Solving Holographic Defects

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NCSR "Demokritos"

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Section 1

Introduction

Conformal field theory

- A well-known result in CFT is that the form of 2 and 3-point functions of quasi-primary scalars is completely determined by conformal symmetry, while 1-point functions are zero:

$$\langle \phi_1(x_1) \rangle = 0 \qquad (\text{except } \langle c \rangle = c)$$

$$\langle \phi_1(x_1)\, \phi_2(x_2) \rangle = \frac{C_{12}}{x_{12}^{2\Delta}}, \quad \Delta \equiv \Delta_1 = \Delta_2, \quad x_{12} \equiv |x_1 - x_2|$$

$$\langle \phi_1(x_1)\, \phi_2(x_2)\, \phi_3(x_3) \rangle = \frac{C_{123}}{x_{12}^{\Delta_1+\Delta_2-\Delta_3}\, x_{23}^{\Delta_2+\Delta_3-\Delta_1}\, x_{31}^{\Delta_3+\Delta_1-\Delta_2}},$$

- If we have more than 3 points we may construct conformally invariant cross ratios, as e.g. in the case of 4 points:

$$\frac{x_{12}x_{34}}{x_{13}x_{24}} \quad \& \quad \frac{x_{12}x_{34}}{x_{14}x_{23}}.$$

- The corresponding n-point function ($n \geq 4$) has an arbitrary dependence on them, e.g. for $n = 4$:

$$\langle \phi_1(x_1)\, \phi_2(x_2)\, \phi_3(x_3)\, \phi_4(x_4) \rangle = f\left(\frac{x_{12}x_{34}}{x_{13}x_{24}}, \frac{x_{12}x_{34}}{x_{14}x_{23}}\right) \cdot \prod_{i<j}^{4} x_{ij}^{\Delta/3-\Delta_i-\Delta_j}, \qquad \Delta \equiv \sum_{i=1}^{4} \Delta_i.$$

Operator product expansion (OPE)

- Generally, we don't need a Lagrangian to define a CFT. A CFT is defined by its local operators and their n-point correlation functions:

$$\{\mathcal{O}_k(x)\} \qquad \langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\ldots\mathcal{O}_n(x_n)\rangle\,.$$

- The latter can be determined by using the operator product expansion (OPE). E.g. for scalars:

$$\phi_1(x_1)\,\phi_2(x_2) = \sum_k \frac{C_{12k}}{C_{kk}}\cdot\mathcal{P}_k(x_{12},\partial_2)\,\phi_k(x_2)\,,$$

where the sum is over all the primary operators of the CFT.

- In general, the $(n+2)$-point function can be computed recursively:

$$\Big\langle \phi_1(x_1)\,\phi_2(x_2)\prod_{i=3}^{n}\phi_i(x_i)\Big\rangle = \sum_k \frac{C_{12k}}{C_{kk}}\cdot\mathcal{P}_k(x_{12},\partial_2)\Big\langle \phi_k(x_2)\prod_{i=3}^{n}\phi_i(x_i)\Big\rangle.$$

- The CFT is fully specified by the CFT data: $\{\Delta_k, \ell_k, f_k, C_{ij}, C_{ijk}\}$.

Defect conformal field theory (dCFT)

Now consider a CFT_d and introduce a boundary at $z = 0$, where $x_\mu = (z, \mathbf{x})$ (Cardy, 1984).

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The subgroup of the d-dimensional (Euclidean) conformal group $SO(d+1,1)$ that leaves the plane $z = 0$ invariant contains:

- $(d-1)$ dimensional translations: $\mathbf{x}' = \mathbf{x} + \mathbf{a}$
- $(d-1)$ dimensional rotations $SO(d-1)$
- d dimensional rescalings $x'_\mu = \alpha\, x_\mu$ & inversions $x'_\mu = x_\mu / x^2$

That is the conformal group in $d-1$ dimensions, $SO(d,1)$.

