A Taste of Complexity

Eliezer Rabinovici - Hebrew University, Jerusalem

19th September 2019


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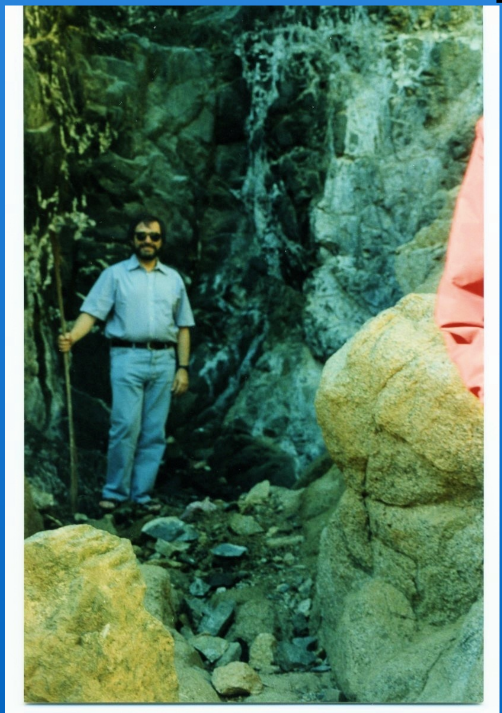
0<<COMPLEXITY (FARHAD)=?

1999-2000



2001

Old Cultures:
Greece, India, Iran, Israel

Why? Where?

2001- Kolymbari

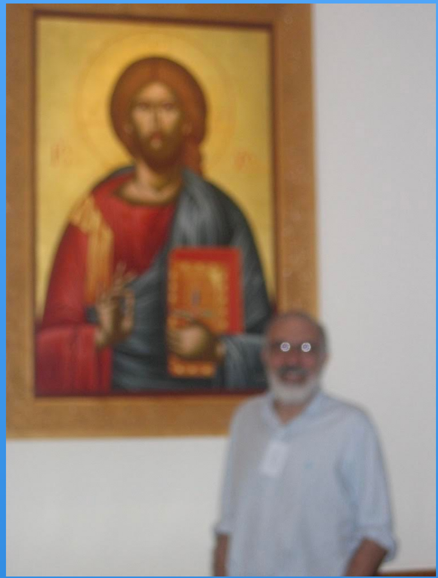
Rather Failed Opration BUT: Rather Bueautiful

2003

2005

2005

Not an Easy Time




AYPA

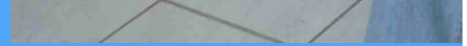
2007

Lets Discuss
Why THE……..???


2009

Not an Easy Year

2011




A1
REGIONAL MEETING
IN STRING THEORY


6 th Regional Meeting in String Theory
19 - 26 June 2011, MILOS


WC
Milos

2013


Samaria

2015-Nafplion

2015 NOT EASY FOR OUR HOSTS

2017


Br
Ba


ALGIDA

<<< 72

Wishing you lots of HEALTH,

good food and drinks

wherever you want it!

Good company and friends

Wherever and whoever you want.

شاد†باش†و†شاد†زی

מזל טוב- Mazal Tov-

HERE AS GOOD LUCK

A Taste of Complexity

J.Barbon, S.Bolognesi, S. Roy, R.Shir(today), R. Sinha

Eliezer Rabinovici - Hebrew University, Jerusalem

19th September 2019

What Effects Can Semi-classical Geometry Capture?

Geometry Can Capture Inclusive Exp(-S)

Effects and Reproduce Average Results.

Geometry May Well Miss Some Exclusive Exp(-S)

Features.

It is About Lower Non-Perturbative Bounds:

It is About Lower Non-Perturbative Bounds:

One Temporal and One Spatial

It is About Lower Non-Perturbative Bounds:

One Temporal and One Spatial

Both Try to Probe Behind the

Horizon

Aspects of Long Time Scales in Field Theory

Classical		**Quantum**
Compact Phase Space	$\Longleftrightarrow$	Discrete Spectrum
Volume Conservation	$\Longleftrightarrow$	Unitarity

Then, If

$$G(t_0) =< \theta_1(t_0, x_1)|\theta_2(0, x_2) >$$

for any ϵ there is a $t^P(\epsilon)$ such that

$$|G(t^P(\epsilon)) - G(t^0)| < \epsilon$$

You See It All!

Consider

$$G(t) = Tr\left[\rho A(t) A(0)\right]$$

For very large time scale

Consider

$$L(t) = \left| \frac{G(t)}{G(0)} \right|^2$$

$$\bar{L} = \lim_{T \to \infty} \frac{1}{T} \int_0^T dt \, L(t)$$

The CFT is unitary and has a Gap

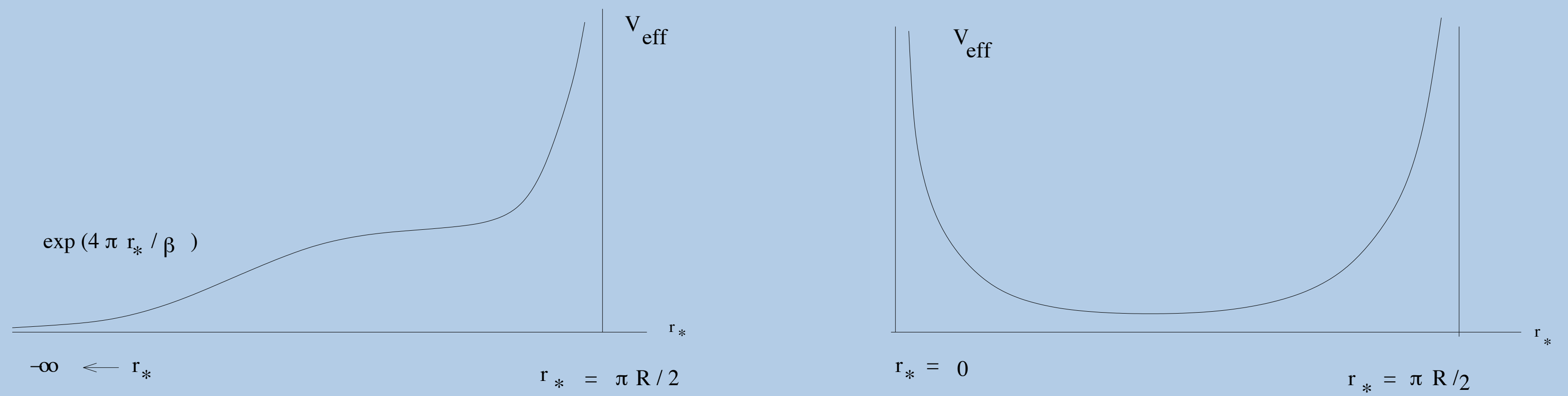
$$\bar{L} \sim \frac{\Delta L}{\Gamma t_H} \sim \exp(-S(\beta))$$

