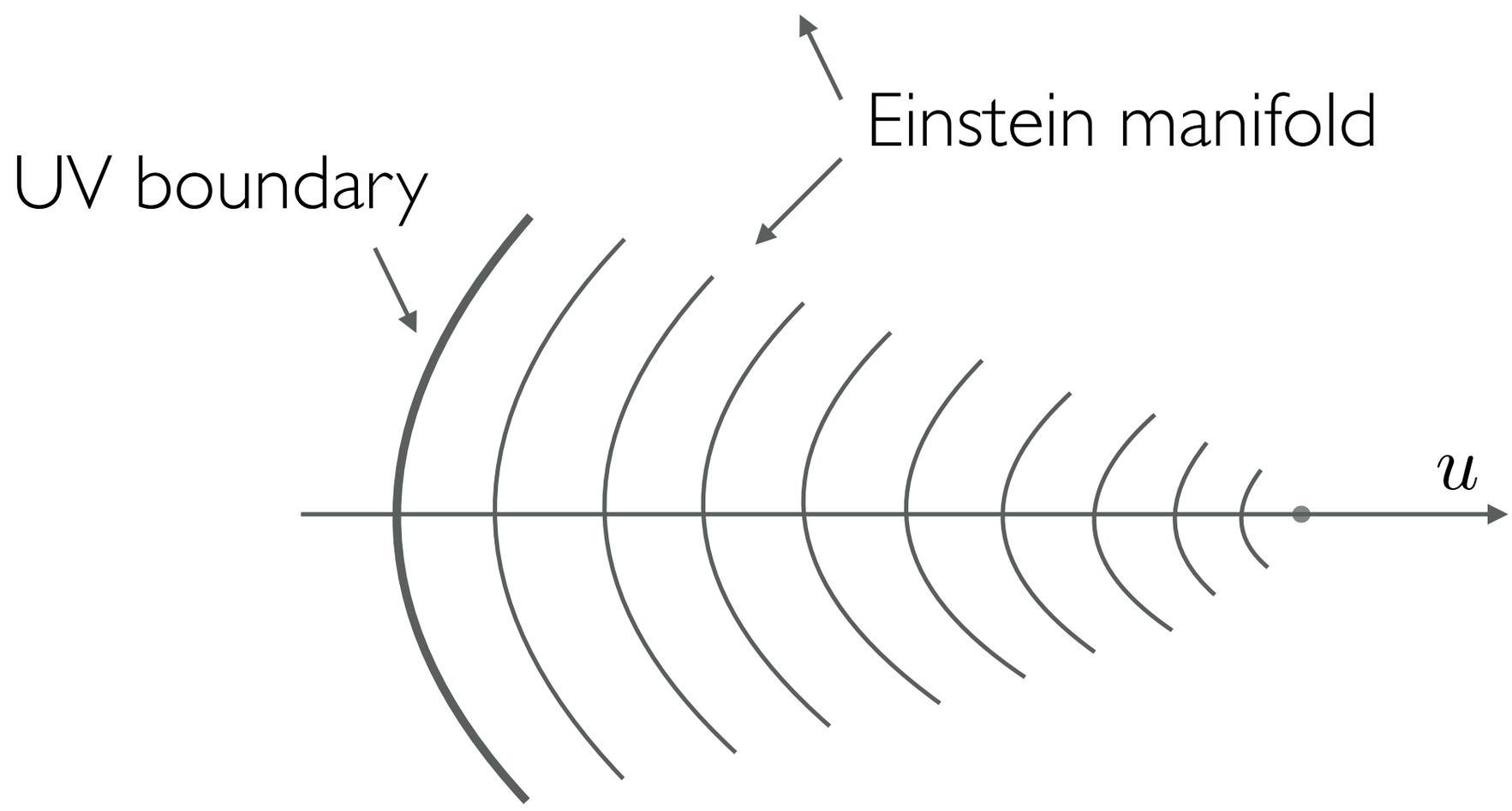

[Ghosh, Kiritsis, Nitti, LW 2017]

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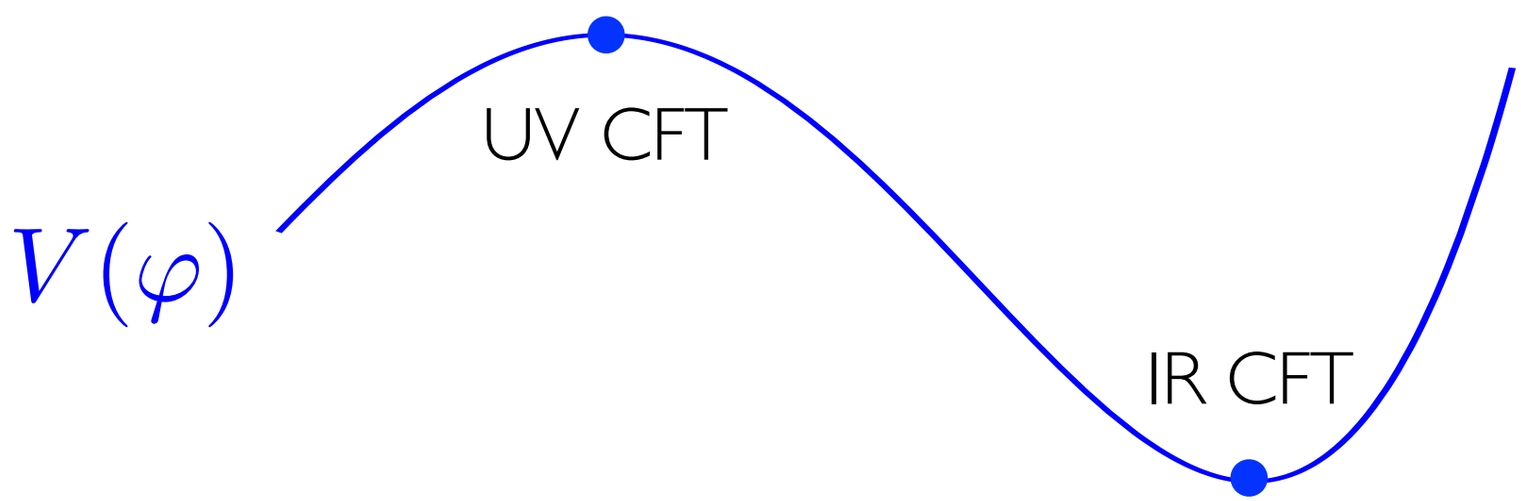
Dilaton potential:



