Anisotropic RG Flows and Strongly Coupled Systems

Dimitrios Giataganas

University of Athens, Greece

Based on works with:
C.S. Chu(NTHU,NCTS), J-P. Derendinger(Bern), U. Gursoy(Utrecht), J. Pedraza(Amsterdam), H. Soltanpanahi(S.China Normal).

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Outline

Motivation I.

Since the discovery of the initial correspondence, there is an extensive research towards to more realistic gauge/gravity dualities (confinement, no susy, temperature, quarks, phase transitions...).

✓ This talk: Theories with Broken Rotational Symmetry in Gauge/Gravity correspondence.

Why?
The existence of strongly coupled anisotropic systems.

- The expansion of the Quark-Gluon plasma at the earliest times after the collision, momentum anisotropic plasmas.
- Strong Magnetic Fields in strongly coupled theories.
- New interesting phenomena in presence on such fiels, i.e. inverse magnetic catalysis.

  eg: (Bali, Bruckmann, Endrodi, Fodor, Katz, Krieg et al. 2011)
- Anisotropic low dimensional materials in condensed matter.

Why? More:

- Weakly coupled vs strongly coupled anisotropic theories.
  (Dumitru, Strickland, Romatschke, Baier,...)
- Properties of top-down supergravity Black hole solutions that are AdS in UV **flowing** to Lifshitz-like in IR :
  $\star$ Fixed scaling parameter z for such anisotropic solutions or even isotropic flows?
  (Azeyanagi, Li, Takayanagi, 2009; Mateos, Trancanelli, 2011;...)
  $\star$ New flows to alternative IR fixed points?

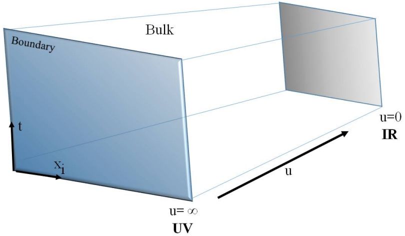
- New Features! Several Universality Relations for the isotropic theories are violated in aniso!
  Shear viscosity η over entropy density s: takes parametrically low values wrt degree of anisotropy $\frac{\eta}{s} < \frac{1}{4\pi}$.
  (Rebhan, Steineder 2011; D.G. 2012; Jain, Samanta, Trivedy 2015; D.G., Gursoy, Pedraza, 2017)

Reminding Slide:

- The anisotropic hyperscaling violation metric

$$ds^2 = u^{-\frac{2\theta}{d}}\left(-u^{2z}\left(dt^2 + dy_i^2\right) + u^2 dx_i^2 + \frac{du^2}{u^2}\right)$$

exhibits a critical exponent z and a hyperscaling violation exponent θ.



- $\theta = 0, \;\; z = 1 \Rightarrow$ AdS.
- $\theta = 0 \Rightarrow$ scale invariant theory.
- In general no scale invariance.

$$t \to \lambda^z t, \qquad y \to \lambda^z y, \qquad x \to \lambda x, \qquad u \to \frac{u}{\lambda}, \qquad ds \to \lambda^{\frac{\theta}{d}} ds \; .$$

How is Anisotropy introduced? A Pictorial Representation:

- For the Lifshitz-like IIB Supergravity solutions

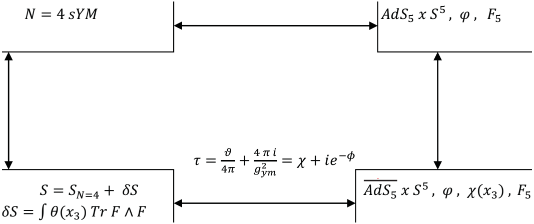
$$ds^2 = u^{2z}(dx_0^2 + dx_i^2) + u^2 dx_3^2 + \frac{du^2}{u^2} + ds_{S^5}^2 .$$

Introduction of additional branes: *(Azeyanagi, Li, Takayanagi, 2009)*

Spacetime

		x_0	x_1	x_2	x_3	u	S^5
AdS →	**D3**	x	x	x	x		
	D7	x	x	x			x

- Which equivalently leads to the following AdS/CFT deformation.



- $dC_8 \sim \star d\chi$ with the non-zero component $C_{x_0 x_1 x_2 S^5}$.