,

$$\bar{L} \sim \exp(-N^2 \ldots) \sim \exp\left(-\frac{1}{G_N} \ldots\right)$$

Non Perturbative from Gravity Point of View

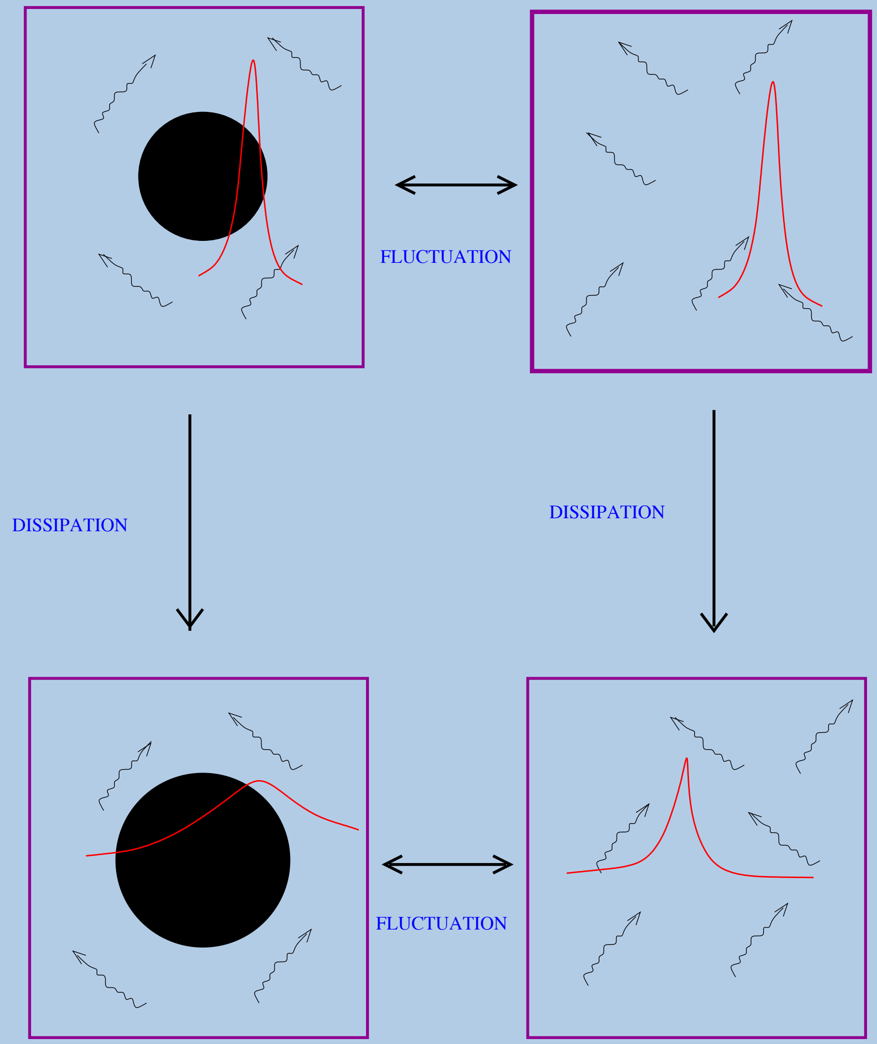
For BH background $\bar{L} \to 0$, Reason:
No Gap in the presence of a BH.