[Ghosh, Kiritsis, Nitti, LW 2017]

Setup: Summary

We will consider **large-N_c** theories at **infinite coupling** of the following type and integrate out via **holography**:

1.) CFTs

2.) RG flow QFTs:

RG flows driven by a relevant operator $\mathcal{O}$ of dimension Δ^{UV} from a UV fixed point to a IR fixed point.

The QFTs are defined on **Einstein manifolds**.

$$S_{\mathrm{eff}}[g] = S_0[g] + S_{\mathrm{grav}}^{\mathrm{on\text{-}shell}}[g]$$

Setup: UV divergences

Integrating out a QFT typically leads to UV divergences.

Regulate UV divergences via a UV cutoff.

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$$S_{\text{grav}}^{\text{on-shell}}[g] = \int d^4x \sqrt{|g|}\, f_{\text{QFT}}(R)$$

$$\sim \int \left(a_1 \Lambda^4 + a_2 \Lambda^2 R + a_3 R^2 \log R\Lambda^{-2} \right)$$

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Regulate UV divergences via a UV cutoff.

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$$\sim \int \left(a_1 \Lambda^4 + a_2 \Lambda^2 R + a_3 R^2 \log R\Lambda^{-2} \right)$$

- The term $\sim \Lambda^4$ renormalizes the cosmological constant
- The term $\sim \Lambda^2 R$ renormalizes the Planck scale.
- The term $\sim R^2 \log R\Lambda^{-2}$ renormalizes the R^2-term.

Setup: Case I

The system of bare grav. theory and QFT is "UV-complete".

$\Lambda \to \infty$ Take cutoff to infinity

$$S_0[g] = \int d^4x \sqrt{|g|} \left[\frac{M_0^2}{2} R - M_0^2 \lambda_0 + a_0 R^2 \right]$$

$$S_{\text{grav}}^{\text{on-shell}}[g] = \int d^4x \sqrt{|g|}\, f_{\text{QFT}}(R)$$

Absorb the divergent terms in renormalized quantities:

$$M_{\text{ren}}^2 \lambda_{\text{ren}} = M_0^2 \lambda_0 + f_{\text{QFT}}\big|_{R=0,\, \Lambda\to\infty}$$

$$\frac{M_{\text{ren}}^2}{2} = \frac{M_0^2}{2} + \frac{d}{dR} f_{\text{QFT}}\big|_{R=0,\, \Lambda\to\infty}$$

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Absorb the divergent terms in renormalized quantities:

$$S_{\text{eff}}[g] = \int d^4x \sqrt{|g|}\, f(R \,|\, M_{\text{ren}}, \lambda_{\text{ren}}, m)$$

Setup: Case 2

In **Case 2** it is assumed that S_0 is an effective theory at some scale Λ.

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Then couple a QFT with UV cutoff Λ to the background described by $g_{\mu\nu}$.

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The combined system is described by:

$$S_{\text{eff}}[g] = \int d^4x \sqrt{|g|}\, f(R \,|\, M_0, \lambda_0, \Lambda, m)$$

Results

Results: UV complete case (I)

The system of bare grav. theory and QFT is "UV-complete".

$\Lambda \to \infty$ Absorb divergent terms in renormalized quantities.

Equation for constant-curvature solutions:

$$M_{\rm ren}^2 R - 4M_{\rm ren}^2 \lambda_{\rm ren} + \langle T_\mu^{{\rm ren},\mu} \rangle = 0$$

$$\langle T_\mu^{{\rm ren},\mu} \rangle = -\frac{2}{\sqrt{|g|}} g^{\mu\nu} \frac{\delta}{\delta g^{\mu\nu}} \ln S_{\rm grav}^{\text{on-shell,ren}}$$

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1.) CFT: $\langle T_\mu^{{\rm ren},\mu} \rangle = -\frac{\tilde{a}}{48} R^2$

2.) RG flow QFT: $\langle T_\mu^{{\rm ren},\mu} \rangle = -\frac{\tilde{a}_{\rm UV}}{48} R^2 + (4 - \Delta^{\rm UV}) m^{4-\Delta^{\rm UV}} \langle \mathcal{O} \rangle (R)$