The resulting setup that contains a CFT_d and a codimension 1 boundary/interface/domain wall/defect upon which a CFT_{d-1} lives, is known as a defect Conformal Field Theory (dCFT) .

dCFT correlators: bulk

Due to the presence of the $z = 0$ boundary we may form cross ratios from only 2 bulk points:

$$\xi = \frac{x_{12}^2}{4\left|z_1\right|\left|z_2\right|} \qquad \& \qquad v^2 = \frac{\xi}{\xi+1} = \frac{x_{12}^2}{x_{12}^2 + 4\left|z_1\right|\left|z_2\right|}$$

This means that 1-point bulk functions are nonzero and the only ones fully determined by symmetry:

$$\langle\phi\left(z,\mathbf{x}\right)\rangle = \frac{C}{\left|z\right|^{\Delta}}$$

n-point bulk functions $(n \geq 2)$ will contain an arbitrary dependence on the cross ratio ξ. E.g. the 2-point bulk function of two scalars will be:

$$\langle\phi_1\left(z_1,\mathbf{x}_1\right)\phi_2\left(z_2,\mathbf{x}_2\right)\rangle = \frac{f_{12}\left(\xi\right)}{\left|z_1\right|^{\Delta_1}\left|z_2\right|^{\Delta_2}},$$

McAvity-Osborn, 1995

i.e. it will not vanish if $\Delta_1 \neq \Delta_2$.

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- 1-point functions are fundamental building blocks of dCFTs (along with the CFT data).

Subsection 2

Holography and dCFTs

Holographic dCFTs

Holographic dCFTs can be realized in the context of the $\text{AdS}_5/\text{CFT}_4$ correspondence:

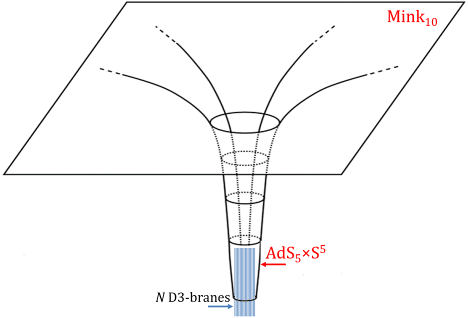
$$\left\{ \text{Type IIB String Theory in } \text{AdS}_5 \times \text{S}^5 \right\} \longleftrightarrow \left\{ \mathcal{N} = 4,\ \mathfrak{su}(N) \text{ Super Yang-Mills Theory in 4d} \right\}$$

Maldacena, 1998

as shown by Karch and Randall in 2001, in an attempt to provide an explicit realization of gravity localization on an AdS_4 brane (Karch-Randall, 2001a).

The D3-D5 system: bulk geometry

IIB string theory on $AdS_5 \times S^5$ is encountered very close to a system of N coincident D3-branes:



The D3-branes extend along x_1, x_2, x_3...

	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	•	•	•	•						

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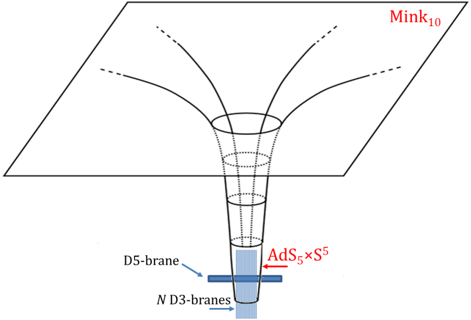

Now insert a single (probe) D5-brane at $x_3 = x_7 = x_8 = x_9 = 0$...