Behind the Horizon

In a Thermal AdS Background a gap is formed and now

$$\bar{L}_{Bulk} \approx \exp(-S) > 0$$

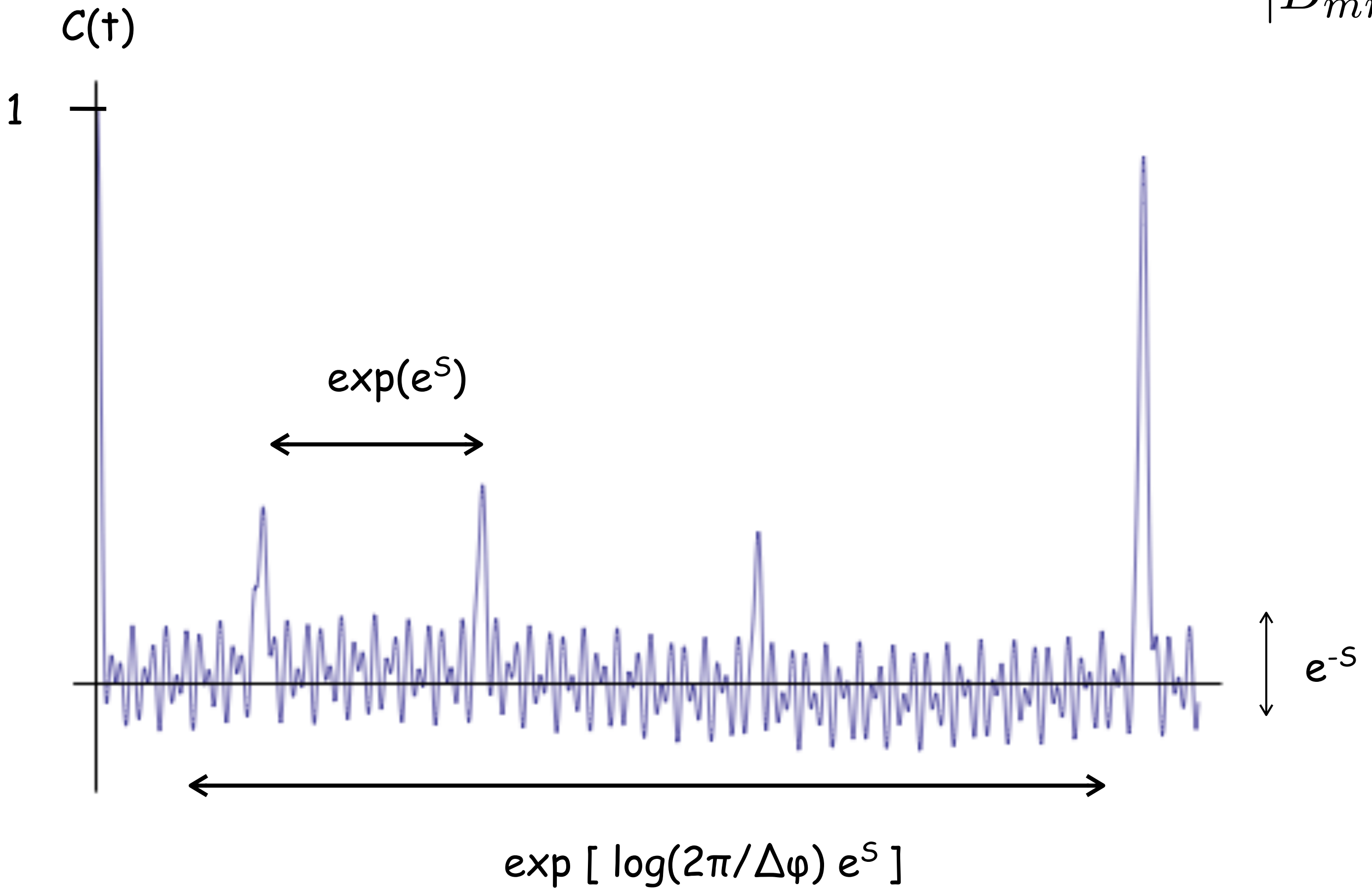

FLUCTUATION
DISSIPATION
DISSIPATION
FLUCTUATION

$$C(t) = \sum_{mn}^{e^{2S}} \rho_m \, |B_{mn}|^2 \, e^{i(E_m - E_n)t}$$

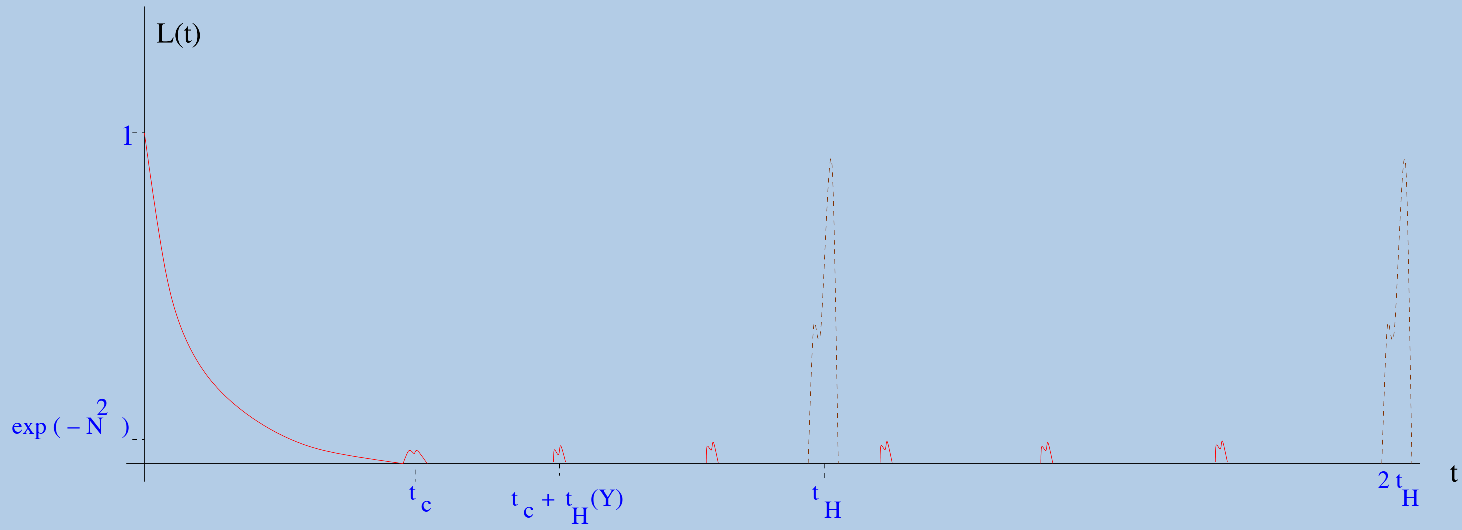
$$C(0) = 1$$

$$\rho_m \sim e^{-S}$$

$$|B_{mn}| \sim e^{-S/2}$$



detailed Poincaré time scale



L(t)
1
exp (– N
2
)
t
c
t
c
+ t
H
(Y)
t
H
2 t
H
t

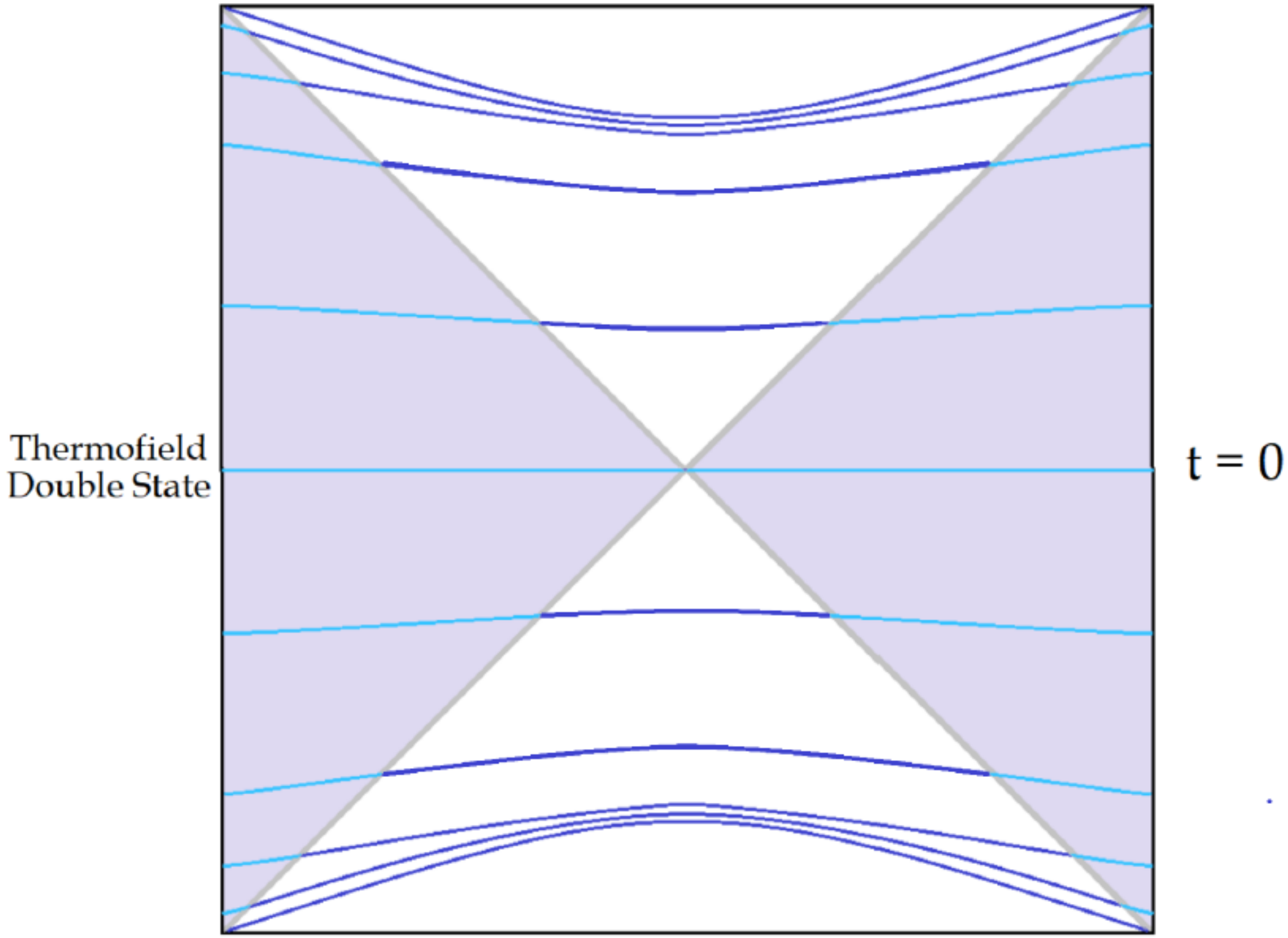
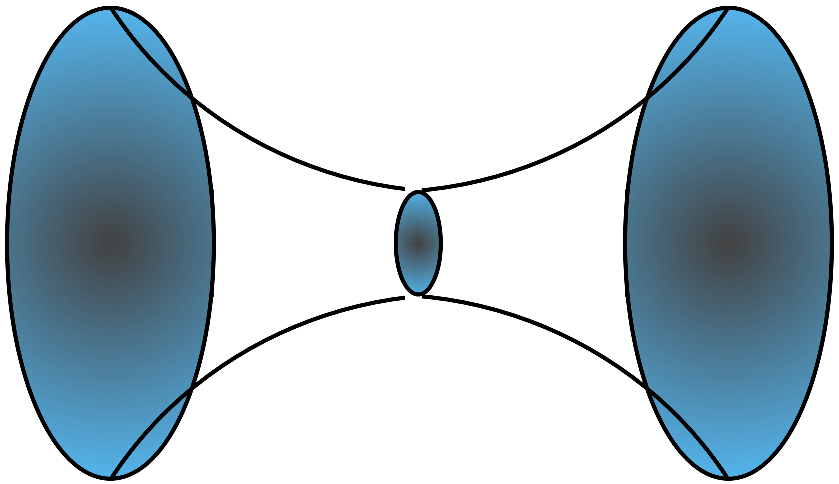