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I.) CFT:

$$R = \frac{24}{\tilde{a}} M_{\rm ren}^2 \left(1 \pm \sqrt{1 - \frac{\tilde{a}}{3} \frac{\lambda_{\rm ren}}{M_{\rm ren}^2}} \right)$$

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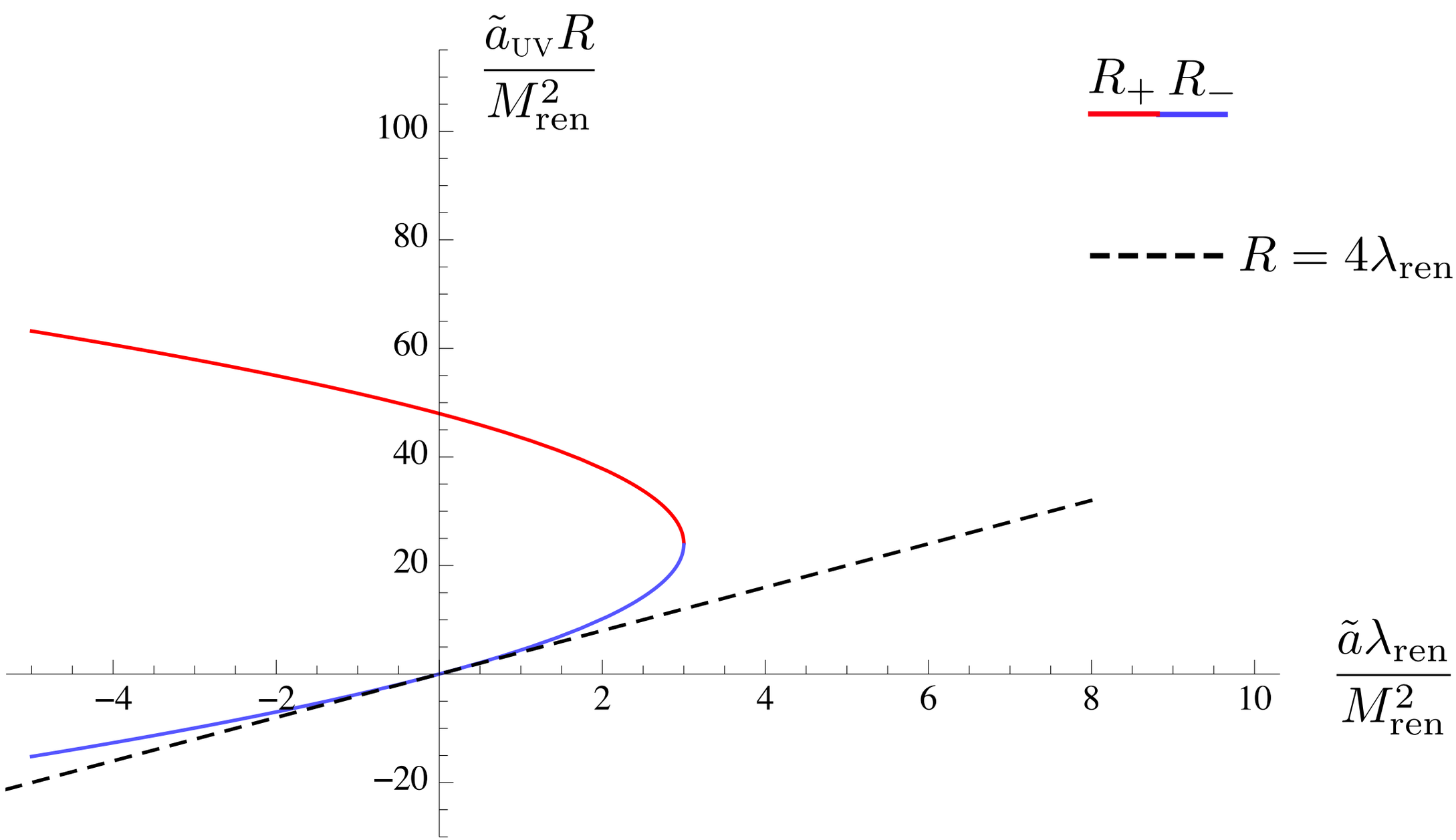