	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	•	•	•	•						
D5	•	•	•		•	•	•			

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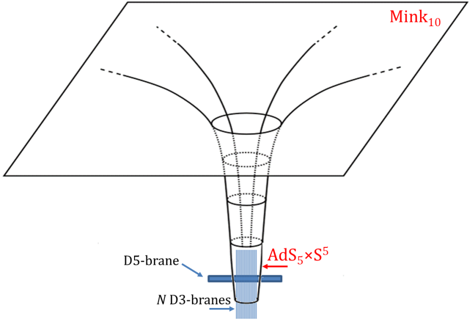
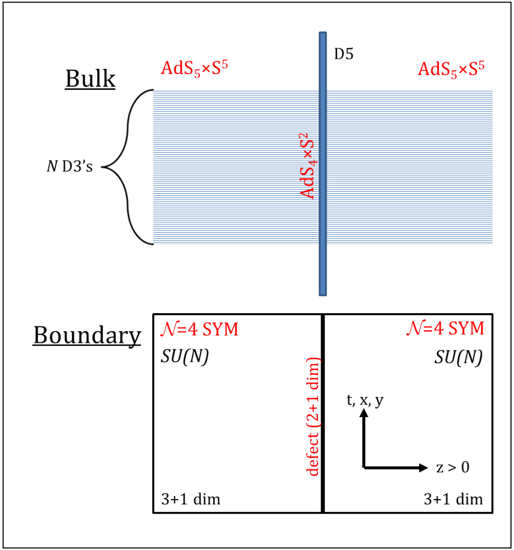


Now insert a single (probe) D5-brane at $x_3 = x_7 = x_8 = x_9 = 0$... its geometry will be $\mathrm{AdS}_4 \times \mathrm{S}^2$...

	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	•	•	•	•						
D5	•	•	•		•	•	•			

The D3-D5 system: description



- In the bulk, the D3-D5 system describes IIB string theory on $AdS_5 \times S^5$ bisected by a D5 brane with worldvolume geometry $AdS_4 \times S^2$.
- The dual field theory is still $SU(N)$, $\mathcal{N} = 4$ SYM in $3+1$ dimensions, that interacts with a CFT living on the $2+1$ dimensional defect:
$$S = S_{\mathcal{N}=4} + S_{2+1}.$$
DeWolfe-Freedman-Ooguri, 2001
- Due to the presence of the defect, the total bosonic symmetry of the system is reduced from $SO(4,2) \times SO(6)$ to $SO(3,2) \times SO(3) \times SO(3)$.
- The corresponding superalgebra $\mathfrak{psu}(2,2|4)$ becomes $\mathfrak{osp}(4|4)$.

Section 2

One-point Functions in the D3-D5 System

Subsection 1

The (D3-D5)$_k$ system

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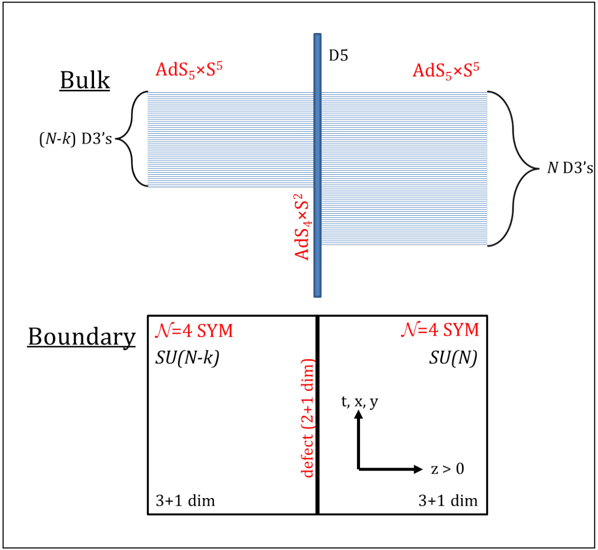
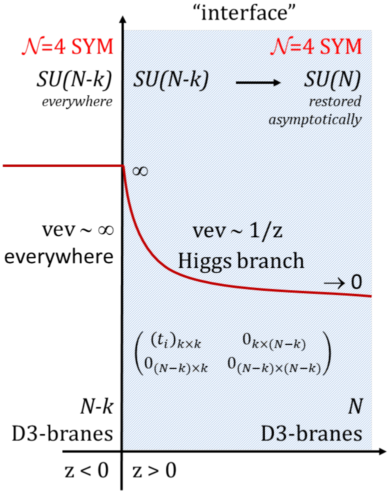