A Theory with Phase Transitions in One Page:

- How the Field Theory looks like?
  - ✓ 4d $SU(N)$ Strongly coupled anisotropic gauge theory.
  - ✓ Its dynamics are affected by a scalar operator $\mathcal{O}_\Delta$.
  - ✓ Anisotropy is introduced by another operator $\tilde{\mathcal{O}} \sim \theta(x_3)\, TrF \wedge F$ with a space dependent coupling.

- The gravity dual theory is an Einstein-Axion-Dilaton theory in 5 dimensions with a non-trivial potential.
  - ✓ A "backreacting" scalar field depending on spatial directions, the axion; and a non-trivial dilaton.
  - ✓ Solutions are non-trivial RG flows: Conformal fixed point in the UV $\Rightarrow$ Anisotropic (Hyperscaling Lifshitz-like) in IR.

- The vacuum state confines color and there exists a phase transition at finite T_c above which a deconfined plasma state arises.

(D.G., Gursoy, Pedraza, 2017)

An Anisotropic Theory

The generalized Einstein-Axion-Dilaton action with a potential for the dilaton and an arbitrary coupling between the axion and the dilaton:

$$S = \frac{1}{2\kappa^2}\int d^5x\,\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{2}Z(\phi)(\partial\chi)^2\right].$$

Where

$$Z(\phi) = e^{2\gamma\phi}\ , \qquad V(\phi) \sim -12\cosh(\sigma\phi) - \left(\frac{m(\Delta)^2}{2} - 6\sigma^2\right)\phi^2\ .$$

((Gubser, Nellore), Pufu, Rocha 2008a,b;Gursoy, Kiritsis, Nitti, 2007;...)

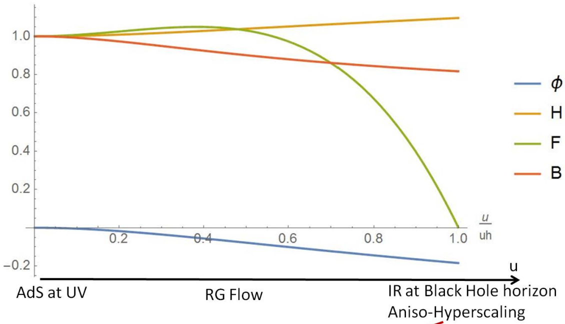
Remark: For $\sigma = 0, \gamma = 1, m(\Delta) = 0$ the action and the solution of eoms, are reduced of IIB supergravity.

A Solution : The RG Flow

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}(u)\mathcal{B}(u)\,dt^2 + dx_1^2 + dx_2^2 + \mathcal{H}(u)dx_3^2 + \frac{du^2}{\mathcal{F}(u)}\right),$$

$$\chi = \alpha x_3\,, \qquad \phi = \phi(u)\,, \qquad \mathcal{F}(u_h) = 0\,.$$

$$\Downarrow \qquad \Downarrow$$



$$ds^2 = u^{-\frac{2\theta}{3}}\left(-u^{2z}\left(f(u)dt^2 + dx_{1,2}^2\right) + \tilde{\alpha}\,u^2 dx_3^2 + \frac{du^2}{f(u)u^2}\right)$$

We have obtained the theories, are they physical and stable?

$$\Downarrow$$
$$\Downarrow$$

✓ Energy Conditions Analysis: $T_{\mu\nu} N^{\mu} N^{\nu} \geq 0 \ , \quad N^{\mu} N_{\mu} = 0 \ .$

AND

✓ Local Thermodynamical Stability Analysis

$$\Downarrow$$
$$\Downarrow$$

YES!

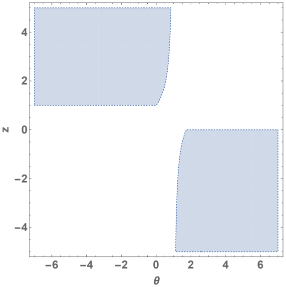
Three conditions that constrain (z, θ) and as a result (γ, σ).