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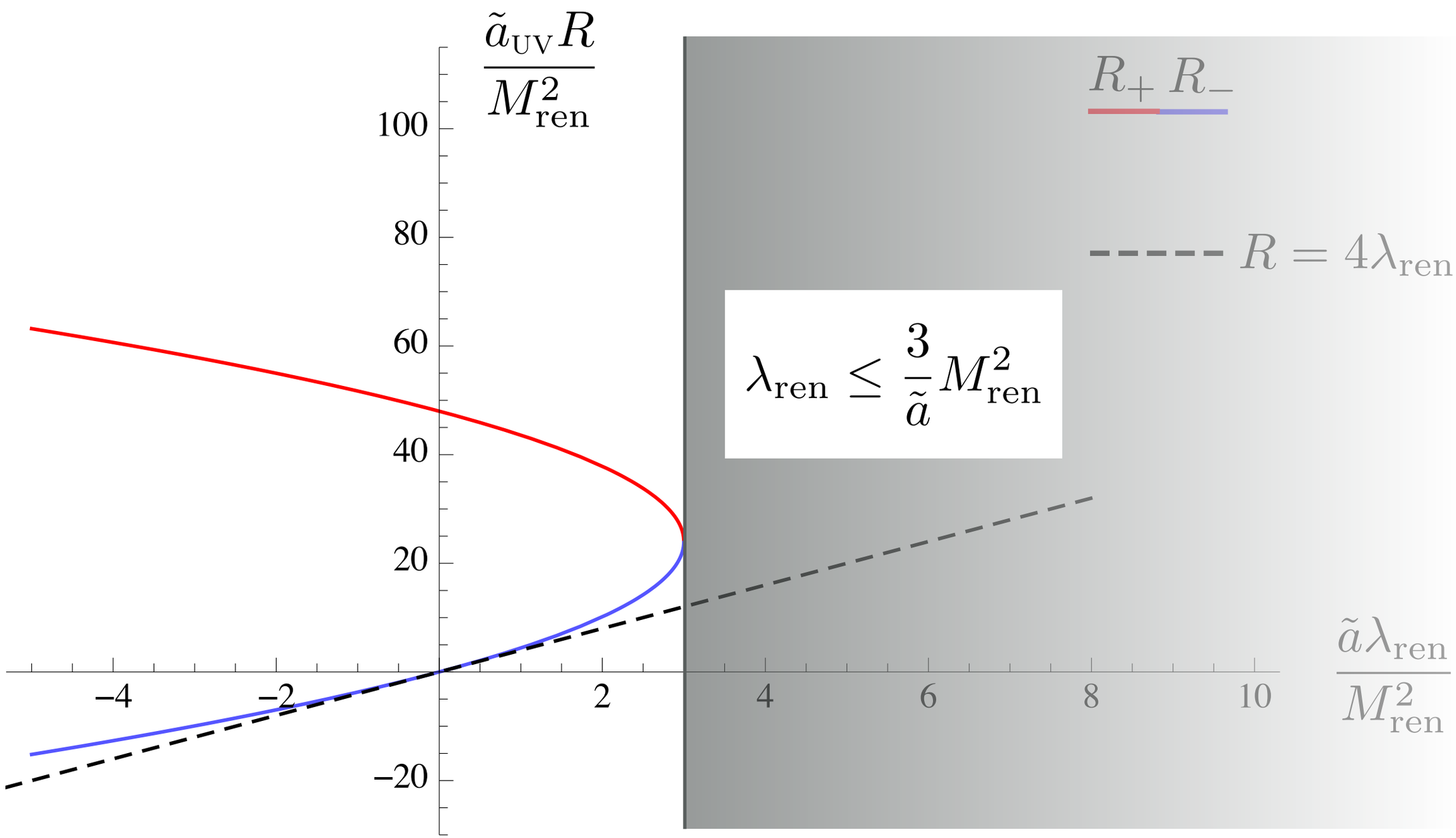

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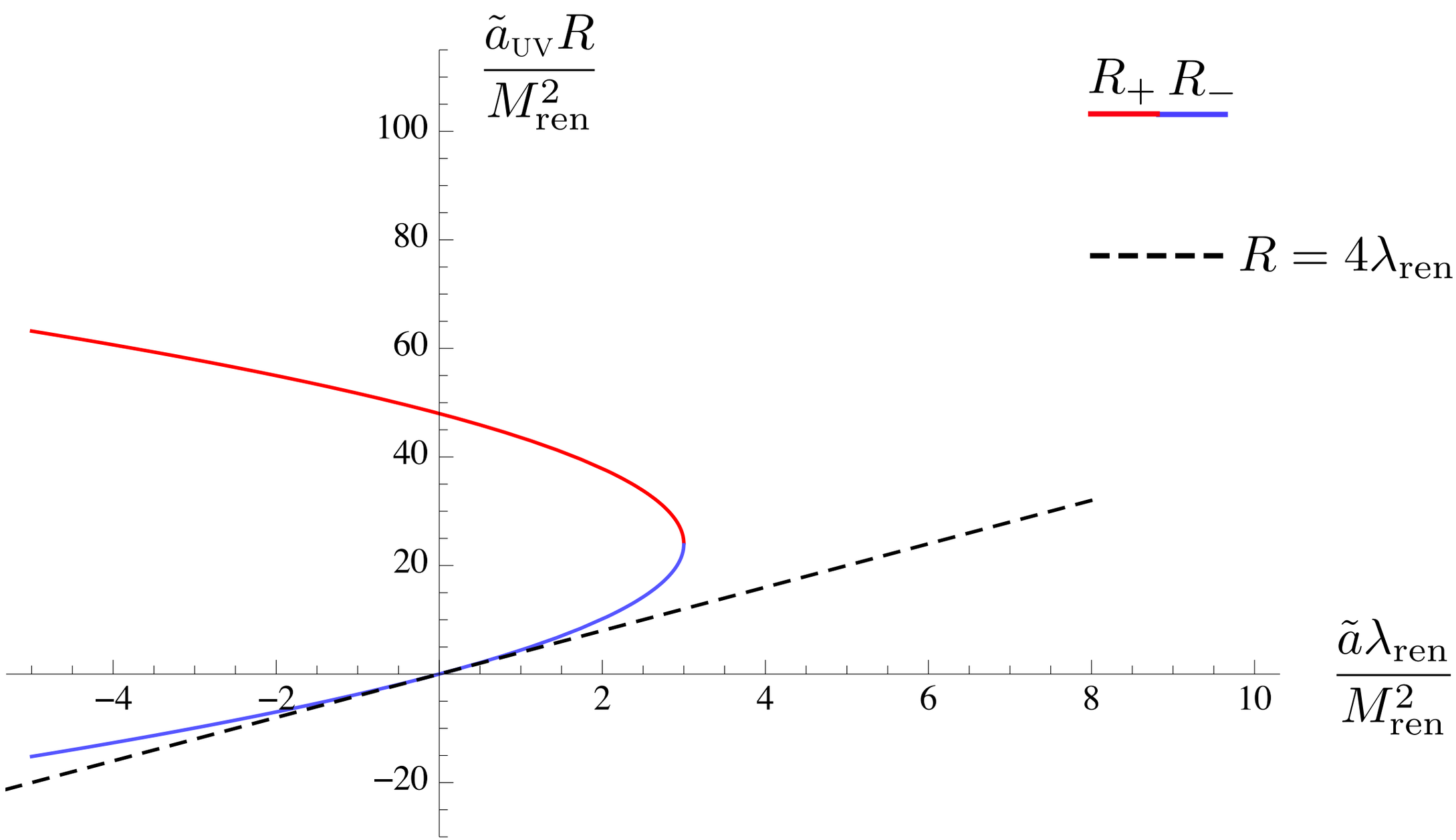