- Add k units of background $U(1)$ flux on the S^2 component of the $AdS_4 \times S^2$ D5-brane.
- Then k of the N D3-branes ($N \gg k$) will end on the D5-brane.
- On the dual SCFT side, the gauge group $SU(N) \times SU(N)$ breaks to $SU(N-k) \times SU(N)$.
- Equivalently, the fields of $\mathcal{N} = 4$ SYM develop nonzero vevs...

(Karch-Randall, 2001b)

The dCFT interface of D3-D5



- An interface is a wall between two (different/same) QFTs
- It can be described by means of classical solutions that are known as "fuzzy-funnel" solutions (Constable-Myers-Tafjord, 1999 & 2001)
- Here, we need an interface to separate the $SU(N)$ and $SU(N-k)$ regions of the $(\text{D3-D5})_k$ dCFT...
- For no vectors/fermions, we want to solve the equations of motion for the scalar fields of $\mathcal{N}=4$ SYM:

$$A_\mu = \psi_a = 0, \qquad \frac{d^2\Phi_i}{dz^2} = [\Phi_j, [\Phi_j, \Phi_i]], \quad i, j = 1, \ldots, 6.$$

- A manifestly $SO(3) \simeq SU(2)$ symmetric solution is given by $(z > 0)$:

$$\Phi_{2i-1}(z) = \frac{1}{z}\begin{bmatrix} (t_i)_{k\times k} & 0_{k\times(N-k)} \\ 0_{(N-k)\times k} & 0_{(N-k)\times(N-k)} \end{bmatrix} \quad \& \quad \Phi_{2i} = 0,$$

Nagasaki-Yamaguchi, 2012

where the matrices t_i furnish a k-dimensional representation of $\mathfrak{su}(2)$:

$$[t_i, t_j] = i\epsilon_{ijk} t_k.$$

1-point functions

Following Nagasaki & Yamaguchi (2012), the 1-point functions of local gauge-invariant scalar operators

$$\langle \mathcal{O}(z,\mathbf{x})\rangle = \frac{C}{z^{\Delta}}, \qquad z>0,$$

can be calculated within the D3-D5 dCFT from the corresponding fuzzy-funnel solution, for example:

$$\boxed{\mathcal{O}(z,\mathbf{x}) = \Psi^{i_1\ldots i_L}\,\mathrm{Tr}\left[\Phi_{2i_1-1}\ldots\Phi_{2i_L-1}\right] \xrightarrow[\text{interface}]{SU(2)} \frac{1}{z^L}\cdot\Psi^{i_1\ldots i_L}\,\mathrm{Tr}\left[t_{i_1}\ldots t_{i_L}\right]}$$

where $\Psi^{i_1\cdots i_L}$ is an $\mathfrak{so}(6)$-symmetric tensor and the constant C is given by (MPS=*matrix product state*)

$$C = \frac{1}{\sqrt{L}}\left(\frac{8\pi^2}{\lambda}\right)^{L/2}\cdot\frac{\langle \mathrm{MPS}|\Psi\rangle}{\langle\Psi|\Psi\rangle^{\frac{1}{2}}}, \qquad \left\{\begin{array}{ll}\langle \mathrm{MPS}|\Psi\rangle \equiv \Psi^{i_1\ldots i_L}\,\mathrm{Tr}\left[t_{i_1}\ldots t_{i_L}\right] & (\text{"overlap"})\\ \langle\Psi|\Psi\rangle \equiv \Psi^{i_1\ldots i_L}\Psi_{i_1\ldots i_L} & \end{array}\right\},$$

which ensures that the 2-point function will be normalized to unity ($\mathcal{O}\rightarrow(2\pi)^L\cdot\mathcal{O}/(\lambda^{L/2}\sqrt{L})$)

$$\langle \mathcal{O}(x_1)\,\mathcal{O}(x_2)\rangle = \frac{1}{|x_1-x_2|^{2\Delta}}$$

within $SU(N)$, $\mathcal{N}=4$ SYM (i.e. without the defect).

