Instability of BTZ black holes in parity even massive gravity

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Motivation

-3D Einstein gravity has no bulk propagating dof

-TMG, NMG and 3D-BI allow massive graviton excitations

-From AdS/CFT point of view 3D-BI gravity can be used to study intermediate coupling behaviour

-What is the QNM structure for BTZ black holes in 3D-BI theory?

Outline

Introduction to 3D-BI gravity

Linearized equations of motion

Analytical analysis (review)

Numerical studies

- Insight to new analytical results

Summary and outlook

BI-3D gravity

$$I_{\rm BI} = -\frac{4m^2}{\kappa_3^2}\int d^3x\sqrt{-\det g}\left[\sqrt{\det(\mathbf{1}+\tfrac{\sigma}{m^2}g^{-1}G)}-\lambda\right]$$

Gullu, Sisman, Tekin, '10

$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ $\quad m^2 \geq 0$ $\quad \sigma = \pm 1$

Expansion in $1/m^2$:

$$I_{\rm NMG} = -\frac{4m^2}{\kappa^2}\int d^3x\sqrt{-\det g}\Big[\underbrace{(1-\lambda) - \frac{\sigma}{4m^2}R - \frac{1}{32m^4}(8R_{\alpha\beta}R^{\alpha\beta} - 3R^2)}_{\text{NMG}} + O(R^3)\Big]$$

+extensions

Effective cosmological constant: $\Lambda = 2\sigma m^2(1-\lambda)$

BI has a single vacuum solution while truncated models may have several vacua!

BI-3D gravity

Equations of motion:

$$\frac{\sigma}{2m^2} P_{\mu\nu\alpha\beta}\mathcal{V}^{\alpha\beta} + g_{\mu\nu}\left[\sqrt{\det(\mathbf{1} + \frac{\sigma}{m^2} g^{-1}G)} - \lambda\right] = 0$$

$$P_{\mu\nu\alpha\beta} := g_{\mu\nu}R_{\alpha\beta} - g_{\alpha\mu}g_{\beta\nu}(R+\Box) + g_{\beta\mu}\nabla_\alpha\nabla_\nu + g_{\beta\nu}\nabla_\alpha\nabla_\mu - g_{\mu\nu}\nabla_\alpha\nabla_\beta + g_{\mu\nu}g_{\alpha\beta}\Box - g_{\alpha\beta}\nabla_\mu\nabla_\nu$$

$$\mathcal{V}^\mu{}_\nu := \sqrt{\det(\mathbf{1} + \tfrac{\sigma}{m^2} g^{-1}G)}\left(\frac{1}{\mathbf{1} + \frac{\sigma}{m^2} g^{-1}G}\right)^\mu{}_\nu$$

Locally AdS spacetime: $$R_{\alpha\beta\mu\nu} = -\frac{1}{\ell^2}(g_{\alpha\mu}g_{\beta\nu} - g_{\beta\mu}g_{\alpha\nu})$$

$$\lambda = \sqrt{1 + \tfrac{\sigma}{m^2\ell^2}} = \underbrace{1 + \tfrac{\sigma}{2\ell^2 m^2}}_{\text{NMG}} - \tfrac{1}{8\ell^4 m^4} + \mathcal{O}(1/m^6)$$

Linearized equations of motion

$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ + TT-gauge: $\nabla_\mu h^{\mu\nu} = h = 0$ + tedious massage

$$\frac{1}{\sqrt{1+\frac{\sigma}{\ell^2 m^2}}}\Big[-\tfrac{2}{\ell^2}(1+3\tfrac{\sigma}{\ell^2 m^2})h_{\mu\nu} - (1+\tfrac{5\sigma}{\ell^2 m^2})\Box h_{\mu\nu} - \tfrac{\sigma}{m^2}\Box^2 h_{\mu\nu}\Big]$$

$$= \boxed{\frac{1}{\sqrt{1+\frac{\sigma}{\ell^2 m^2}}}}(\Box + \sigma m^2 + \tfrac{3}{\ell^2})(\Box + \tfrac{2}{\ell^2})h_{\mu\nu} = 0$$

Massless mode

Massive mode: $\mathbf{m}^2 = -\sigma m^2(1+\frac{\sigma}{m^2\ell^2}) = -\sigma m^2 \lambda^2$

To find the mass for NMG and its generalization one has to start from the linearized equation and the pre-factor plays a role!