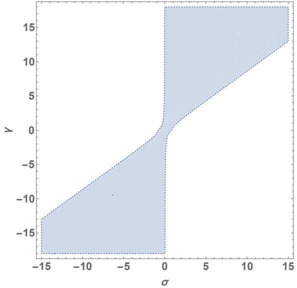
$$(z-1)(1-\theta+3z) \geq 0 \ , \qquad z = \frac{2+4\gamma^2-3\sigma^2}{2\gamma(2\gamma-3\sigma)} \ ,$$

$$\theta^2 - 3 + 3z(1-\theta) \geq 0 \ , \qquad \theta = \frac{3\sigma}{2\gamma} \ ,$$

$$1-\theta+2z \geq 0 \ .$$

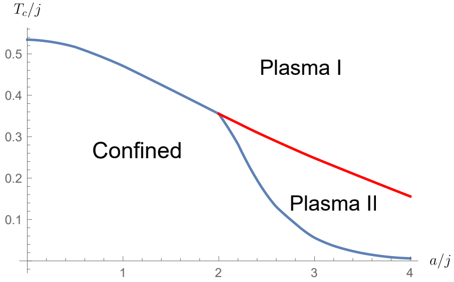
The blue region is the acceptable for the theory parameters.





Phase Transitions

- Competition for dominance between different gravitational backgrounds.
- The Critical Temperature of the theories vs the anisotropy gives:



- The T_c is reduced in presence of anisotropies of the theory.

(D.G., Gursoy, Pedraza, 2017)

A Proposal

- The $Tc(\alpha)$ decrease with anisotropy α.
- No charged fermionic degrees of freedom in our case; our plasma is neutral.
- Anisotropy causes lower T_c= "Inverse Anisotropic Catalysis".

Universal Results: η/s in Theories with Broken Symmetry

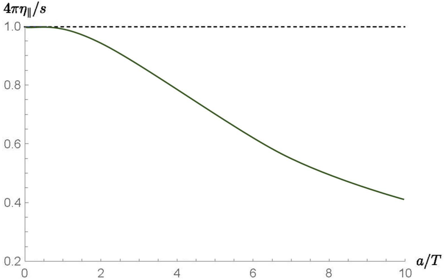
Consider a finite T theory in the deconfined phase:

$$ds^2 = g_{tt}(u)dt^2 + g_{11}(u)(dx_1^2 + dx_2^2) + g_{33}(u)dx_3^2 + g_{uu}(u)du^2$$

- The anisotropic shear viscosity violates the isotropic “bound” of $1/4\pi$:

$$\eta_{ij,kl} = \lim_{\omega\to 0} \frac{1}{\omega} \mathrm{Im} \int dt dx e^{i\omega t} \theta(t) \langle [T_{ij}(t,x), T_{kl}(0,0)] \rangle$$

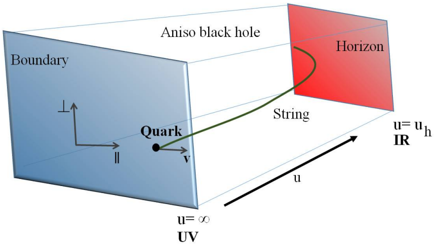
"Frictionless" Anisotropic Plasma.

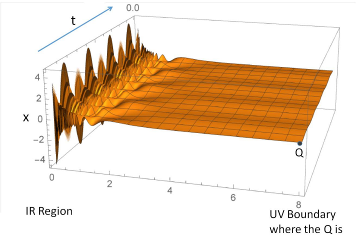


- The Ratio:

$$4\pi \frac{\eta_{\|}}{s} = \frac{g_{11}}{g_{33}}\Big|_{u=u_h} \sim \left(\frac{T}{\alpha}\right)^p , \qquad p = 2 - \frac{2}{z} \sim [0,\infty) , \quad \alpha \gg T .$$

Langevin Dynamics and Brownian Motion





$$\frac{\kappa_{\parallel}}{\kappa_{\perp}} = \frac{\left(g_{00} g_{\parallel\parallel}\right)'}{g_{\perp\perp} g_{\parallel\parallel} \left(\frac{g_{00}}{g_{\parallel\parallel}}\right)'}\Bigg|_{u=u_{wh}}, \qquad \left\langle p^2_{\parallel,\perp} \right\rangle \sim \kappa_{\parallel,\perp} \mathcal{T}$$

A Universal Inequality for Isotropic Theory:

$\kappa_{\parallel} \geq \kappa_{\perp}$ for any isotropic strongly coupled plasma!

Can be inverted in the anisotropic theories: $\kappa_{\parallel} \geq < \kappa_{\perp}$.

