

Cutoff function in holographic RG flow

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ADS/CFT Duality

$$Z_B \equiv \int D\varphi_{(z,x)} e^{-s(\varphi)} = \int D\Phi(x) e^{-S(\Phi) + \int dx \varphi_0(x) \mathcal{O}(x)}, \quad (1)$$

$$\langle \varphi(x_1) \dots \varphi(x_n) \rangle = \frac{\delta}{\delta J(x_1)} \dots \frac{\delta}{\delta J(x_n)} Z_J, \quad (2)$$

$$Z_J \equiv \int D\Phi e^{-S(\Phi) + \int J\Phi dx}. \quad (3)$$

Integrating High momenta $p > \Lambda$

K. Wilson

$$Z = \int_{p < \Lambda} D\Phi e^{-S_\Lambda(\Phi)}, \quad (4)$$

Wegner + Houghton 1973

Smooth Cutoff

Wilson 1974

Smooth Cutoff-Almost Integrating out

Wilson 1974

Cutoff Function

Polchinski 1984

$$\wedge \frac{d}{\Phi \wedge} S_{int} = -\frac{1}{2} \int dp \wedge \frac{d}{d \wedge} K \left[\frac{\delta S_{int}}{\delta \varphi(-p)} \frac{\delta S_{int}}{\delta \varphi(p)} + \frac{\delta^2 S_{int}}{\delta \varphi(-p) \delta \varphi(p)} \right], \quad (5)$$

$$S_{\wedge} = -\frac{1}{2} \int \frac{\varphi(-p) \varphi(p)}{K} + S_{int}. \quad (6)$$

Concrete form of "Almost Integration" of WILSON

Alternative Methods for Momentum Cutoff

$$\varphi(p) \longrightarrow h(p)\varphi(p), \quad (7)$$

$$\wedge \frac{d}{d\wedge} S_\wedge = -\frac{1}{2} \int dp \wedge \frac{d}{d\wedge} h \left[\frac{\delta S_\wedge}{\delta \varphi(p)} \frac{\delta S_\wedge}{\delta \varphi(-p)} + \frac{\delta^2 S_\wedge}{\delta \varphi(-p) \delta \varphi(p)} \right] \quad (8)$$

RG flow on field theory side in AdS/CFT corresponds to evolution along the radical direction in the bulk

$$Z_{B,z_0} \equiv \int_{z>z_0} D\varphi(z,x) e^{-s(\varphi)} = \int D\Phi(x) e^{-S_\Lambda(\Phi) + \int \varphi_0(x) \mathcal{O}(x)} \quad (9)$$

$$\Lambda \sim \frac{1}{z_0}. \quad (10)$$

Precise relation between the scalar?

Cutoff function in the bulk?

Percursor

HKLL construction Hamilton, Kabat, Lipshitz, Lowe hep-th/0606141

$$\varphi(z, x) = \int dx' K(z, x \mid x') \mathcal{O}(x') \quad (11)$$

**Use the Precursor map to find the correspondence
between the cutoff functions and scales on the two
sides**

$$\tilde{\mathcal{O}}(p) \longrightarrow h(p)\tilde{\mathcal{O}}(p), \quad (12)$$

$$\tilde{\varphi}_h(p) = \int dp' \tilde{K}(z, p|p') h(p') \tilde{\mathcal{O}}(p'). \quad (13)$$

$$\rho(z)\tilde{\varphi}(p) = \int dp' \tilde{K}(z, p|p') \tilde{\mathcal{O}}_\rho(p), \quad (14)$$

Examples: Higher-spin/O(N)

Koch, Jeviski, Rodrigues, Yoon

arXiv:1408.4800

AdS_4/CFT_3

$$\mathcal{O}(x, y) \equiv \Phi_i(x)\Phi_i(y) \quad (15)$$

Precursors:

$$\begin{aligned} \tilde{\varphi}_s(z, x) = & \int d^4 p \ e^{i(p.x+p^z z)} \int d^2 P_1 d^2 P_2 \delta(p_1^+ + p_2^+ - p^+) \cdot \delta(p_1 + p_2 - p) \\ & \cdot \delta(p_1 \sqrt{\frac{p_2^+}{p_1^+}} - p_2 \sqrt{\frac{p_1^+}{p_2^+}} - p^z) \cdot (\frac{1}{p_1^+} + \frac{1}{p_2^+}) (p_1^+ + p_2^+)^s P_s^{-\frac{1}{2}, -\frac{1}{2}}(\frac{p_2^+ - p_1^+}{p_2^+ + p_1^+}) \cdot \tilde{\mathcal{O}}(p_1, p_1^+; p_2, p_2^+), \end{aligned} \quad (16)$$

of HKLL form:

$$\varphi_s(\tilde{z}, x) = \int_{p^2 < 0} d^3 p e^{ip.x} J_{-\frac{1}{2}}(z \sqrt{-p^2}) \cdot \sqrt{\frac{\pi z}{2}} \sqrt{-p^2} \frac{s!}{\Gamma(s + \frac{1}{2})} \frac{1}{(p^+)^s} \tilde{\mathcal{O}}_s(p), \quad (17)$$

$$\tilde{\mathcal{O}}_s(p) = \int d^2 P_1 d^2 P_2 \left(\frac{1}{p_1^+} + \frac{1}{p_2^+} \right) \delta(p_1^+ + p_2^+ - p^+) \delta(p_1 + p_2 - p). \quad (18)$$

$$(p_1^+ + p_2^+)^s P_s^{-\frac{1}{2}, -\frac{1}{2}}(\frac{p_2^+ - p_1^+}{p_2^+ + p_1^+}) \tilde{\mathcal{O}}(P_1, P_2)$$