Linearized equations of motion

NMG: $$(\Box + \sigma m^2 + \tfrac{5}{2\ell^2} + \mathcal{O}(\tfrac{1}{m^2}))(\Box + \tfrac{2}{\ell^2})h_{\mu\nu} = 0$$

$$\mathbf{m}^2_{\rm NMG} = -\sigma m^2(1 + \tfrac{\sigma}{2m^2\ell^2})$$

NMG$^{(2)}$: $$\left(\Box + \sigma m^2 + \frac{3}{\ell^2} + \frac{3\sigma}{8m^2\ell^4} + \mathcal{O}(\frac{1}{m^4})\right)(\Box + \frac{2}{\ell^2})h_{\mu\nu} = 0$$

$$\mathbf{m}^2_{{\rm NMG}^{(2)}} = -\sigma m^2(1 + \tfrac{\sigma}{m^2\ell^2} + \tfrac{3}{8m^4\ell^4})$$

NMG$^{(n)}$: $$\mathbf{m}^2_{{\rm NMG}^{(n)}} = -\sigma m^2\left(1 + \frac{\sigma}{m^2\ell^2} + \frac{c_n}{(\ell^2 m^2)^n}\right) \qquad |c_n| < 1$$

-Any truncation leads to only 3 terms in the mass expansion

-For m l>1 and n→ infinity we recover the BI-3D with only 2 terms

In all cases (parity even theories)
the linearized massive mode is given by

$$(\Box + a)h_{\mu\nu} = 0$$

$$\downarrow$$

$$a(\mathbf{m}^2, \ell^2)$$

Analytical analysis

One can decompose the massive mode equation in TT-gauge as

$$\epsilon_\alpha{}^{\mu\nu}\nabla_\nu h_{\mu\beta} \pm \frac{M}{\ell}\, h_{\alpha\beta} = 0 \qquad M = \sqrt{3 - a\,\ell^2} \qquad M^2_{\mathrm{NMG}^{(n)}} = -\sigma m^2\left(1 + \frac{c_n}{(\ell m)^{2n}}\right)$$

Using the following form of the BTZ metric

$$ds^2 = \ell^2\left(-\sinh^2\rho dt^2 + \cosh^2\rho d\phi^2 + d\rho^2\right)$$

Left/Right moving QNMs:

$$h^r_{\mu\nu} = e^{-ik(t+\phi)}e^{-2h_r t}(\sinh\rho)^{-2h_r}(\tan\rho)^{-ik}\begin{pmatrix} 1 & 0 & \frac{2}{\sinh 2\rho} \\ 0 & 0 & 0 \\ \frac{2}{\sinh 2\rho} & 0 & \frac{4}{\sinh^2 2\rho}\end{pmatrix}, \quad h_r = \frac{1}{2}(\mp M - 1)$$

$$h^l_{\mu\nu} = e^{-ik(t+\phi)}e^{-2h_l t}(\sinh\rho)^{-2h_l}(\tan\rho)^{ik}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & \frac{2}{\sinh 2\rho} \\ 0 & \frac{2}{\sinh 2\rho} & \frac{4}{\sinh^2 2\rho}\end{pmatrix}, \quad h_l = \frac{1}{2}(\pm M - 1)$$

$$h^{(n)}_{\mu\nu} = (L_{-1}\bar{L}_{-1})^n h_{\mu\nu} \quad \rightarrow \quad \omega_n = -i(M - 1 + 2n) \pm k$$