(Gursoy, Kiritsis, Mazzanti, Nitti, 2010; D.G, Soltanpanahi, 2013a,b; D.G. 2018)

Anisotropic candidate of c-function

- A proposed c-function is
  (Chu, Giataganas, 2019;(2d) Casini, Huerta 2006; (iso 2d+) Ryu, Takayanagi 2006; Myers, Singh 2012; (nrcft) Cremonini, Dong 2014)

$$c_x := \beta_x \frac{l_x^{d_x - 1}}{H_x^{d_1 - 1} H_y^{d_2}} \frac{\partial S_x}{\partial \ln l_x} \,, \qquad d_x := d_1 + d_2 \frac{n_2}{n_1}$$

  where H is the infrared regulator, d_1 (x_i), d_2 (y_i) are the spatial dimensions and n_1, n_2 are defined at the fixed point:

$$[t] = L^{n_t}, \quad [x_i] = L^{n_1}, \quad [y_j] = L^{n_2} \ .$$

- A relativistic "c-theorem" is guaranteed as long as the NEC: $T_r^r - T_0^0 \geq 0$ is satisfied:

$$\frac{dc}{dr} \propto \int_0^l dx A'^{-2} \left(T_r^r - T_0^0 \right) \geq 0 \ .$$

- How about the Anisotropic Theories?

- Not a one-to-one correspondence between NEC and c-function monotonicity, but not surprising!
- The NEC can be written as $f_i'(r) > 0$, where $f_i(r)$ are functions of metric elements.
- Observation: For an anisotropic theory with a conformal UV fixed point and $d_1 = d_2$ the metric boundary condition

$$f_{i\ UV,\ r=\infty} \leq 0 \ ,$$

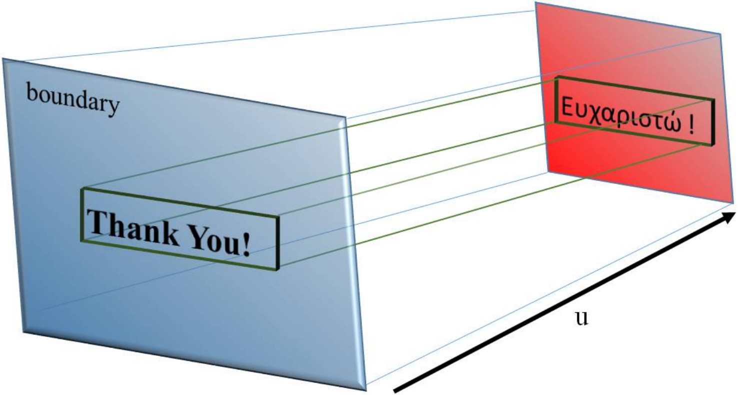
guarantees the right monotonicity for the c-functions along the RG flow

$$\frac{dc}{dr} \propto -\int f_i(r) \ .$$

Conclusions

- ✓ Observation: In strongly coupled theories many phenomena are more sensitive to the presence of the anisotropy than the source that triggers it.
- ✓ Strongly Coupled Confining Anisotropic theories with confinement /deconfinement phase transition.
- ✓ The phase transitions occur at lower critical Temperature as the anisotropy is increased = Inverse Anisotropic Catalysis!
- ✓ Several Universal Isotropic relations are anisotropically violated.
- ✓ Holographic monotonic functions and conditions of monotonicity for (anisotropic) RG flows.
- Are there any other observables that form functions, such that to have monotonic behavior along the RG flow?

(Chu, Derendinger, DG in progress)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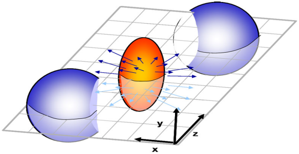
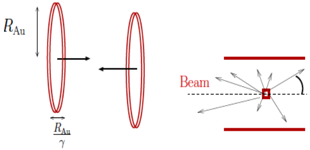
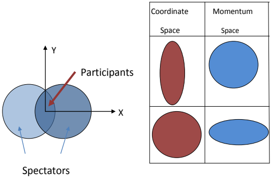
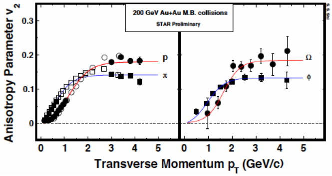
An Anisotropy of the Quark-Gluon Plasma





Pressure gradients for non-central collisions along the short axis of the elliptic flow are higher than the long axis. The expansion along the short axis is more rapid leading to anisotropic momentum distribution.