Figure 17: Penrose diagram for an AdS eternal black hole. The diagram is foliated by maximal slices. The darker blue portions of the slices represent the wormhole behind the horizons. The volume of the slices “bounces,” decreasing in the lower part of the diagram, and increasing in the upper part.

ERB



Volume and Length Grow linearly with time.

Can grow up to time/size of order e^S.

Upper Bound Exp(#S)

Reason:

A SPATIAL Correlation Function has a Lower Bound

Due to “Recurrences”

$$\mathrm{Exp}(-L(RB)) = \mathrm{Exp}(-S)$$

Which Translates into an Upper Bound

A Very Large Upper Bound

for the Volume of the ERB

This All Goes on Well After Thermalisation Time Scales .

What is the Boundary Phenomenon? Time Scale?

Behind the Horizon

Complexity of state F

Given:

An Initial Simple State- I

Simple Operations

Definition: The complexity of the state is the minimal

number of given operations to construct F from I
(also Complexity of the operator)

Universality classes?

Classical Complexity

(0,0,0….,0)

0 to 1 , 1 to 0

On S Sites

Maximal Classical Complexity is Proportional to the Entropy- S

Quantum Complexity

$$|\psi\rangle = \sum_{1}^{2^K} \alpha_i |i\rangle$$

Number of cells is

$$\left(\frac{1}{\epsilon}\right)^{e^S}.$$

There are O(S) choices at each step so the number of states one can reach in n steps is of order S^n

$$S^{C_\epsilon} \leq \left(\frac{1}{\epsilon}\right)^{e^S}$$

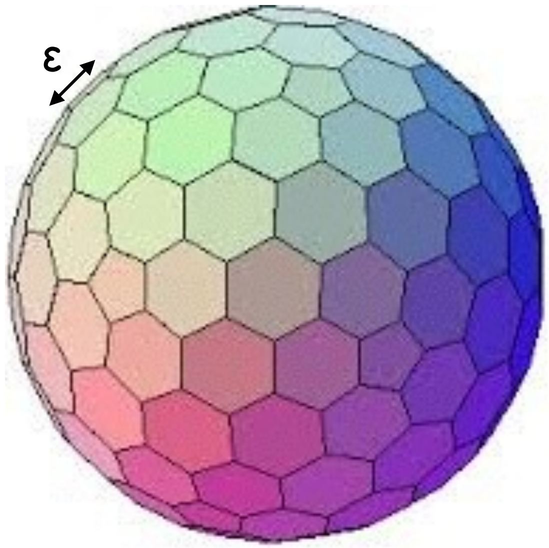
The complexity is bound by

Exp(S)log(1/ ϵ)

No "Real" Upper Bound

Manifold of pure states with ε-resolution

$$\frac{U(e^S)}{U(e^S-1)\times U(1)}$$



$$\text{Complexity}_\epsilon \leq \log[\#_{\text{cells}}] \sim e^S \log(1/\epsilon)$$

Claim: Most states have this maximal Complexity.

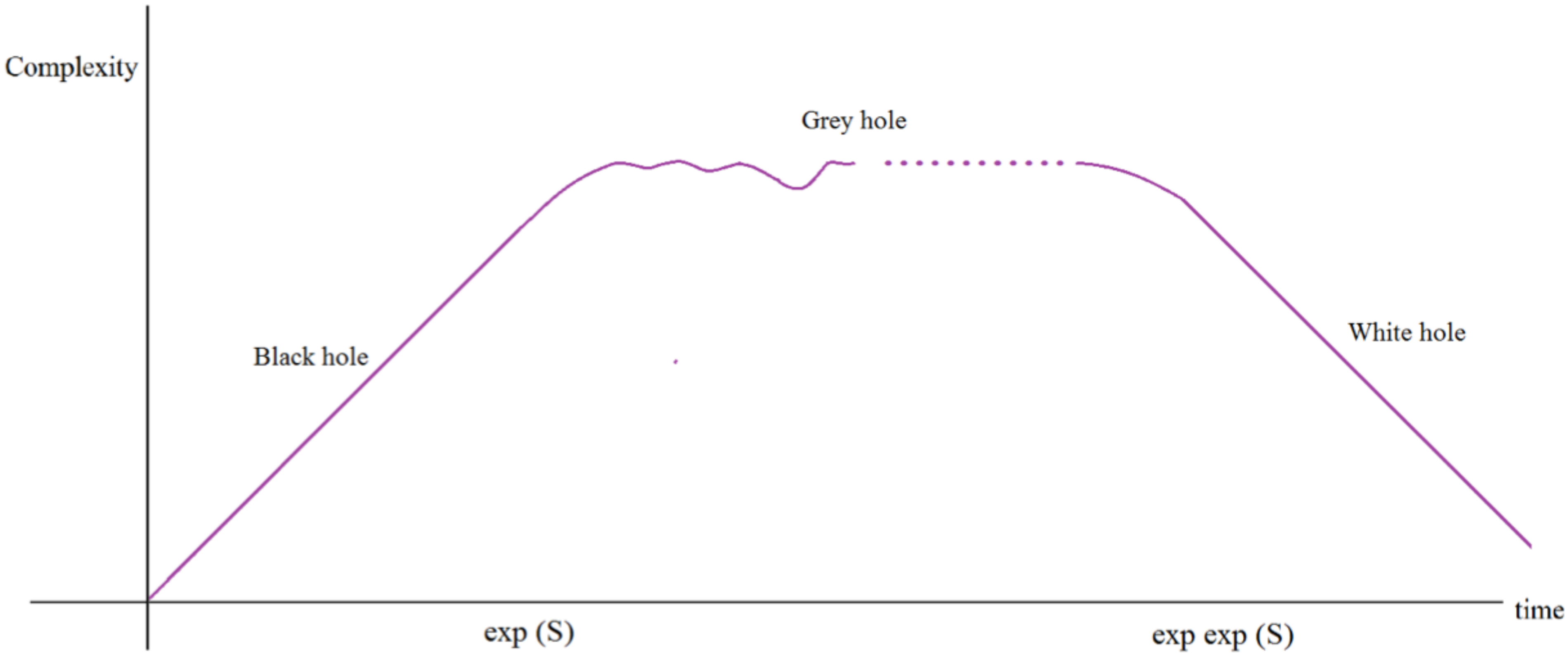
Claim- BH: Complexity increases till time

Exp(S) and reaches the value Exp(S)

Well beyond log S and S scrambling

and thermalisation times.