Bethe eigenstates

We will only consider the 1-point functions of Bethe eigenstates $|\Psi\rangle$ of the integrable $\mathfrak{so}(6)$ spin chain:

$$\mathbb{D} = L \cdot \mathbb{I} + \frac{\lambda}{8\pi^2} \cdot \mathbb{H} + \sum_{n=2}^{\infty} \lambda^n \cdot \mathbb{D}_n, \qquad \mathbb{H} = \sum_{j=1}^{L} \left(\mathbb{I}_{j,j+1} - \mathbb{P}_{j,j+1} + \frac{1}{2}\,\mathbb{K}_{j,j+1}\right), \qquad \lambda = g_{\text{YM}}^2 N,$$

Minahan-Zarembo, 2002

which describes the mixing of single-trace operators $\mathcal{O}(x)$ up to one loop in $\mathcal{N} = 4$ SYM. We've set:

$$\mathbb{I} \cdot |\ldots \Phi_a \Phi_b \ldots\rangle = |\ldots \Phi_a \Phi_b \ldots\rangle$$

$$\mathbb{P} \cdot |\ldots \Phi_a \Phi_b \ldots\rangle = |\ldots \Phi_b \Phi_a \ldots\rangle$$

$$\mathbb{K} \cdot |\ldots \Phi_a \Phi_b \ldots\rangle = \delta_{ab} \sum_{c=1}^{6} |\ldots \Phi_c \Phi_c \ldots\rangle .$$

The above result is unaffected by the presence of a defect in the SCFT (DeWolfe-Mann, 2004).

Subsection 2

Determinant formulas

M. de Leeuw, C. Kristjansen, G. Linardopoulos, *Scalar One-point functions and matrix product states of AdS/dCFT*, Phys.Lett. **B781** (2018) 238 [arXiv:1802.01598]

1-point functions in $\mathfrak{su}(2)$

In the $\mathfrak{su}(2)$ sector our goal is to calculate the one-point function coefficient:

$$C = \frac{1}{\sqrt{L}} \left(\frac{8\pi^2}{\lambda}\right)^{L/2} \cdot \frac{\langle \mathrm{MPS}|\mathbf{p}\rangle}{\langle \mathbf{p}|\mathbf{p}\rangle^{\frac{1}{2}}}, \qquad k \ll N \to \infty.$$

where the $k \times k$ matrices $t_{1,3}$ form a k-dimensional representation of $\mathfrak{su}(2)$:

$$\langle \mathrm{MPS}|\mathbf{p}\rangle = \mathfrak{N} \cdot \sum_{\sigma \in S_M} \sum_{1 \le x_k \le L} \exp\left[i \sum_k p_{\sigma(k)} x_k + \frac{i}{2} \sum_{j<k} \theta_{\sigma(j)\sigma(k)}\right] \cdot \mathrm{Tr}\left[t_3^{x_1-1} t_1 t_3^{x_2-x_1-1} \ldots\right].$$

<u>Overlap properties</u>:

- The overlap $\langle \mathrm{MPS}|\mathbf{p}\rangle$ vanishes if $M \equiv N_1$ or L is odd: $\mathrm{Tr}\left[t_3^{x_1-1} t_1 t_3^{x_2-x_1-1} \ldots\right]\Big|_{M \text{ or } L \text{ odd}} = 0$
- The overlap $\langle \mathrm{MPS}|\mathbf{p}\rangle$ vanishes if $\sum p_i \neq 0$: due to trace cyclicity
- The overlap $\langle \mathrm{MPS}|\mathbf{p}\rangle$ vanishes if momenta are not fully balanced $(p_i, -p_i)$: due to $Q_3 \cdot |\mathrm{MPS}\rangle = 0$