The elliptic flow parameter

$$v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

can be measured experimentally through the particle distributions.

A New Analytic Black-Hole Solution

The potential and the axion-dilaton coupling

$$V(\phi) = 6e^{\sigma\phi}, \qquad Z(\phi) = e^{2\gamma\phi}.$$

A Lifshitz-like anisotropic hyperscaling violation background which may accommodate a black hole

(D.G., 2018)

$$ds_s^2 = \alpha^2 C_R e^{\frac{\phi(u)}{2}} u^{-\frac{2\theta}{3z}} \left(-u^2 \left(f(u) dt^2 + dx_i^2 \right) + C_Z u^{\frac{2}{z}} dx_3^2 + \frac{du^2}{f(u)\alpha^2 u^2} \right) ,$$

where

$$f(u) = 1 - \left(\frac{u_h}{u} \right)^{3+(1-\theta)/z} , \qquad e^{\frac{\phi(u)}{2}} = u^{\frac{\sqrt{\theta^2+3z(1-\theta)-3}}{\sqrt{6}z}} ,$$

$$C_R = \frac{(3z-\theta)(1+3z-\theta)}{6z^2} , \quad C_Z = \frac{z^2}{2(z-1)1+3z-\theta} ,$$

$$z = \frac{4\gamma^2 - 3\sigma^2 + 2}{2\gamma(2\gamma - 3\sigma)} , \qquad \theta = \frac{3\sigma}{2\gamma} .$$

Axion-Dilaton Coupling and Potential, rule the Scaling Coefficients

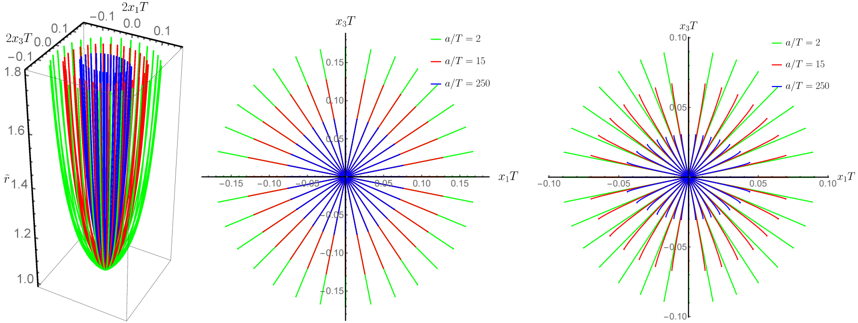
- The values of (θ, z) dependence on (γ, σ)

$$z = \frac{4\gamma^2 - 3\sigma^2 + 2}{2\gamma(2\gamma - 3\sigma)} \ , \qquad \theta = \frac{3\sigma}{2\gamma} \ .$$

- Special case: $(\sigma = 0, \ \gamma = 1)$ supergravity truncated action with a single solution $(\theta = 0, \ z = 3/2)$.
- The scaling factors z and θ are determined by the constants in the Axion-Dilaton Coupling and the Potential. This is the reason that in the particular setup the supergravity solutions have them fixed.

Baryons in Theories with External Fields

- The quark distribution for baryons in theories with anisotropic dynamics:



- Baryon on the transverse plane and Baryon on the plane that the field lies. (D.G. 2018)
- System of fundamental F1 strings with a vertex Dp-brane, in an anisotropic gravity theory.
- Similar effect on Q-distribution, for speeding baryons in strong coupled isotropic plasma. (Athanasiou, Liu, Rajagopal 2008)

Local Thermodynamic Stability

- The necessary and sufficient conditions for local thermodynamical stability in the canonical ensemble are