State in General then Black Hole Bulk



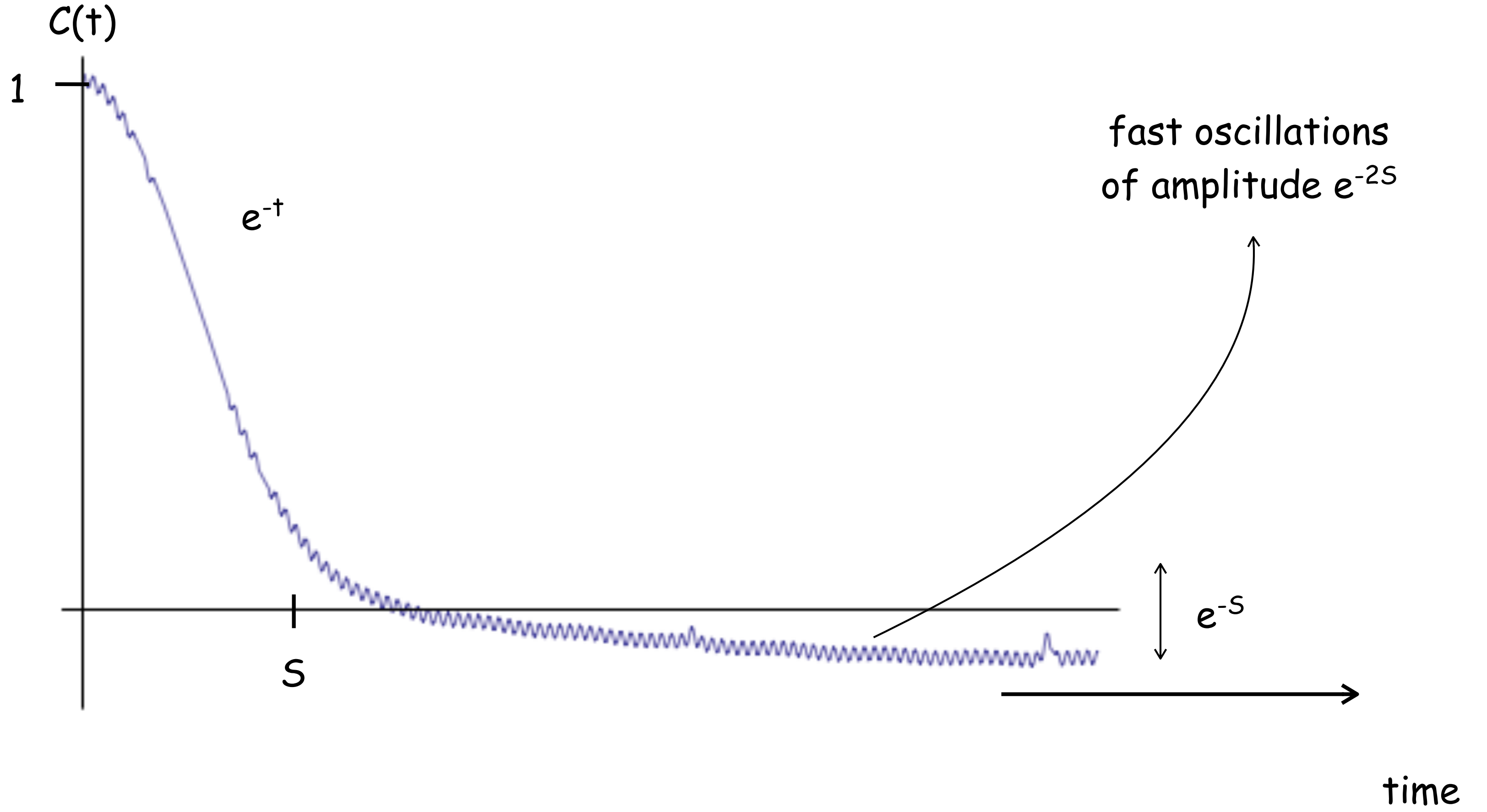
Note the Similarity

$$C(t) = \sum_{mn}^{e^{2S}} \rho_m \, |B_{mn}|^2 \, e^{i(E_m - E_n)t}$$

$$C(0) = 1$$

$$\rho_m \sim e^{-S}$$

$$|B_{mn}| \sim e^{-S/2}$$



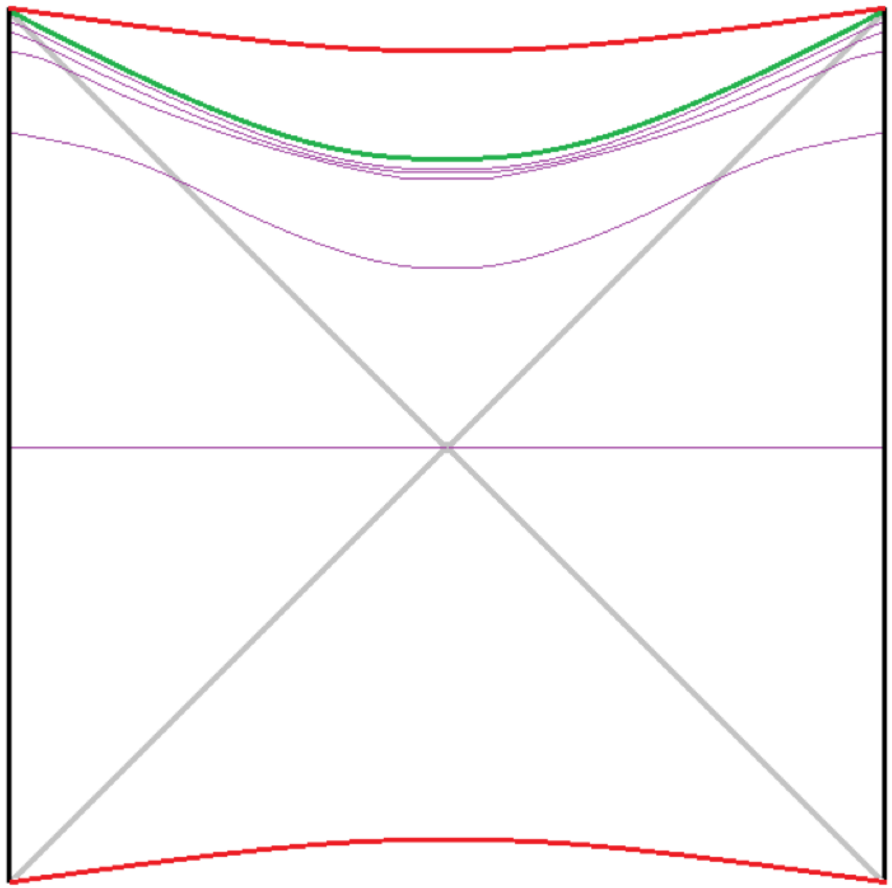
Page time scale

$$\frac{dC}{dt} \sim T\,S$$

- **BULK**

$$C(t) \propto \frac{\mathrm{Vol}(\Sigma_t)}{G\ell}$$

In the BULK Maximal Volume



$$\frac{dC}{dt} \sim T\,S$$

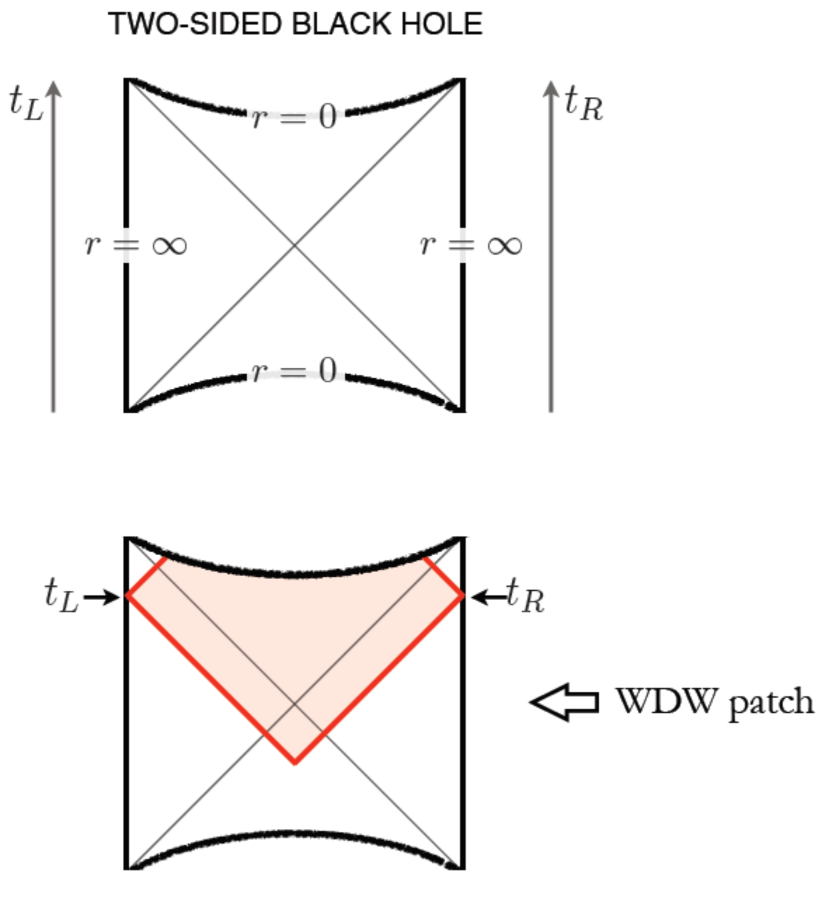
- **BULK**

$$C(t) \propto \frac{\mathrm{Vol}(\Sigma_t)}{G\ell}$$

$$\text{Complexity} = \frac{\text{Action}}{\pi\hbar}$$

$$
\begin{aligned}
\mathcal{A} &= \mathcal{A}_{EH} + \mathcal{A}_{YGH} \\