$M \geq 0$ → stable frequencies

Numerical calculation

Tackle the 2nd order differential equation

$$(\Box + a)h_{\mu\nu} = 0 \qquad h_{\mu\nu}(r,t,\phi) = \mathcal{Z}_{\mu\nu}(r)e^{-i\frac{\omega}{r_0}t + i\frac{k}{r_0}\phi}$$

Using the Eddington-Finkelstein form of the BTZ metric

$$ds^2 = \frac{\ell^2}{r^2}\left[-(1-\frac{r^2}{r_0^2})dt^2 - 2dtdr + d\phi^2\right]$$

k=0 → two sets of perturbations $(h_{t\phi}, h_{r\phi})$ $(h_{rr}, h_{tr}, h_{tt}, h_{\phi\phi})$

TT-gauge → two decoupled equations for $\mathcal{Z}_1 = h_{r\phi}, \quad \mathcal{Z}_2 = h_{\phi\phi}$

Dirichlet b.c. & Ingoing b.c. → QNMs

Numerical calculation

Tackle the 2nd order differential equation

$$u := r/r_0$$

$$\left[(M-3)(M+1)(M-1)^2 u - iM\omega\left(-2M^2+3(M-3)u^2+3M-1\right)+3(2M-1)u\omega^2\right]\mathcal{Z}_1(u)$$
$$+\left[2iu\omega\left(-M^2+3(M-2)u^2-3M+1\right)+\left(1-M^2\right)\left((2M-5)u^2-2M+1\right)-6u^2\omega^2\right]\mathcal{Z}_1'(u)$$
$$+\left[\left(M^2-1\right)u\left(u^2-1\right)-3iu^2\left(u^2-1\right)\omega\right]\mathcal{Z}_1''(u)=0$$

$$\Big[\left(M^2-1\right)^2 u\left(u^2-(M-2)M\right)+\omega^2\left(-M^2\left(u^2+2\right)u+2Mu+u^3\right)$$
$$+i\left(M^2-1\right)\omega\left(-2M^3+M^2+2Mu^2+u^2\right)-i(2M+1)u^2\omega^3\Big]\mathcal{Z}_2(u)+\Big[2i\left(M^2-1\right)u\omega\left(M^2-u^2\right)$$
$$+\left(M^2-1\right)\left(-2M^3+M^2+\left(2M^3-3M^2+2M+1\right)u^2+(1-2M)u^4\right)+2iu^3\omega^3$$
$$+u^2\omega^2\left((2M-1)u^2-2M-1\right)\Big]\mathcal{Z}_2'(u)+\left[\left(M^2-1\right)u\left(u^2-1\right)\left(u^2-M^2\right)+\left(u^3-u^5\right)\omega^2\right]\mathcal{Z}_2''(u)=0$$

Numerical calculation (k=0)

QNMs Chebyshev polynomials (with N~ 250, precision~1000)

$$\mathcal{Z}_1 \rightarrow \omega_n^{(1,1)} = -i\,(M-1+2\,n)\ , \qquad \omega_n^{(1,2)} = -i\,(M+3+2\,n)\ , \qquad n=0,1,2,\cdots$$

$$\mathcal{Z}_2 \rightarrow \omega_n^{(2,1)} = -i\,(M-1+2\,n)\ , \qquad \omega_n^{(2,2)} = -i\,(M+3+2\,n)\ , \qquad n=0,1,2,\cdots\ ,$$

$$\boxed{\omega^{(2,3)} = -i\,\left(1-M^2\right)}$$

The first two sets in Z_2 channel coincide with the two sets of Z_1 channel and, the first set is in perfect agreement with the analytical expressions

Except the first two modes in $\omega^{(i,1)}$ set there is a degeneracy in other modes between first and second sets namely ,$\omega^{(i,1)}=\omega^{(i,2)}$ for n=0,1,2,⋯

For $0 < M < 1$, the zeroth mode in the first family, $\omega^{(i,1)}$, is unstable and all other modes are stable.
For $M > 1$, the single mode in the third family $\omega^{(2,3)}$ is unstable and all other modes are stable.