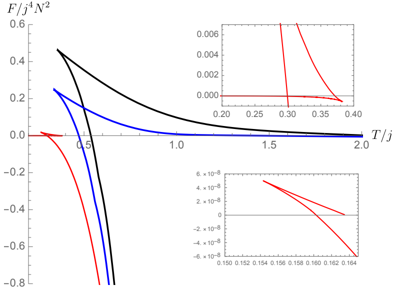
$$c_\alpha = T\left(\frac{\partial S}{\partial T}\right)_\alpha \geq 0 \, , \qquad \Phi' = \left(\frac{\partial \Phi}{\partial \alpha}\right)_T \geq 0$$

  c_α is the specific heat: increase of the temperature leads to increase of the entropy.
  Φ' is derivative of the potential: the system is stable under infinitesimal charge fluctuations.
- In the GCE these conditions should be equivalent of having no positive eigenvalues of the Hessian matrix of the entropy with respect to the thermodynamic variables. *(Gubser, Mitra 2001)*
- In the IR the positivity of the specific heat imposes

$$c_\alpha = 1 - \theta + 2z \geq 0$$

Confinement/Deconfinement Phase transitions

- Competition for dominance between different gravitational backgrounds.
- The free energy of the theories vs the temperature T for different anisotropy (α/j=0,1,3):



- Horizontal Axis: Confining Phase.
- Upper Branch: Black hole A:Deconfining Plasma Phase.
- Lower Branch: Black hole B:Deconfining Plasma Phase.
- $\alpha/j \simeq 2$: A critical value above which a richer structure in the phase diagram exist.

η/s for our theories

- Shear Viscosity over Entropy Density

$$\eta_{ij,kl} = -\lim_{\omega\to 0} \frac{1}{\omega} \mathrm{Im} \int dt dx e^{i\omega t} \langle T_{ij}(t,x), T_{kl}(0,0)\rangle$$

$$s = \frac{2\pi}{\kappa^2} A \ .$$

The two-point function is obtained by calculating the response to turning on suitable metric perturbations in the bulk.

- The relevant part of the perturbed action is mapped to a Maxwell system with a mass term.

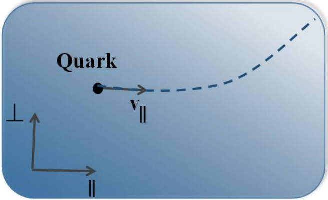
$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(-\frac{1}{4g_{eff}^2} F^2 - \frac{1}{4} m^2(u) A^2 \right) \ ,$$

where

$$m^2(u) = Z(\phi + \frac{1}{4} \log g_{33})\alpha^2 \ , \quad \frac{1}{g_{eff}^2} = g_{33}^{3/2}(u) \ , \quad A_\mu = \frac{\delta g_{\mu 3}}{g_{33}}$$

Langevin Dynamics and Brownian Motion

Langevin coefficients κ: Consider a heavy quark ($M \gg T$) moving along the "$\|$" direction in a strongly coupled plasma.



The Macroscopic Langevin equation:

$$\dot{p}_i(t) = -\eta_D p_i(t) + \xi_i(t) \ ,$$

p: the momentum of the particle, η_D: the friction coefficient, ξ: the random force.

$$\left\langle \xi_{\|,\perp}(t) \right\rangle = 0 \ , \qquad \left\langle \xi_{\|,\perp}(t) \xi_{\|,\perp}(t') \right\rangle = \kappa_{\|,\perp} \delta(t-t') \ , \qquad \left\langle p^2_{\|,\perp} \right\rangle = 2\kappa_{\|,\perp} \mathcal{T}$$

Parts of the Theory Timeline-Related bibliography:

Non-Confining Anisotropic Theories:

(Azeyanagi, Li, Takayanagi, 2009; Mateos, Trancanelli, 2011; Jain, Kundu, Sen, Sinha, Trivedi, 2015;...)

Confining Anisotropic Theories:

(D.G., Gursoy, Pedraza, 2017)

Similar ideas in different context. For example:

(Gaiotto, Witten 2008; Chu, Ho, 2006; Choi, Fernadez, Sugimoto 2017;...)

- A proposed c-function is (Chu, Giataganas, 2019)

$$c_x := \beta_x \frac{l_x^{d_x-1}}{H_x^{d_1-1} H_y^{d_2}} \frac{\partial S_x}{\partial \ln l_x} \,, \qquad d_x := d_1 + d_2 \frac{n_2}{n_1}$$

with the dimensions n_1, n_2 are defined at the fixed point

$$[t] = L^{n_t}, \quad [x_i] = L^{n_1}, \quad [y_j] = L^{n_2} \,.$$

- For the holographic dual consider the background form:

$$ds_{d+2}^2 = -e^{2B(r)} dt^2 + dr^2 + e^{2A_1(r)} dx_i^2 + e^{2A_2(r)} dy_i^2 \,.$$

where $d_1 + d_2 = d$ the space dimensions.