&= \frac{1}{16\pi G}\int_{W\,dW} \sqrt{|g|}(R - 2\Lambda) + \frac{1}{8\pi G}\int_{\partial W\,dW} \sqrt{|\gamma|}K
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A} &= \mathcal{A}_{EH} + \mathcal{A}_{YGH} \\
&= \frac{1}{16\pi G}\int_{W\,dW}\sqrt{|g|}(R-2\Lambda) + \frac{1}{8\pi G}\int_{\partial W\,dW}\sqrt{|\gamma|}K
\end{aligned}
$$



SKY Model and others -explicit calculations

Claim- BH: When Complexity increases

When the future is NOT singular the increase is Robust

When the future is singular (BH)

the Complexity decreases

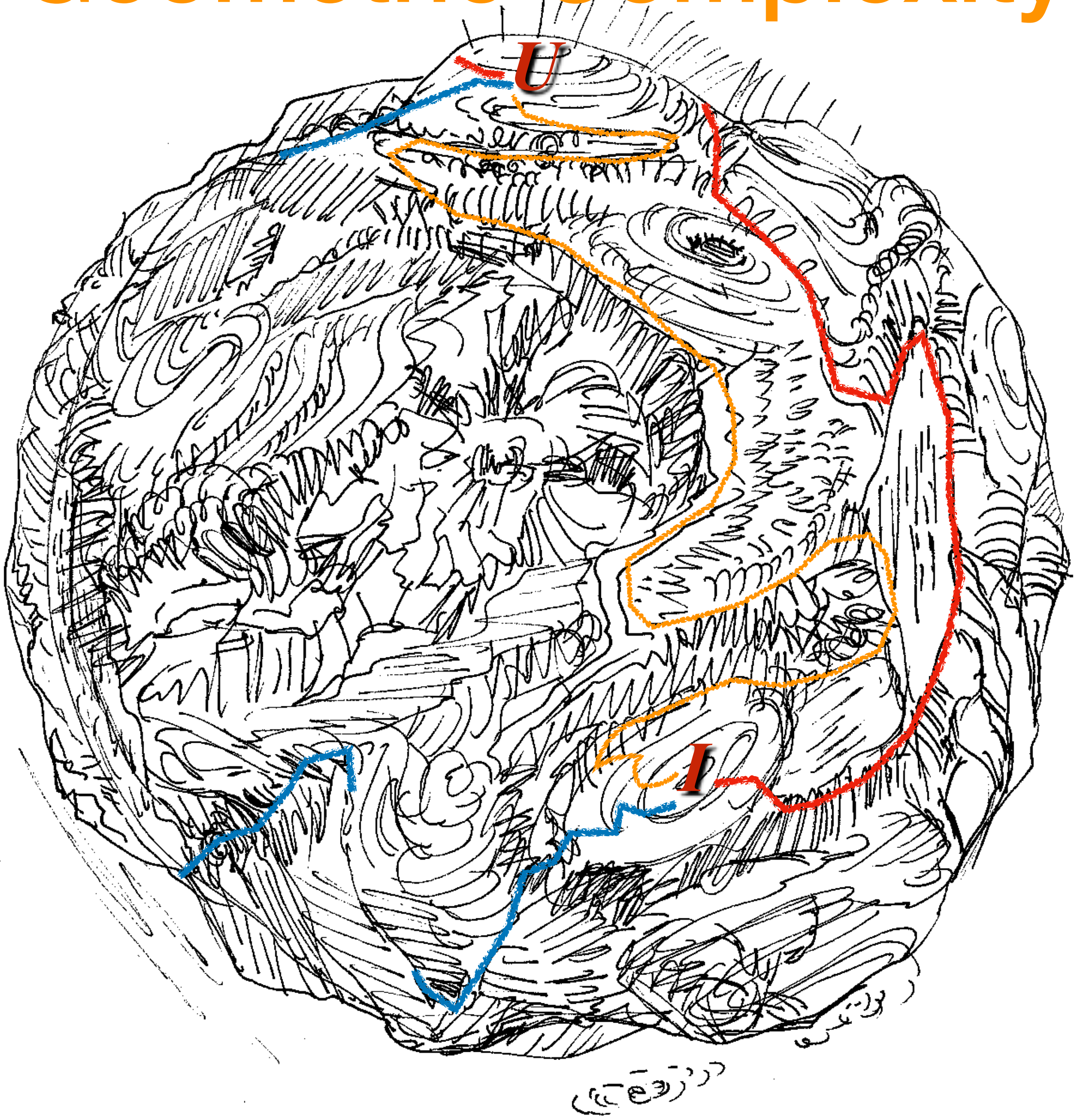
Not robust

Volume of ER bridge

slogan: As complexity of the state evolves

the geometry behind the horizon is "created"