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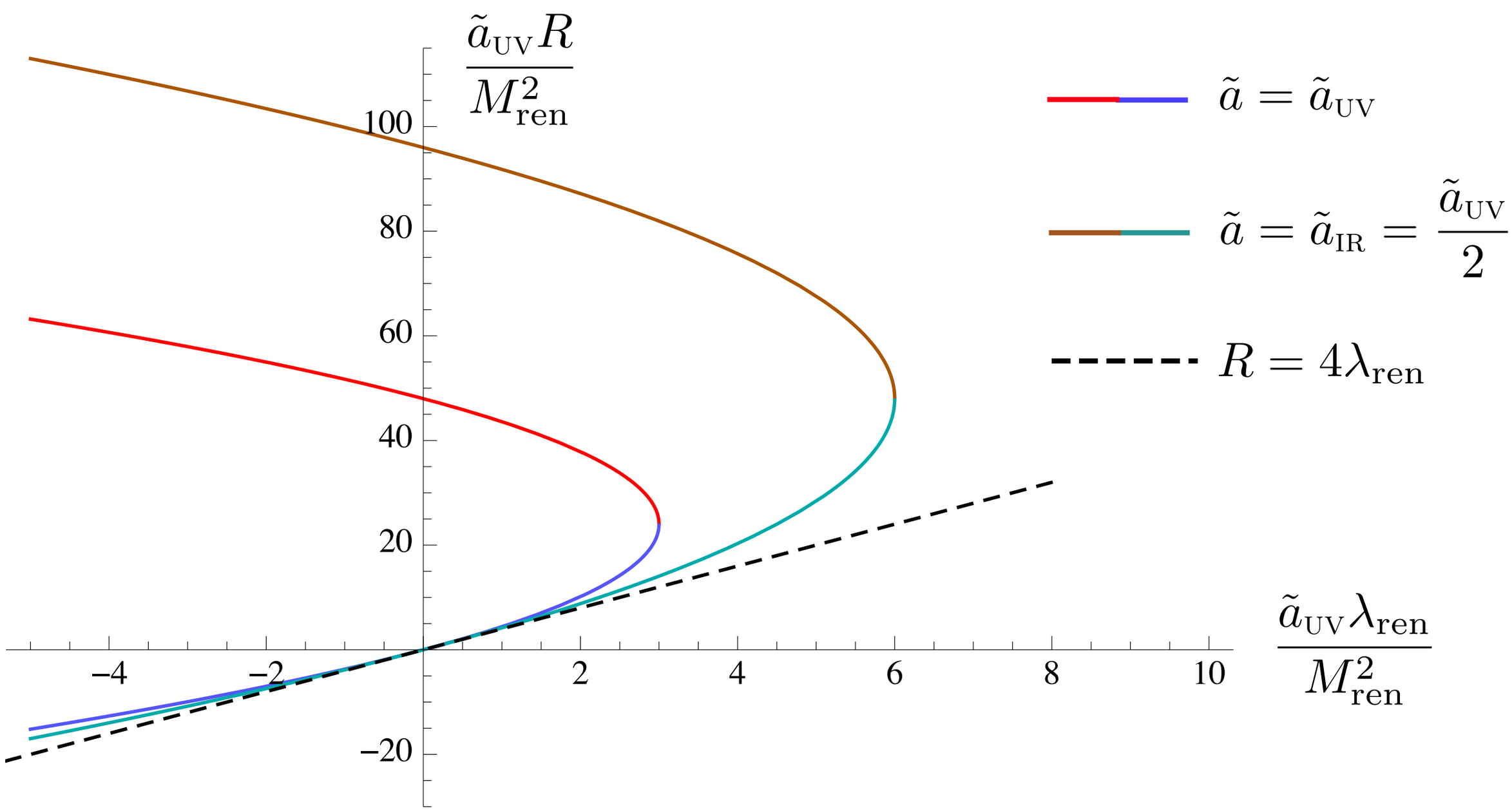