Cutting off the bulk field

$$\varphi_s(z, x) \rightarrow \varphi_s^\rho(z, x) \equiv \rho(z)\varphi_s(z, x), \quad (19)$$

Gives cutoff on the boundary operator

$$\tilde{\mathcal{O}}_s(p) \rightarrow (\tilde{\rho} o \tilde{\mathcal{O}}_s)(p), \quad (20)$$

and

$$\tilde{\tilde{\rho}}(r) \tilde{\tilde{\mathcal{O}}}(r), \quad (21)$$

r is the "dual Fourier" component of p, relative coordinate

Another example

AdS_3/CFT_2 , scalar field

$$S = \int dx^2 dz (\partial_\mu \varphi \partial_\mu \varphi + m^2 \varphi^2), \quad (22)$$

Precursor:

$$\varphi(z, x) = cz \int_{p^2 < 0} dp^2 e^{ip.x} \frac{1}{p\nu} J_\gamma(pz) \tilde{\mathcal{O}}(p), \quad \nu = \Delta - 1 \quad (23)$$

$$\int_0^\infty zp J_\nu(pz) J_\nu(p'z) dz = \delta(p - p'), \quad (24)$$

$$\frac{1}{p\nu} \tilde{\mathcal{O}}_\rho(p) = \int_0^\infty dp' I_\rho(p, p') \frac{1}{p'^\nu} \tilde{\mathcal{O}}(p'), \quad (25)$$

AdS/CFT equivalence conjecture:

$$Z_\rho(\varphi_0) = e^{-W_\rho(\varphi_0)}, \quad (26)$$

$$Z_\rho(\varphi_0) = \int D\varphi(z, x) e^{-s(\rho\varphi)}, \quad (27)$$

$$e^{-W_\rho(\varphi_0)} = \int D\Phi(x) e^{-S(\Phi_\rho) + \int \varphi_0 \rho \Phi_\rho}. \quad (28)$$

where

$$\varphi_{0,\rho} = I_\rho \varphi_0$$

$$\varphi_0(x) = \varphi(z = z_0, x), \quad (29)$$

z_0 the inflection point of ρ .

$$\Phi_\rho(p) = \int dp' I_\rho(p, p') \Phi(p'), \quad (30)$$

or, generally,

$$\Phi_\rho \equiv I_\rho \Phi, \quad (31)$$

Varying z_0 :

In the bulk:

$$\dot{Z}_\rho(\varphi_0) = \mathcal{H}(\varphi_0, \frac{\partial}{\partial \varphi_0}) Z_\rho(\varphi_0). \quad (32)$$

\mathcal{H} is the "Hamiltonian" derived from the bulk action s .

On the boundary

$$(e^{-W_\rho(\varphi_0)}) = \int D\Phi [-\frac{\partial}{\partial z_0} S(\frac{\partial}{\partial \varphi_0}) - \dot{I}_\rho \Phi S'(\Phi_\rho) + \varphi_0 (I_\rho^2) \dot{\Phi}] e^{-S(\Phi_\rho) + \varphi_0 \rho \Phi_\rho} \\ = \int dp [-\frac{\partial}{\partial z_0} S(\frac{\partial}{\partial \varphi_0}) - \dot{I}_\rho \frac{\partial}{\partial \varphi_0} S'(\frac{\partial}{\partial \varphi_0}) + 2\varphi_0 \dot{I}_\rho \frac{\partial}{\partial \varphi_0}] e^{-W_\rho(\varphi_0)}, \quad (33)$$

$$\frac{\partial}{\partial z_0} S = -\frac{1}{2} \int dp [\frac{\partial}{\partial \varphi} S \dot{I}_\rho \frac{\partial}{\partial \varphi} S + \frac{\partial}{\partial \varphi} \dot{I}_\rho \frac{\partial}{\partial \varphi} S]. \quad (34)$$

under the path integral:

$$\mathcal{H}(\varphi_0, \frac{\partial}{\partial \varphi_0}) = -\frac{\partial}{\partial z_0} S(\frac{\partial}{\partial \varphi_0}) - \dot{I}_\rho \frac{\partial}{\partial \varphi_0} S'(\frac{\partial}{\partial \varphi_0}) + 2\varphi_0 \dot{I}_\rho \frac{\partial}{\partial \varphi_0}. \quad (35)$$

$T\bar{T}$ deformation of CFT

Zamalachikov,

Hartman, Kruthoff, Shagholian, Tajdini (HKST)

$$\frac{d}{dr_c} = -\mathcal{H}(\Phi, \mathcal{O}) \quad (36)$$

Compare to Eq. 35
only one direction in the field space.