de Leeuw-Kristjansen-Zarembo, 2015

The $\mathfrak{su}(2)$ determinant formula

Vacuum overlap:

$$\langle \text{MPS}|0\rangle = \text{Tr}\left[t_3^L\right] = \zeta\left(-L, \frac{1-k}{2}\right) - \zeta\left(-L, \frac{1+k}{2}\right), \qquad \zeta(s, a) \equiv \sum_{n=0}^{\infty} \frac{1}{(n+a)^s},$$

where $\zeta(s, a)$ is the Hurwitz zeta function. For M balanced excitations the overlap becomes:

$$\boxed{C_k(\{u_j\}) \equiv \frac{\langle \text{MPS}|\{u_j\}\rangle_k}{\sqrt{\langle\{u_j\}|\{u_j\}\rangle}} = C_2(\{u_j\}) \cdot \sum_{j=(1-k)/2}^{(k-1)/2} j^L \left[\prod_{l=1}^{M/2} \frac{u_l^2\left(u_l^2 + k^2/4\right)}{\left[u_l^2 + (j-1/2)^2\right]\left[u_l^2 + (j+1/2)^2\right]}\right]}$$

where

$$C_2(\{u_j\}) \equiv \frac{\langle \text{MPS}|\{u_j\}\rangle_{k=2}}{\sqrt{\langle\{u_j\}|\{u_j\}\rangle}} = \left[\prod_{j=1}^{M/2} \frac{u_j^2 + 1/4}{u_j^2} \frac{\det G^+}{\det G^-}\right]^{\frac{1}{2}},$$

and the $M/2 \times M/2$ matrices $G_{jk}^{\pm}$ and $K_{jk}^{\pm}$ are defined as:

$$G_{jk}^{\pm} = \left(\frac{L}{u_j^2 + 1/4} - \sum_n K_{jn}^{+}\right)\delta_{jk} + K_{jk}^{\pm} \qquad \& \qquad K_{jk}^{\pm} = \frac{2}{1+(u_j-u_k)^2} \pm \frac{2}{1+(u_j+u_k)^2}.$$

Buhl-Mortensen, de Leeuw, Kristjansen, Zarembo, 2015

The $\mathfrak{su}(3)$ determinant formula

Moving to the $\mathfrak{su}(3)$ sector, let us define the following Baxter functions Q and R :

$$Q_1(x) = \prod_{i=1}^{M} (x - u_i)\,, \qquad Q_2(x) = \prod_{i=1}^{N_+} (x - v_i)\,, \qquad R_2(x) = \prod_{i=1}^{2\lfloor N_+/2 \rfloor} (x - v_i)\,.$$

All the one-point functions in the $\mathfrak{su}(3)$ sector are then given by

$$\boxed{C_k(\{u_j; v_j\}) = T_{k-1}(0) \cdot \sqrt{\frac{Q_1(0)\, Q_1(i/2)}{R_2(0)\, R_2(i/2)} \cdot \frac{\det G^+}{\det G^-}}}$$

de Leeuw-Kristjansen-GL, 2018

where $u_i \equiv u_{1,i}$, $v_j \equiv u_{2,j}$ and

$$T_n(x) = \sum_{a=-n/2}^{n/2} (x + ia)^L \frac{Q_1(x + i(n+1)/2)Q_2(x + ia)}{Q_1(x + i(a + 1/2))Q_1(x + i(a - 1/2))}.$$

The validity of the $\mathfrak{su}(3)$ formula has been checked numerically for a plethora of $\mathfrak{su}(3)$ states.