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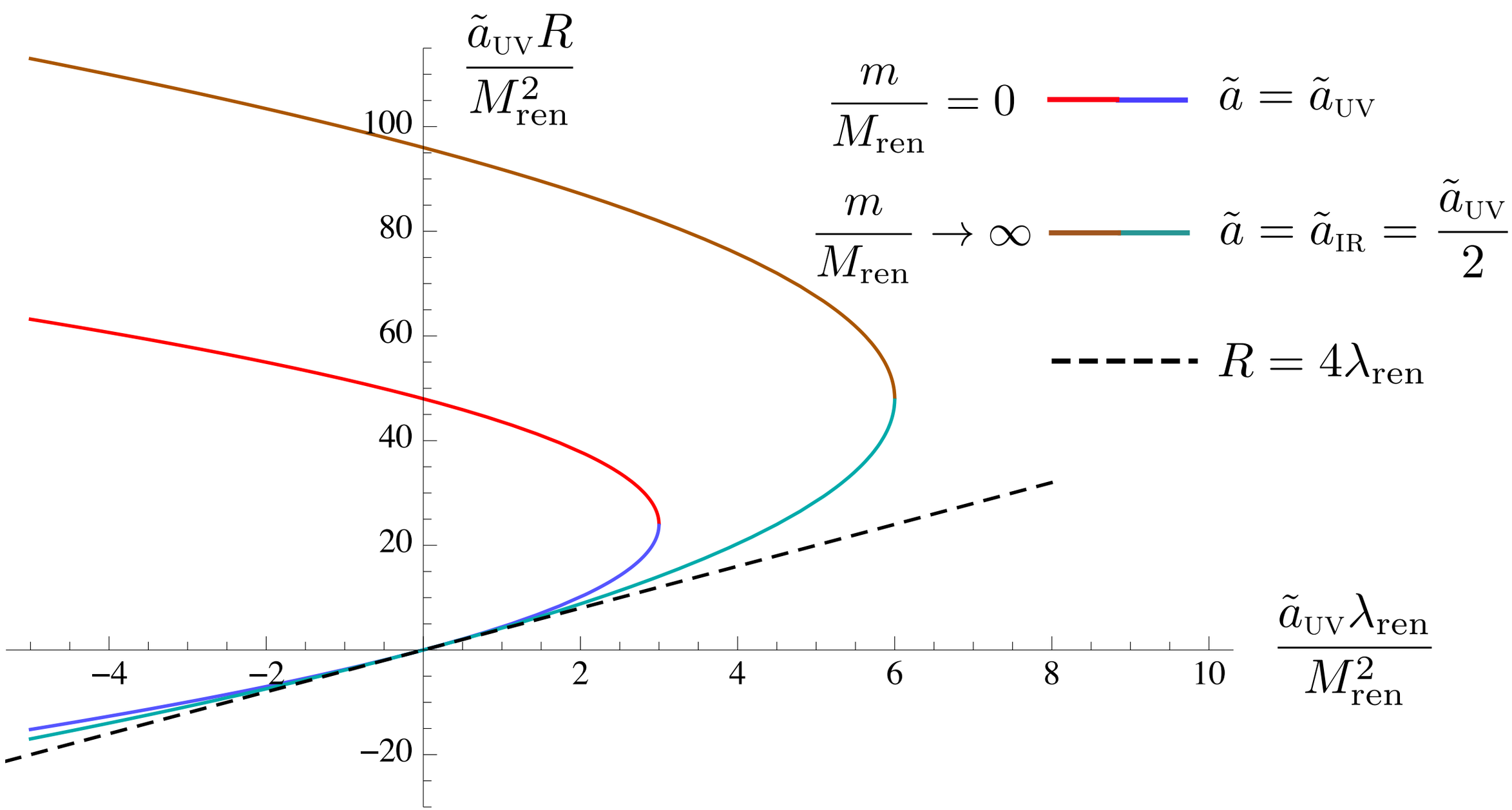