Geometric Complexity



Problem: Depends on Metric

ε-resolution

$$\mathcal{C} = \int_{\mathbb{K}}^{U} \sqrt{G_{MN} dX^M dX^N}$$

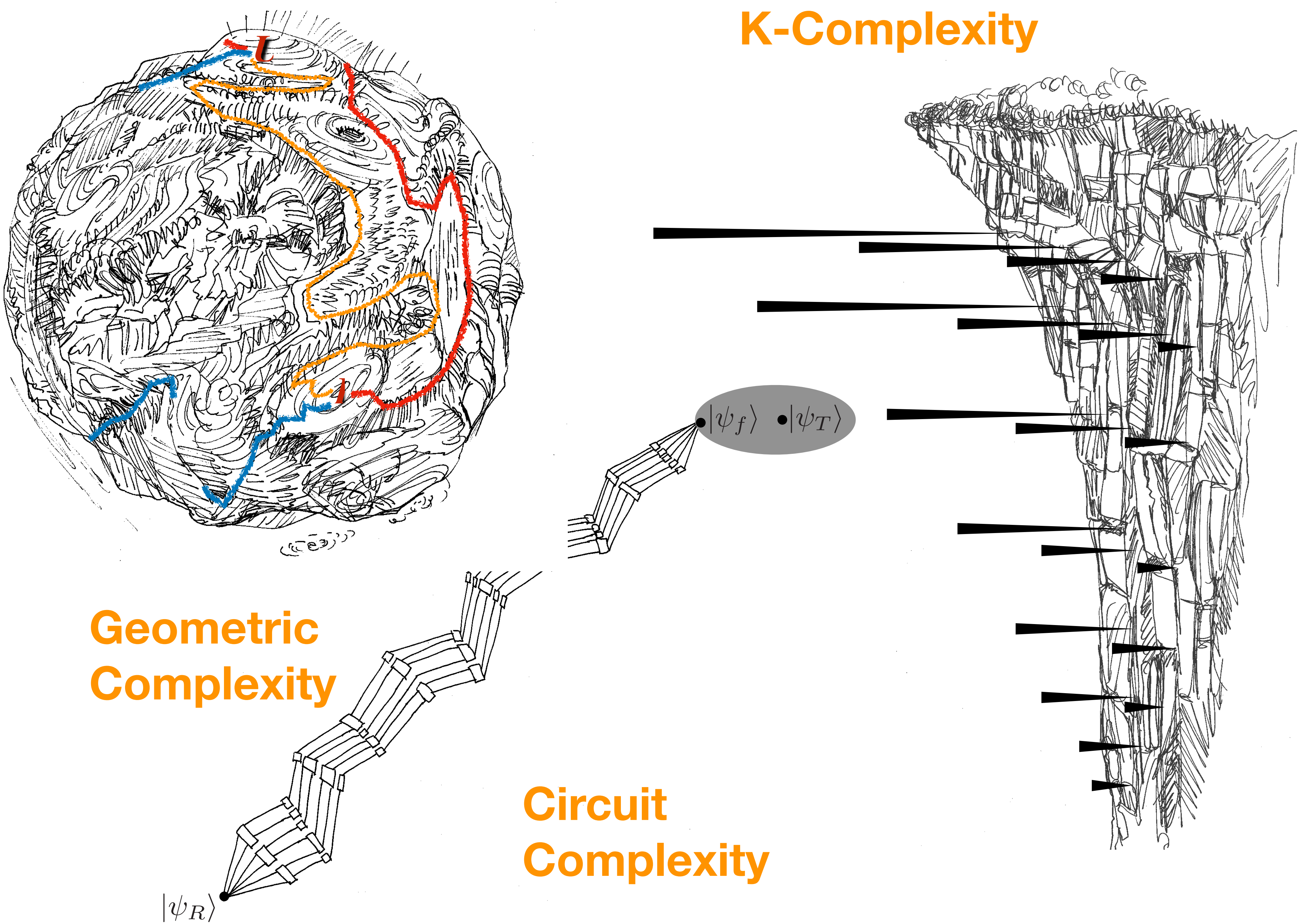
Problem: Depends on Measure and Metric

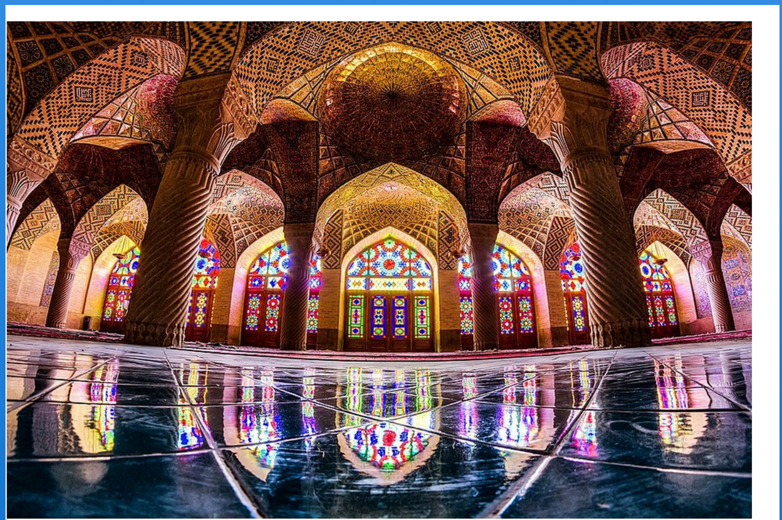
We Search for a Definition

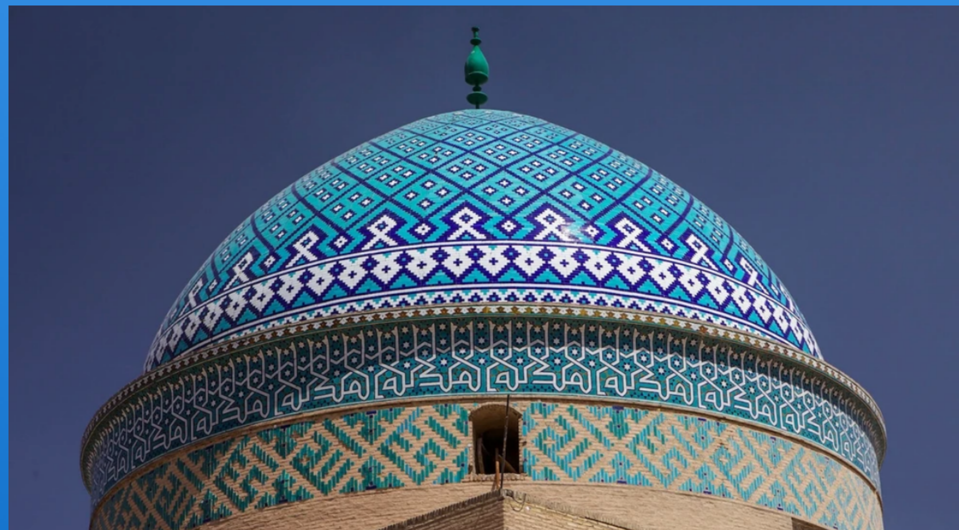
of Complexity Without Some Arbitrary Tolerance