- Then the effective dimensions are

$$d_x := d_1 + d_2 \alpha \,, \qquad \alpha := \lim_{r \to \infty} \frac{A_2(r)}{A_1(r)} \,.$$

- The derivative of the c-function along the RG flow is

$$\frac{4G_N^{(d+2)}}{\beta_x}\frac{\partial c_x}{\partial_{r_m}} = e^{k_m} l_x^{d_x-1} d_x \left[k'_m \int_0^{l_x} dx \frac{1}{k'(r)} \left(\frac{k'(r)}{d_x} - A'_1(r) - \frac{k''(r)}{k'(r)} \right) \right] ,$$

where $k(r) := d_1 A_1(r) + d_2 A_2(r)$.

- The null energy conditions rewritten in terms of monotonic functions are

$$g'_1(r) := \Big((B'(r) - A'_1(r)) e^{B(r)+k(r)} \Big)' \geq 0 ,$$
$$g'_2(r) := \Big((B'(r) - A'_2(r)) e^{B(r)+k(r)} \Big)' \geq 0 ,$$
$$f'(r) e^{-k(r)/(d_1+d_2)+B(r)} - \frac{d_1 d_2}{d_1+d_2} (A'_1(r) - A'_2(r))^2 \geq 0 ,$$

where

$$f(r) := -k'(r) e^{k(r)/(d_1+d_2)-B(r)} .$$

- Not a one-to-one correspondence between NEC and c-function monotonicity, but not surprising!

- The sufficient conditions of c-function monotonicity are

$$f(r) \leq 0 \ ,$$

$$f'(r)e^{B(r)-\frac{k(r)}{d}} + \frac{f(r)e^{-\frac{(d+1)k(r)}{d}}}{d_1+d_2}\left((d_1-d_2)g_1(r)+2d_2g_2(r)\right) + \frac{k'(r)^2d_2(1-\alpha)}{(d_1+d_2)(d_1+d_2\alpha)} \geq 0,$$

$$f'(r)e^{B(r)-\frac{k(r)}{d}} + \frac{f(r)e^{-\frac{(d+1)k(r)}{d}}}{d_1+d_2}\left((d_2-d_1)g_2(r)+2d_1g_1(r)\right) - \frac{k'(r)^2d_1(1-\alpha)}{(d_1+d_2)(d_1+d_2\alpha)} \geq 0,$$

where $f(r)$ and $g_{1,2}(r)$ are the monotonically increasing functions.

- It is possible to impose boundary data for a generic background to guarantee monotonic c-function for the whole flow!
- For example, for a theory with $d_1 = d_2$, the boundary condition

$$g_{i\ UV} \leq 0 \quad \text{and} \quad \alpha = 1 \ ,$$

guarantees the right monotonicity the c-functions along the RG flow.

- A simple example: A theory with Lifshitz-like anisotropic symmetry:

$$ds^2 = r^{2z}\left(-dt^2 + d\vec{x}^2\right) + r^2 d\vec{y}^2 + \frac{dr^2}{r^2}\ , \rightarrow$$

$$ds^2 = e^{2zr}\left(-dt^2 + d\vec{x}^2\right) + e^{2r} d\vec{y}^2 + dr^2\ ,$$

$$t \rightarrow \lambda^z t, \qquad x \rightarrow \lambda^z x, \qquad y \rightarrow \lambda x, \qquad r \rightarrow \frac{r}{\lambda}\ .$$

z measures the degree of Lorentz symmetry violation and anisotropy.

- The entanglement entropy is

$$S_{EEx} = N^2 H_x^{d_1-1} H_y^{d_2}\left[\frac{\beta_1}{\epsilon^{d_1-1+\frac{d_2}{z}}} - \frac{\beta_2}{l_x^{d_1-1+\frac{d_2}{z}}}\right]\ ,$$

$$S_{EEy} = N^2 H_x^{d_1} H_y^{d_2-1}\left[\frac{\tilde{\beta}_1}{\epsilon^{d_1+\frac{d_2-1}{z}}} - \frac{\tilde{\beta}_2}{l_y^{d_1 z+(d_2-1)}}\right]\ .$$

- $d_x = d_1 + d_2/z$ and $d_y = d_1 z + d_2$ appear in the entanglement entropy formula.
- Sufficient conditions $z = 1$; Necessary conditions $z \geq 1$.