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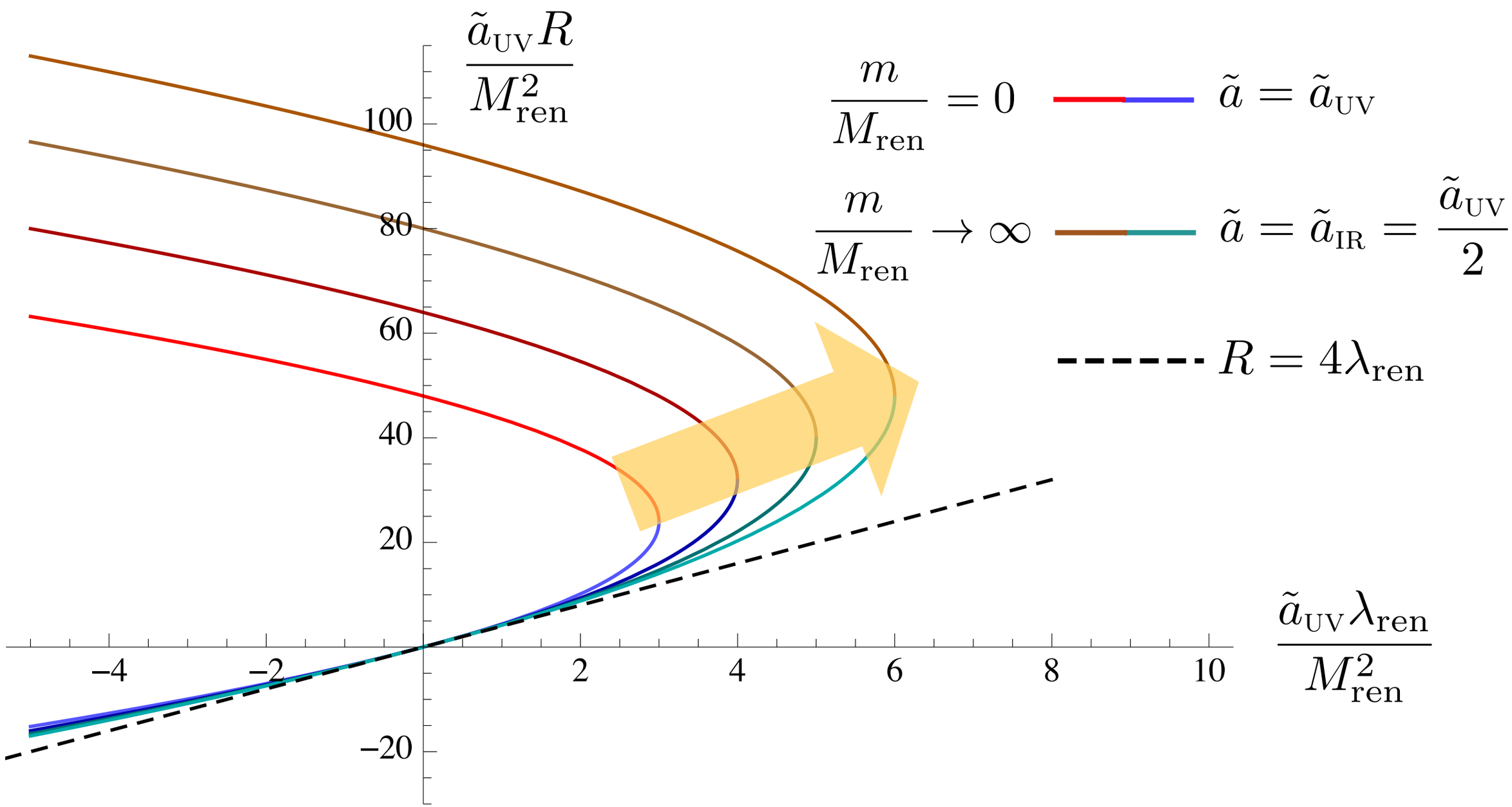

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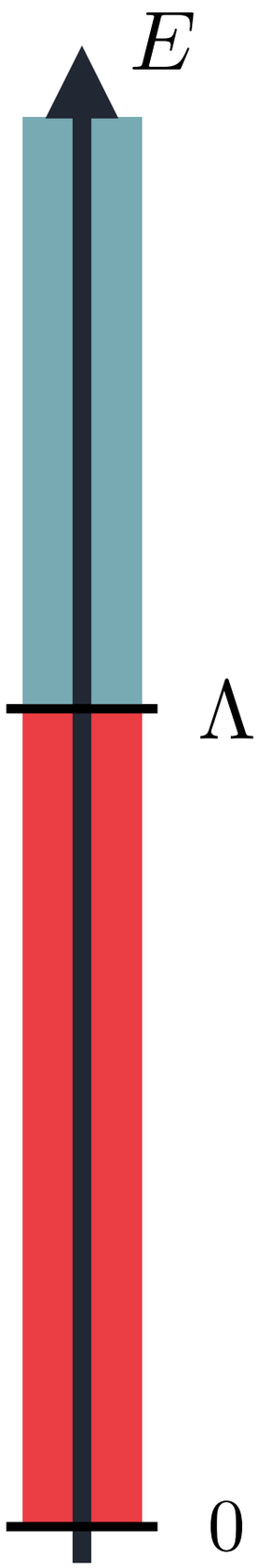

In **Case 2** it is assumed that S_0 is an effective theory at some scale Λ.

$$S_0[g] = \int d^4x \sqrt{|g|} \left[\frac{M_0^2}{2} R - M_0^2 \lambda_0 + a_0 R^2 \right]$$

Then couple a QFT with UV cutoff Λ to the background described by $g_{\mu\nu}$.