The results are TOO accurate!!!!

Numerical calculation (k=0)

2nd QNM equation simplifies for single mode $\omega = -i\,(1-M^2)$

$$\mathcal{Z}''(u) + \frac{\left(2M^2u-2M(u^2-1)+(u-2)u-1\right)}{u(u^2-1)}Z'(u) + \frac{(M^2-1)\left(-2M(u+1)+u^2+1\right)}{(u-1)u(u+1)^2}Z(u) = 0$$

This has analytical solutions (with ingoing bc):

$$\mathcal{Z}(u) = u^{2M}(u+1)^{M^2-1}\,{}_2F_1\left(\frac{M(M+1)}{2}, \frac{(M^2+M-2)}{2}; M+1; u^2\right)$$

For other modes one can also find the analytic solutions after plugging the modes in the QNM equations.

Numerical calculation (k>0)

Perturbations are coupled even in TT-gauge

$$\omega_n^{(1)}(k) = -i\,(M-1+2\,n) \pm k\ , \qquad \omega_n^{(2)} = -i\,(M+3+2\,n) \pm k\ , \qquad n = 0, 1, 2, \cdots\ ,$$
$$\omega^{(3)} = -i\,\left(1 - M^2 + (\pm k)^2\right)\ .$$

The third set has completely different dependency not only in the parameter M but also in angular momentum k such that they are purely imaginary for any value of the parameters in the domain of our interest.

If $1 \leq M \leq \sqrt{1+(\pm k)^2}$ all QNMs are stable!!!

-Maybe rotating BTZ is stable for k=0!!

Summary and outlook

-Non-rotating BTZ is not stable in 3D parity even massive gravity theory
(regardless of bulk-boundary clash)

-There is a new SINGLE QNM which is not in a Christmas tree

-This unstable BTZ decay to ....?

-What about rotating BTZ?

-Is this unique for 3D massive gravity (parity even)

-How can we understand this in first order formulation?

Thank you for your attention

BI-3D gravity

Boundary theory:

Wald formula: $$c = \frac{\ell}{2G_3} g_{\mu\nu} \frac{\partial \mathcal{L}}{\partial R_{\mu\nu}} = \frac{3\ell}{2G_3} \sigma\lambda$$

Unitarity of the boundary theory → $\sigma = 1$ & $\lambda > 1$

Hamilton-Jacobi analysis:

Brown-York stress tensor: $$T^{ab} = \sigma\lambda(K^{ab} - \hat{h}^{ab} K)$$

Extrinsic curvature Boundary metric

→ Expansion in $1/m^2$ agrees with NMG and its extension.

Linearized equations of motion

$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ + TT-gauge: $\nabla_\mu h^{\mu\nu} = h = 0$ + tedious massage

$$\frac{1}{\sqrt{1+\frac{\sigma}{\ell^2 m^2}}}\Big[-\tfrac{2}{\ell^2}(1+3\tfrac{\sigma}{\ell^2 m^2})h_{\mu\nu} - (1+\tfrac{5\sigma}{\ell^2 m^2})\Box h_{\mu\nu} - \tfrac{\sigma}{m^2}\Box^2 h_{\mu\nu}\Big]$$

$$= \frac{1}{\sqrt{1+\frac{\sigma}{\ell^2 m^2}}}(\Box + \sigma m^2 + \tfrac{3}{\ell^2})(\Box + \tfrac{2}{\ell^2})h_{\mu\nu} = 0$$

Massless mode

Massive mode: $\mathbf{m}^2 = -\sigma m^2(1+\frac{\sigma}{m^2\ell^2}) = -\sigma m^2 \lambda^2$? $m^2_{\rm BF} = -1/\ell^2$

$\sigma = -1$: $\mathbf{m}^2 > m^2_{\rm BF}$ nonunitary bdry theory

$\sigma = 1$: $\mathbf{m}^2 < m^2_{\rm BF}$ unitary bdry theory

} Bulk-boundary clash