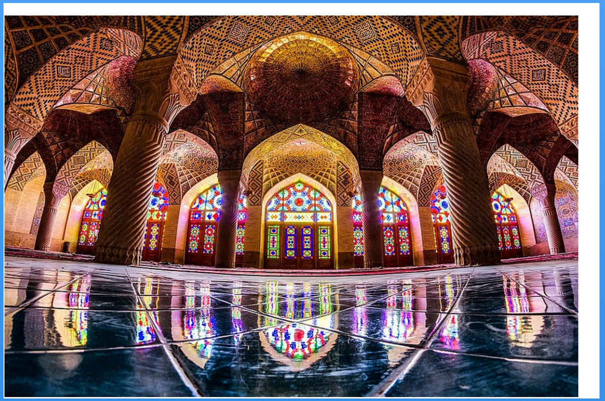
Ruth Shir's Talk

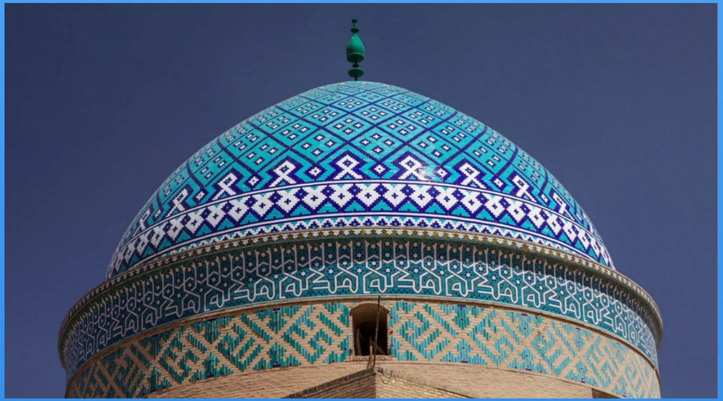
K Complexity

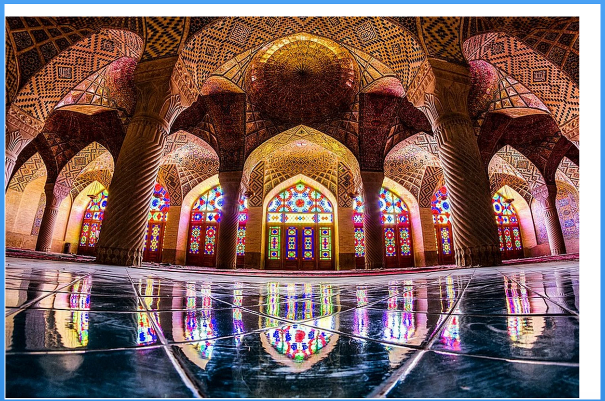
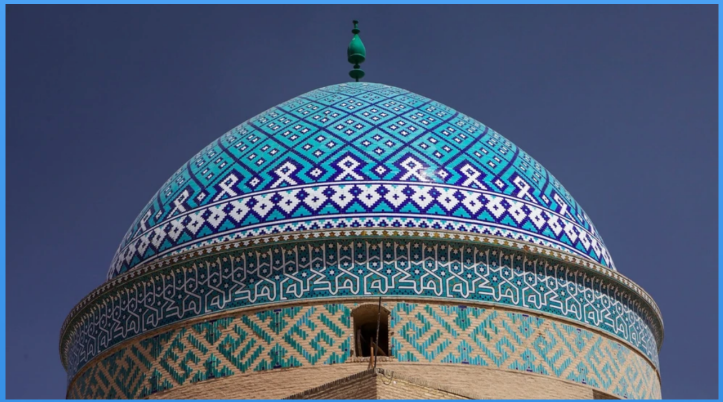

K-Complexity
$|\psi_f\rangle$
$|\psi_T\rangle$
Geometric Complexity
Circuit Complexity
$|\psi_R\rangle$

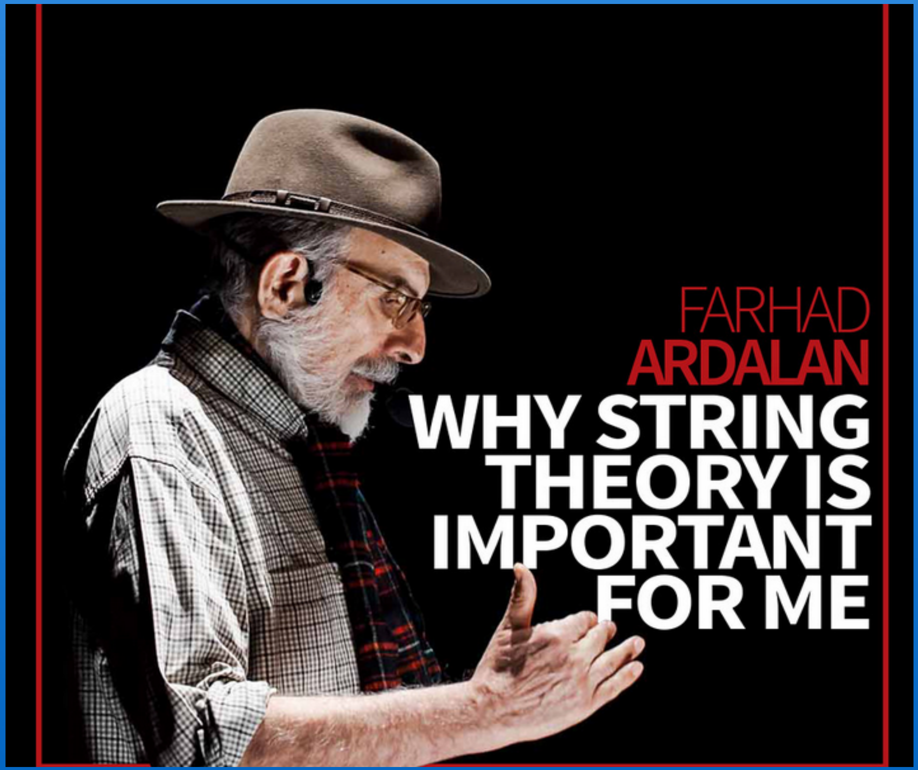









FARHAD
ARDALAN
WHY STRING
THEORY IS
IMPORTANT
FOR ME