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For a CFT: $$f_{\text{CFT}} = \tilde{a} \left[6\Lambda^4 \sqrt{1 + \frac{R}{12\Lambda^2}} + \frac{R\Lambda^2}{4} \sqrt{1 + \frac{R}{12\Lambda^2}} + \frac{R^2}{48} \log\left(\sqrt{1 + \frac{12\Lambda^2}{R}} - \sqrt{\frac{12\Lambda^2}{R}} \right) \right]$$

Results: cutoff QFT (2)

Eq. for const.-curv. sol.: $$M_0^2 R - 4M_0^2\lambda_0 + 24\tilde{a}\Lambda^4\sqrt{1+\frac{R}{12\Lambda^2}} = 0$$

Solution: $$R = 4\lambda_0 - 24\tilde{a}^2\frac{\Lambda^6}{M_0^4}\left(\sqrt{1+\frac{M_0^4}{\tilde{a}^2\Lambda^4}+\frac{M_0^4\lambda_0}{3\tilde{a}^2\Lambda^6}}-1\right)$$

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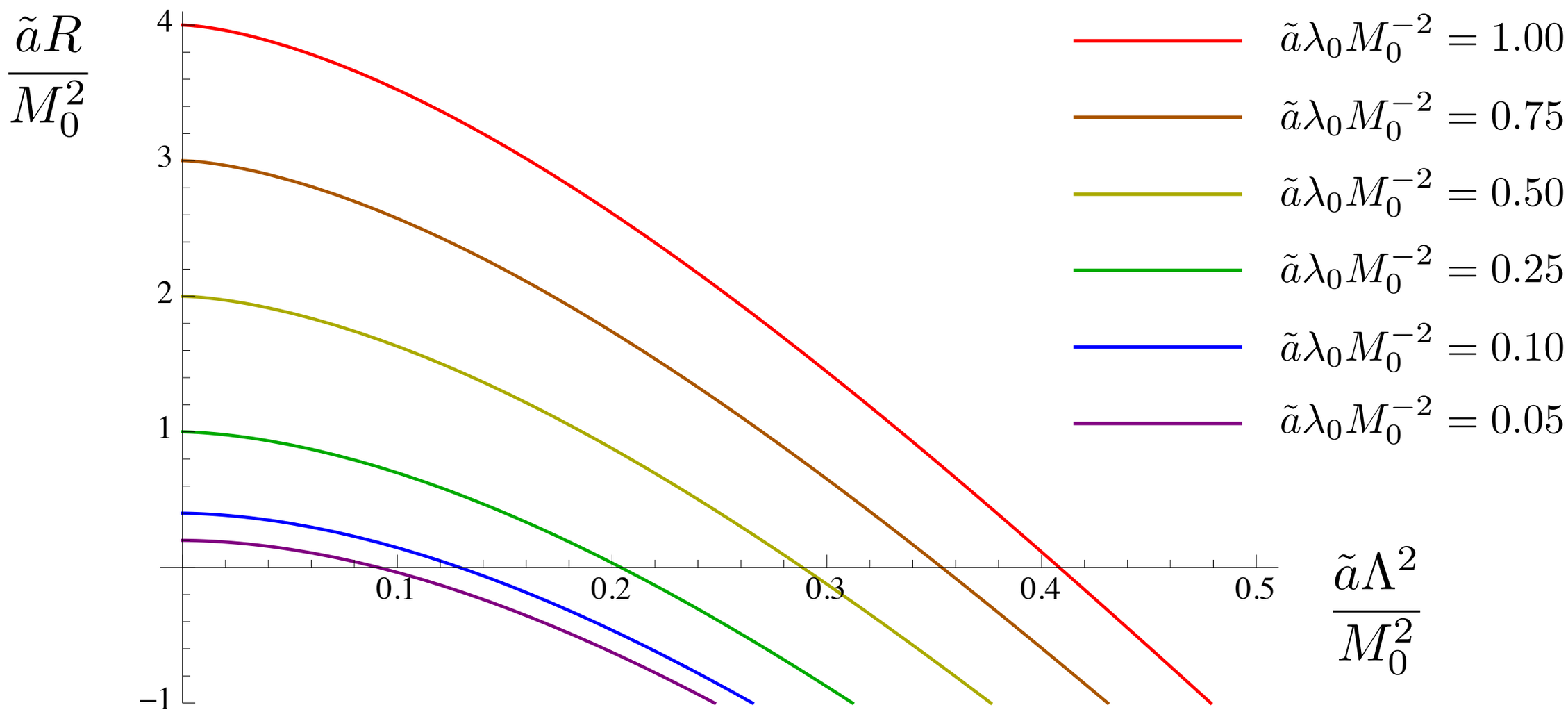

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- For $\Lambda = 0$ have $R(\Lambda = 0) = 4\lambda_0$.
- Increasing Λ always decreases R .
- For sufficiently large Λ the curvature R becomes negative.
- The (thermal) entropy of dS space scales as $S_{\rm th} \sim R^{-1}$. Increasing Λ thus increases $S_{\rm th}$ of dS which may be naively expected. Can this entropic argument be made precise?

Summary

0.) Advantages from holography:

- Integrating out a QFT via its gravity-dual is highly tractable.
- Get explicit results for large-N theories at infinite coupling.

1.) UV complete setting (case 1)

- **CFTs**: only have solution if $\lambda_{\rm ren} \leq \frac{3}{\tilde{a}} M_{\rm ren}^2$.
- **RG flow QFTs**: back-reaction effect interpolates between that of the UV CFT and the IR CFT.

2.) Cutoff QFT (case 2)

- Increasing the UV cutoff always reduces the curvature R .

Open Questions

- Is it possible to develop a precise and quantitative **entropic** understanding of the back-reaction effect?

- Are the solutions found stable under small **perturbations**. To what extent can this question be addressed in the simplified setup considered here with $\nabla_\rho R_{\mu\nu} = 0$?

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Many thanks for your attention!