The $\mathfrak{so}(6)$ determinant formula

The one-point function in the $\mathfrak{so}(6)$ sector is given by

$$\boxed{C_k\left(\{u_j; v_j; w_j\}\right) = \mathbb{T}_{k-1}(0) \cdot \sqrt{\frac{Q_1(0)\,Q_1(i/2)\,Q_1(ik/2)\,Q_1(ik/2)}{R_2(0)\,R_2(i/2)\,R_3(0)\,R_3(i/2)} \cdot \frac{\det G^+}{\det G^-}}}$$

where $u_i \equiv u_{1,i}$, $v_j \equiv u_{2,j}$, $w_k \equiv u_{3,k}$ and

$$\mathbb{T}_n(x) = \sum_{a=-n/2}^{n/2} (x + ia)^L \frac{Q_2(x+ia)\,Q_3(x+ia)}{Q_1(x+i(a+1/2))\,Q_1(x+i(a-1/2))}.$$

de Leeuw-Kristjansen-GL, 2018

This formula has also been verified numerically. The $M/2 \times M/2$ matrices $G^{\pm}_{jk}$ and $K^{\pm}_{jk}$ are defined as:

$$G^{\pm}_{ab,jk} = \delta_{ab}\delta_{jk}\left[\frac{Lq_a^2}{u_{a,j}^2 + q_a^2/4} - \sum_{c=1}^{3}\sum_{l=1}^{\lceil N/2 \rceil} K^{+}_{ac,jl}\right] + K^{\pm}_{ab,jk}, \qquad K^{\pm}_{ab,jk} = \mathbb{K}^{-}_{ab,jk} \pm \mathbb{K}^{+}_{ab,jk}$$

$$\mathbb{K}^{\pm}_{ab,jk} \equiv \frac{M_{ab}}{(u_{a,j} \pm u_{b,k})^2 + \frac{1}{4}M_{ab}^2}.$$

The $\mathfrak{so}(6)$ determinant formula

The one-point function in the $\mathfrak{so}(6)$ sector is given by

$$\boxed{C_k(\{u_j; v_j; w_j\}) = \mathbb{T}_{k-1}(0) \cdot \sqrt{\frac{Q_1(0)\, Q_1(i/2)\, Q_1(ik/2)\, Q_1(ik/2)}{R_2(0)\, R_2(i/2)\, R_3(0)\, R_3(i/2)} \cdot \frac{\det G^+}{\det G^-}}}$$

where $u_i \equiv u_{1,i}$, $v_j \equiv u_{2,j}$, $w_k \equiv u_{3,k}$ and

$$\mathbb{T}_n(x) = \sum_{a=-n/2}^{n/2} (x+ia)^L \frac{Q_2(x+ia)\, Q_3(x+ia)}{Q_1(x+i(a+1/2))\, Q_1(x+i(a-1/2))}.$$

de Leeuw-Kristjansen-GL, 2018

More properties of one-point functions in $\mathfrak{so}(6)$:

- One-point functions vanish if M or $L + N_+ + N_-$ is odd.
- Because $Q_3 \cdot |\mathrm{MPS}\rangle = 0$, all 1-point functions vanish unless all the Bethe roots are fully balanced:

$$\{u_1, \ldots, u_{M/2}, -u_1, \ldots, -u_{M/2}, 0\}$$
$$\{v_1, \ldots, v_{N_+/2}, -v_1, \ldots, -v_{N_+/2}, 0\}, \qquad \{w_1, \ldots, w_{N_-/2}, -w_1, \ldots, -w_{N_-/2}, 0\}.$$

Section 3

Outlook & Applications

Outlook

Surface critical phenomena are described by means of dCFTs and BCFTs... the surface critical exponents are related to the conformal dimensions of boundary operators...

Applications

- Boundary conformal bootstrap (Liendo-Rastelli-van Rees, 2012): The insertion of a boundary in the bulk of a CFT can be used to constrain both the dCFT and the original CFT...
- D3-D7 system proposed as a holographic model of graphene (Rey, 2009) and topological insulators (Kristjansen-Semenoff, 2016)....
- Relation to the quench action approach (Piroli-Vernier-Calabrese-Pozsgay, Bertini-Tartaglia-Calabrese, 2018)...
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Ευχαριστώ!

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